

**INDUCTION – REVIEW SET 2**  
**CSC 335**

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Again, I strongly recommend David Liben-Nowell's excellent text, Discrete Mathematics for Computer Science, published by Wiley. These problems are from his Chapter 5.

- (1) In chess, a knight at position  $\langle r, c \rangle$  can move in an L-shaped pattern to any of eight positions: moving over one row and up/down two columns, or two rows over and one column up/down. A *knight's walk* is a sequence of legal moves, starting from a square of your choice, that visits *every* square of the board. Prove by induction that there exists a knight's walk for any  $n$ -by- $n$  chessboard, for any  $n \geq 4$ . Note that the knight (horse) may visit squares more than once.
- (2) A string of balanced parentheses  $[ , ]$  is one of the following:
  - (a) the empty string;
  - (b) a string  $[S]$  where  $S$  is a string of balanced parentheses; or
  - (c) a string  $S_1S_2$  where  $S_1$  and  $S_2$  are both balanced strings of parentheses.

Prove by structural induction

- (a) Every string of balanced parentheses has exactly the same number of open parentheses as close parentheses;
- (b) Every prefix of a string of balanced parentheses has at least as many open parentheses as it does close parentheses