AN ELECTROMAGNETIC THEORY OF GRAVITATION.

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SYNOPSIS.

An Electromagnetic Theory of Gravitation.—An electric system in a medium whose specific inductive capacity k varies from point to point tends to move in the direction of increasing k. It is suggested that if we *assume the specific inductive capacity of the ether to vary near matter*, gravitation may be explained as a result of this tendency. In a medium in which at a distance r from a mass m, $k = 1 + m/r$, it is shown that a rigid electrostatic system would be acted on by a force directed toward m and equal to mm'/r^2, where m' is the electromagnetic mass of the system. But in order to *explain the observed deflection of light by the sun* we must have $k = 1 + 2m/r$; and this will not give the force mm'/r^2 unless the system contracts in the ratio of $1 : 1 - m/r$. A physical explanation of this assumed contraction is suggested. If the system with the mass m' is also supposed to modify k, it is necessary to take into account the energy changes in m and in the ether. The *effect of gravitation on the frequency of the light emitted by an atom*, which was predicted by Einstein, can be easily deduced from the present theory.

THE recent discovery of the deflection of light by the gravitational field of the sun shows for the first time a connection between electromagnetic phenomena and gravitation. This deflection shows that the velocity of light and therefore the specific inductive capacity and magnetic permeability of the ether vary with the gravitational potential.

In an electrostatic field an insulator tends to move to regions where the electric field is stronger and in a medium of varying specific inductive capacity an electrostatic system will tend to move towards places where the specific inductive capacity is greater. The theory proposed in this paper is that matter which is believed to be composed of electrical charges tends to move through the ether in the direction in which the specific inductive capacity and permeability of the ether increase most rapidly and that this tendency is the cause of gravitation.

Consider the case of a small rigid system of massless electrically charged bodies immersed in a medium of specific inductive capacity K. If the charges remain constant the electrostatic energy (E) of the system varies inversely as K. Hence $E = E_1/K$ where E_1 denotes the value of E when $K = 1$. Hence if K is not constant throughout the medium but varies we have

$$F_s = -\frac{\partial E}{\partial s} = \frac{E_1}{K^2}\frac{\partial K}{\partial s} = E\frac{\partial}{\partial s}(\log K),$$

where F_s denotes the force on the system, tending to move it in the direction s due to the variation of the electrostatic energy E inside the system.

Suppose that in the space around a body of large mass m,

$$K = 1 + Gm/rc^2$$

where c denotes the velocity of light in the ether in the absence of a gravitational field, r the distance from the mass m, and G is the constant of gravitation. Then except very close to the mass m, Gm/rc^2 will be extremely small compared to unity[1] so that $\log K = Gm/rc^2$ and

$$F_r = -GEm/r^2c^2.$$

The small mass (m') of the electrical system if due entirely to its electrical energy is given by $m' = E/c^2$ so that

$$F_r = -Gmm'/r^2,$$

which expresses Newton's law of gravitation for the electrical system and the mass m.

Thus if we regard matter as an electrostatic system, having purely electromagnetic mass, the observed gravitational attraction can be explained by supposing that $K = 1 + Gm/rc^2$. If the matter also contains magnetic energy then if we suppose that the magnetic permeability of the ether in the space around a body of mass m is given by

$$\mu = c^2(1 + Gm/rc^2)$$

we get the same force per unit mass on the magnetic energy as on the electrostatic energy so that $F_r = -Gmm'/r^2$ where m' now denotes the total mass due to the electric and magnetic energies.

Taking $c = 1$ and $G = 1$ we have[2] for the refractive index (v) of the ether $v = \sqrt{\mu K} = 1 + m/r$. This value of v gives only half the observed deflection of light by the sun. The deflection found requires that $v = 1 + 2m/r$.[3]

In the above simple theory it is assumed that the size of the electrostatic system remains unchanged when it moves into a region where the specific inductive capacity has a different value; but since the forces between the parts of the system depend on K we should expect the size of the system to vary with K.

The electromagnetic forces alone are not sufficient to determine the

[1] The greatest value of Gm/rc^2 in the solar system is about 2×10^{-6} at the surface of the sun.

[2] Taking $c = 1$ is equivalent to adopting 3×10^{10} cms. as the unit of length if the second is taking as the unit of time. Taking $G = 1$ is equivalent to adopting for the unit of mass, a mass which gives unit gravitational acceleration at unit distance from it. This unit is about 4×10^{38} grams when the unit of time is one second and the unit of length 3×10^{10} cms.

[3] Report on The Relativity Theory of Gravitation, Eddington, page 54.

size of a system composed of positive and negative electrons so that it is easy to see that the size must be determined by the internal forces which hold the parts of the electrons together. If l is a quantity proportional to the linear dimensions of the system and equal to unity when $K = 1$, then the electrostatic energy is inversely proportional to Kl so that $E = E_1/Kl$.

For the energy of an electrostatic system is equal to $\frac{1}{2}\Sigma EV$ where V denotes the potential of a charge E. Also

$$V = \frac{1}{4\pi}\int\frac{\rho dv}{Kr},$$

where ρ denotes the density of electricity in the element of volume dv and r the distance of the element dv from the point at which the potential is V. If the linear dimensions of the whole system change uniformly then ρdv will remain constant so that V varies inversely as Kl and hence the energy also varies inversely as Kl when the charges remain constant. In this case the force on the system, due to the variation of the energy inside it, will be given by

$$F_s = + E\frac{\partial}{\partial s}(\log Kl).$$

Thus to get the correct value of F_r we must have $Kl = K^{1/2}$ or $l = K^{-1/2}$. If $K = 1 + 2m/r$ then l must be equal to $1 - m/r$ so that

$$F_r = + E\frac{\partial}{\partial r}(\log (1 + m/r)) = -\frac{mm'}{r^2}.$$

If $\mu = 1 + 2m/r$ there will be an equal force per unit mass on the magnetic energy also.

The tension inside the Lorentz electron is supposed not to vary when the shape of the electron changes, due to its motion through the ether, but when the specific inductive capacity of the ether changes it may be supposed to change.

The internal tension in the electron may be regarded as a sort of elastic reaction against the electric displacement or polarization at its surface. We should therefore expect the tension to be proportional to the displacement. This gives

$$\frac{1}{Ka^4} \propto \frac{1}{a^2},$$

where a is the radius of the electron, so that $aK^{1/2}$ is constant. It is easy to see that the linear dimensions of an electrostatic system, entirely composed of positive and negative electrons, will be proportional to a so that if $aK^{1/2}$ is constant then $l = K^{-1/2}$ which is the value required to give the observed gravitational attraction. For in an electrostatic system we have $\Delta V = -\rho/K$ and if we put $x = lx'$, $y = ly'$, $z = lz'$, $V = V'/Kl$ and $\rho dxdydy = \rho' dx'dy'dz'$, this becomes $\Delta V' = -\rho'$ which is the equation giving the potential in a system in which $K = 1$. Thus a system having any value of K can be transformed into a corresponding system in which $K = 1$ by changing the linear dimensions by a factor l having any value. We conclude that the actual change of dimensions will be determined by the change in the radii of the electrons.

The internal energy of the electron is inversely proportional to Kl like the external electrostatic energy so that there will be the same force per unit mass on the internal energy as on the electrostatic energy.

It appears therefore that if $K = \mu = 1 + 2m/r$ and the system contracts in the ratio $1 : K^{-1/2}$ or $1 : 1 - m/r$ then $F_r = -mm'/r^2$ where m' now denotes the mass of the total energy of the system including the electrostatic, magnetic and internal energies.

The contraction in the ratio $1 : K^{-1/2}$ agrees with that indicated by Einstein's theory, to the order of approximation used here, for if in the expression for the line element in the four dimensional manifold

$$ds^2 = -\gamma^{-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 + \gamma dt^2,$$

where $\gamma = 1 - 2m/r$, we put $r = r_1 + m$ and neglect squares of m/r_1 it becomes

$$ds^2 = -\gamma^{-1}(dr_1^2 + r_1^2 d\theta^2 + r_1^2 \sin^2\theta d\phi^2) + \gamma dt^2.$$

This indicates a contraction in the ratio $1 : \gamma^{1/2}$ or $1 : 1 - m/r$ in the element of length $\sqrt{dr_1^2 + r_1^2 d\theta^2 + r_1^2 \sin^2\theta d\phi^2}$.

According to the theory outlined above both gravitation and the deflection of light are due to variations in the specific inductive capacity and permeability of the ether. How these variations are produced by the presence of matter in the ether, at distant points, remains to be explained.

The attracting mass m has been supposed to be surrounded by a region in which $K = \mu = 1 + 2m/r$ and the attracted mass m' has been supposed to be an electric system and it has been shown that the electric system will be acted on by a force very approximately equal to the Newtonian gravitational attraction and also it is known that light will be deflected in consequence of the assumed variations in K and μ, in the way observed.

The attracted mass m' however must also be supposed to modify K and μ so that there will be changes produced in the electrical energy in the mass m by the motion of the mass m'. These changes have so far been left out of account.

We have seen that the energy in the mass m' is given by

$$E = E_1/\sqrt{K} = E_1(1 - m/r)$$

and in the same way the energy (F) in the mass m will be given by $F = F_1(1 - m'/r)$.

We have $m_1' = E_1$ and $m_1 = F_1$ approximately so that the total energy in the two masses is approximately equal to

$$m_1'(1 - m/r) + m_1(1 - m'/r)$$

or $m_1' + m_1 - 2mm'/r$ very nearly. The stress tending to diminish r might therefore be expected to be equal to $2mm'/r^2$ instead of mm'/r^2.

We might suppose that $Kl = K^{1/4}$ instead of $Kl = K^{1/2}$ and so diminish the calculated attraction again by one half but I think it is better to adopt a different plan for $Kl = K^{1/2}$ seems a reasonable assumption and it agrees approximately with Einstein's theory.

The modification of the ether represented by the change of K and μ from unity to $1 + 2m/r$ must require some energy for its production so that besides the energy variations inside the attracting masses we ought also to consider the energy variations in the surrounding ether.

Suppose that there is in the ether an amount of energy per unit volume equal to

$$\frac{1}{32\pi}\left\{\left(\frac{\partial K}{\partial x}\right)^2 + \left(\frac{\partial K}{\partial y}\right)^2 + \left(\frac{\partial K}{\partial z}\right)^2\right\}.$$

This expression need not include μ because $K = \mu$ everywhere; it has been chosen so as to give the desired energy variation and is otherwise a pure assumption.

Then the energy outside a mass m of radius a is equal to

$$\int_a^\infty \frac{1}{32\pi}\left(\frac{\partial K}{\partial r}\right)^2 4\pi r^2 dr,$$

so that since $K = 1 + 2m/r$ it is equal to $\frac{1}{2}m^2/a$. This is $\frac{1}{2}m$ multiplied by the gravitational potential m/a.

If we have two masses m and m' with radii a and b the outside energy will be

$$\tfrac{1}{2}m\left(\frac{m}{a} + \frac{m'}{r}\right) + \tfrac{1}{2}m'\left(\frac{m'}{b} + \frac{m}{r}\right) = \frac{m^2}{2a} + \frac{m'^2}{2b} + \frac{mm'}{r}.$$

Thus the total energy inside and outside is equal to

$$m_1 + m_1' + \frac{m^2}{2a} + \frac{m'^2}{2b} - \frac{mm'}{r},$$

so that the stress tending to diminish r is approximately equal to mm'/r^2, as it should be, for the variation of $m^2/2a$ and $m'^2/2b$ is negligible compared with that of mm'/r.

If m is very large compared with m' we may regard m as fixed. In this case when m' moves towards m the electrical energy in m' diminishes by an amount corresponding to the gravitational attraction and also the energy in m diminishes by an equal amount and the energy outside in the ether increases by an equal amount. Thus the loss of energy in the large mass is compensated by an equal gain in the ethereal energy. The total energy of the small mass remains constant and since the ethereal energy is nearly all near the large mass the total energy attracting the small mass also remains almost constant. The effective masses therefore remain constant unless the masses come near each other. This theory makes the energy in the ether positive which is satisfactory. It has always been difficult to believe that the ether in a gravitational field contains less energy per unit volume than the ether at a great distance from matter.

It will be observed that on this theory the gravitational force on either of the two attracting masses is equal to that due to its tendency to move in the direction of increasing specific inductive capacity due to the energy variation inside itself. Thus the force on each body can be attributed to a change of potential energy inside the body itself when it moves and so to an action between the body and the ether inside it.

The change in the frequency of the light emitted by an atom, due to gravitation, predicted by Einstein can be easily deduced from the present theory. We have seen that the electrical energy of any electrical system is equal to $E_1K^{-1/2}$ where E_1 denotes its energy when $K = 1$. According to Bohr's theory of the emission of light the frequency is determined by the energy (ϵ) in the quantum emitted by means of the equation $\epsilon = hn$ where n is the frequency and h is Planck's constant. If the emitting atom is in ether of specific inductive capacity K instead of ether for which $K = 1$ its energy in each of its possible stable states will be diminished by the factor $K^{-1/2}$ so that the energy of any quantum emitted will also be diminished in the same ratio. The frequency will therefore also be diminished by the factor $K^{-1/2}$ or $1 - m/r$ as predicted by Einstein.

In conclusion it may be said that since matter is certainly partly electrical and since the refractive index of the ether certainly varies near large masses, it seems certain that part at least of the observed gravitational forces must be due to an action of the kind considered in this paper, so that it is satisfactory to find that it is possible to explain the whole attraction by means of this kind of action.

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