PREFACE

The Louis de Broglie Foundation (which was created in 1973, for the fiftieth anniversary of the discovery of wave mechanics) and the University of Perugia, have offered an international symposium to Louis de Broglie on his 90th birthday. This publication represents the Proceedings of this conference which was held in Perugia on April 22-30, 1982.

It was an opportunity for the developing of physical conceptions of all origins, which may serve to throw light on the mysterious power of the quantum theory. Quantum Mechanics has reached maturity in its formalism and although no experiment yet has come to challenge its predictions, one may question the limits of its validity. In fact the true meaning of this vision of the microphysical world remains the subject of endless debating, at the heart of which lies "the foundational myth" of wave-particle dualism. Albert Einstein and Louis de Broglie are the two discoverers of this fundamental duality, which they always considered as a deep physical reality rather than a phenomenological artifice.

During the conference a survey has been given of the essential recent experimental results in corpuscular and quantum optics and the most up-to-date theoretical aspects of the specificity of microphysical phenomena : various interpretations of quantum mechanics, "alternative theories" and hidden parameters theories, probabilistic and axiomatic questions and tentative crucial experiments.

The conference took place in the magnificent atmosphere of the villa Colombella lent to us by the Università per Stranieri di Perugia.

Without the organizational activity of Professor V. Aquilanti and his colleagues, especially Professors G. Liuti and G.G. Volpi, this conference would not have met such a success.

Our thanks are for all the organizations which have in a way or another helped us in conducting this entreprise : Università per Stranieri di Perugia, Consiglio Nazionale delle Ricerche, Regione dell'Umbria, Provincia di Perugia, Università di Perugia, Azienda Autonoma del Soggiorno di Perugia, Comune di Perugia.

DE BROGLIE'S INITIAL CONCEPTION OF DE BROGLIE WAVES

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1. INTRODUCTION

In this paper my intention is to present de Broglie's personal view on "matter waves", with some consequences on the theory of measurement. This conception of "matter waves" was at the very origin of wave Mechanics and was, later on, developed by de Broglie and his collaborators during the last 25 years. There are two papers on this subject in this book. The present one combines my own two lectures given during the Perugia Symposium and the second paper (by Daniel Fargue) is especially devoted to the double solution theory and the fundamental idea of using non-linear equations. In connection with this last problem, let me note that I am very glad to see how the approach of physicists has changed during the last few years : 20 or 25 years ago, when we were the only ones to speak about non linear equations and the possibility of generalizing Quantum Mechanics on the basis of this idea, all the official scientific commissions considered that it was an utterly foolish idea devoid of any interest, which only demonstrated that de Broglie had definitely became an old man. Today the possibility of non linear effects in the diffraction or interferences of matter-waves is largely recognized and this is certainly a very important evolution, which may help to find an intelligible feature of the dualistic properties of matter. Nevertheless, it must be confessed that, just like a quarter of century ago, the main difficulty remains to find one non linear equation, on the basis of a general principle and not to stay with an infinity of possible equations, among which the right one may exist. So, it would be a criminal illusion to ignore the fact that, in spite of some practical progress, the path to a true physical solution to the problem of wave-particle dualism is, most probably, as difficult as it was 25

S. Diner et al. (eds.), The Wave-Particle Dualism, 1–25.

years ago. The correct non linear equation will probably only be found as a result of many years of work ... unless (as sometimes happens in the history of science) some young student who was, of course, not invited to Perugia, is in the process of finding a very simple and absolutely new solution to the problem !

2. DE BROGLIE'S MAIN IDEA

Let us come, now, to the very subject of this lecture. De Broglie's main idea is not dualism, but coexistence between waves and particles. Here lies the difference between Bohr and de Broglie : Bohr believed in a kind of double faced physical being which appears to us, in certain circumstances as a particle and in others as a wave ; on the contrary, de Broglie considered that there is only one thing which is always (at the same time) a particle and a wave and which is such that the properties of the particle which we observe are guided -commanded- by the wave structure of the system. And it was his idea from the beginning of wave mechanics. We shall try to show this in a short survey of de Broglie's famous first three papers which appeared in the Comptes Rendus de l'Académie des Sciences in 1923.

The idea was the following. De Broglie first considered a particle ; to this particle, he supposed that (following his own words : "as a consequence of a great law of nature"), one can associate a frequency defined by the equality between "Einstein's energy" and "Planck's energy" written in the proper system :

$$m_0 c^2 = h\,\nu_0 \tag{1}$$

At this stage it is supposed that the "proper frequency belongs to the particle and not to a wave. But de Broglie remarked that if an observer looks at the particle moving before him with a velocity $\underline{v}$, the mass is transformed according to the well known formula :

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

whereas the frequency ν_0, which must be considered as being internal to the particle, obeys the relativistic law of the slowing down of clocks and becomes :

$$\nu_1 = \nu_0 \sqrt{1 - \frac{v^2}{c^2}}\ . \tag{3}$$

So that the equality (1) no longer works ! It is not relativistically invariant. De Broglie was deeply puzzled by this failure and he only reached the following hypothesis after a long period of reflection : he supposed that in the proper system there is not only an "internal clock", with a frequency ν_0, associated to the particle, but also a stationary wave, with the same frequency and the same phase as the clock and written :

$$e^{i\nu_0 t_0} \tag{4}$$

But now, in another galilean system, with a relative velocity v, the Lorentz transformation of time will give :

$$t_0 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\left(t - \frac{vx}{c^2}\right)$$

and the wave will become :

$$e^{i \frac{\nu_0}{\sqrt{1-v^2/c^2}}\left(t - \frac{vx}{c^2}\right)} \tag{5}$$

Now, we can see that the frequency ν of this wave is not the same as the frequency ν_1 defined in (3) but :

$$\nu = \frac{\nu_0}{\sqrt{1-v^2/c^2}} \quad . \tag{6}$$

The variance of the wave's frequency ν is, thus, the same as the variance (2) of the mass and, using (1), (2) and (6), we may write, invariantly :

$$mc^2 = h\nu \quad , \tag{7}$$

which would be impossible with the frequency ν_1 of the "internal clock" of the particle. Besides, we get through (5) the phase velocity V and de Broglie's well know formula :

$$Vv = c^2 \quad . \tag{8}$$

As we can see, V is greater than c, but we know that the associated group velocity may be shown to be equal to v, i.e. equal to the velocity of the particle[1].

After the discovery of the important properties (6) and (7) of the wave frequency ν, de Broglie could have abandoned his initial idea of an internal clock frequency ν_1. But this was not to be the case because he made a second important remark :

In the frame of a galilean observer who sees the particle moving before him with the velocity v, de Broglie calculated the phase of the internal clock of the particle when the latter is in a point x at the instant t. In virtue of (1) and (3), one gets :

$$\phi_{(\text{clock})} = \nu_1 t = \frac{m_0 c^2}{h}\sqrt{1 - \frac{v^2}{c^2}}\,\frac{x}{v} \tag{9}$$

Now, de Broglie calculated at the same moment t and in the same point x at which the particle lies, the value of the phase of the wave. Using (5) and (6) we will find this value to be (remember that x = vt !) :

$$\phi_{wave} = \nu\left(t - \frac{vx}{c^2}\right) = \frac{m_0 c^2}{h}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(\frac{x}{v} - \frac{vx}{c^2}\right) = \frac{m_0 c^2}{h}\sqrt{1-\frac{v^2}{c^2}}\,\frac{x}{v} \qquad (10)$$

It is the same phase ! In other words, we know that the frequency of the internal clock of the particle is smaller than the frequency of the wave, but at the same time, the velocity of the particle is smaller than the phase velocity of the wave, and the result is a kind of compensation of these two discrepancies, so that we can assert : "For any galilean observer, the phase of the "internal clock" of the particle is, at each instant equal to the value of the phase of the wave calculated at the same point at which the particle lies".

This is de Broglie's law of accordance (or harmony) of phases. It is interesting to underline here that de Broglie considered this law and not the discovery of the famous wave-length to be his fundamental achievement. Ten years ago, during the celebration of his eightieth birthday at the Academy of Sciences, he said to us, recalling Bergson's words : "A man never has more than one great idea in his life" and he added : "If I ever had such an idea, it is certainly the law of phase harmony which was expressed in the first chapter of my Thesis in 1924".

The wave-length became famous after the discovery of the diffraction of material corpuscles and thanks to the proeminent role played by Schrödinger's equation. But in fact, the forgotten law of phase harmony, which is no longer quoted in text books, constitutes the basis of the whole problem of wave-particle dualism and contains, in the opinion of de Broglie, the deep mystery which has to be solved in the first place if one is to understand quantum mechanics. De Broglie never considered that, in stating this law, he had given any explanation of wave particle dualism : he only found an important formula which follows from the laws of relativity. But the question is : what property is hidden behind this formula ? What is this mysterious balance between corpuscle and wave (similar to the balance between a surf-rider and a sea wave), which is expressed in the formula ?

In order to explain de Broglie's mode of reasoning it might be interesting to recall how, thanks to his wave, he reached the quantized levels of Bohr's atom. Everybody knows how easy it is to find this result in an elementary way, by stating a resonance condition on de Broglie's wave-length along a circular Kepler's orbit. But this was not the way chosen by de Broglie, who did not make use of the wave-length ! His reasoning was more complicated because it was based on the phase-harmony but in this way he made use of wave-particle dualism.

He said the following : Consider a circular Bohr orbit. The electron starts from a point O and describes the orbit with a velocity v, while the wave starts at the same moment, but with the much greater phase velocity $V = v/c^2$, so that, at a certain time τ the wave will overtake the particle in a point O'. At the instant τ, the particle will have described an arch OO' and for the wave, we must have the same arch plus an entire orbit, from which we get :

$$V\tau = \frac{c^2}{v}\tau = v(\tau+T) \qquad \tau = \frac{v^2}{c^2-v^2}T, \tag{11}$$

where T is the period of the electron on the orbit. From that, we find the internal phase of the particle (the phase of the internal clock !), at the instant τ of the overtaking of the particle by the wave (we make use of (1), (3) and (11)) :

$$2\pi\nu_1\tau = 2\pi\frac{m_0c^2}{h}\sqrt{1-\frac{v^2}{c^2}} \times \frac{v^2}{c^2-v^2}T \tag{12}$$

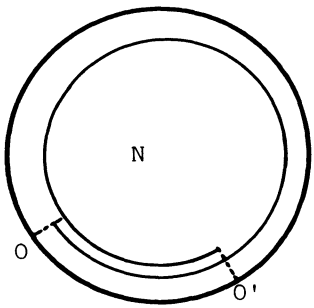

Fig.1 : During the time the electron needs to describe the arch of trajectory OO', it is overtaken by its own wave, the velocity of which is much greater than the velocity of the corpuscle.

Then de Broglie imposed the principle of phase harmony between the internal phase and the phase of the overtaking wave, which is ensured by the condition :

$$2\pi\nu_1\tau = 2n\pi \quad ; \qquad n \in \mathbb{N}. \tag{13}$$

But, from (12) and (13) it follows that :

$$m_0v^2T/\sqrt{1-v^2/c^2} = nh, \tag{14}$$

which is exactly Bohr's condition in a relativistic form.

Introducing the length $L = vT$ of the orbit and the wave-length $\lambda = h/p$, we obtain immediately the formula (which could be set at once as a condition) :

$$L = n\lambda . \tag{15}$$

Of course, de Broglie verified this formula later but it is interesting to note that, even in his thesis, he gave the formula $\lambda = h/p$ one hundred pages further than the Bohr's atom problem (incidentally and at the non-relativistic approximation). For him, the fundamental quantities were not the wave-length but the frequencies of the wave and the clock, and the phase and group velocities (the latter equal to the velocity of the particle) -in other words, the ingredients that enter in the law of phase harmony.

It is important to realize that, in a certain sense, the principal element of de Broglie's reasoning is not the wave, but the clock. And this is why de Broglie always considered that the first fundamental idea of wave mechanics was not in the work of Huygens but in the work of Newton, namely in the famous "theory of fits", in which an internal frequency was already introduced in the particles of light, through the hypothesis of alternacies of fits of reflection and fits of transmission.

To conclude this section, an essential point has to be emphasized : the role played here by the special theory of relativity. We noted that the first problem, for de Broglie, was a discrepancy between the relativistic variance of a clock frequency and a mass. The introduction of the wave itself is motivated by this problem. The principle of phase harmony is of relativistic essence. Let us not forget that the phase velocity, the role of which is absolutely fundamental, is only univocally defined in a relativistic theory for instance, in the Schrödinger equation, this velocity remains undefined.

The importance of the theory of relativity in de Broglie's theory appears in the same manner in his analogy between mechanics and optics.

In de Broglie's thesis there was another way of obtaining a wave formulation of mechanics : it was the analogy between the principle of least action and the principle of Fermat. Although this problem is far better known than the previous one, we must recall it for the very purpose of showing the role of relativity.

Let us write the quadridimensional momentum :

$$J_\alpha = m_0 c\, u_\alpha + e\, A_\alpha \qquad (\alpha=1,2,3,4) \tag{16}$$

where u_α is the quadri-velocity and A_α the electromagnetic potential. The principle of Hamilton takes the form :

$$\delta \int_P^Q J_\alpha\, dx^\alpha = 0 \qquad (\alpha=1,2,3,4), \tag{17}$$

where P and Q are the ends of the universe line of the motion. After that, de Broglie took the case of a conservative system and wrote the principle in the old form of Maupertuis :

$$\delta \int_A^B J_i \, dx^i = 0 \qquad (i=1,2,3) \tag{18}$$

where A and B are, now, the ends of the space trajectory.

Now, de Broglie compared these formulae with the principle of the shortest path in optics. He first wrote the principle in a relativistic way :

$$\delta \int_P^Q d\phi = \delta \int 2\pi O_\alpha dx^\alpha \ (\alpha=1,2,3,4), \tag{19}$$

where P and Q are again the end points of a line of universe (the quadri-opticray) and ϕ the phase of the wave ; O_α is the universe wave-vector. If we consider the case of an index of refraction independant of time, this will be the equivalent of the conservative case in mechanics and we shall get, instead of the "Hamilton form" (19), the "Maupertuis form" :

$$\delta \int_A^B O_i dx^i \qquad (i=1,2,3), \tag{20}$$

which is nothing but the integral of Fermat (of course, A and B are points in $\mathbb{R}^3$).

Now, de Broglie considered the two universe vectors : the quadri-momentum J_α and the quadri-wave vector O_α and he noted that, if we postulate that Planck's formula has a universal meaning, we can find a relation between these vectors. As a matter of fact if they are written more explicitely, we have (dropping here the potential A for the sake of simplicity, but this is not necessary):

$$J_i = m_0 c u_i = \frac{m_0 v_i}{\sqrt{1-v^2/c^2}} = p_i \quad (i=1,2,3) \ ; \ J_4 = \frac{w}{c} \tag{21}$$

(v_i = velocity of the particle ; w = energy)

$$O_i = \frac{\nu}{V} n_i \quad (i=1,2,3) \quad ; \quad O_4 = \frac{\nu}{c} \tag{22}$$

(ν = frequency ; V = phase velocity ; n_i = unitary wave vector). We see immediately (in virtue of Planck' law) that :

$$w = h\nu \Rightarrow J_4 = h \, O_4 . \tag{23}$$

Thus, postulating the relativistic invariance of this relation, de Broglie claimed that we must have :

$$J_\alpha = h \, O_\alpha \qquad (\alpha=1,2,3,4). \tag{24}$$

And so, by comparing (21) and (22), we find de Broglie's formula :

$$p = \frac{h\nu}{V} = \frac{h}{\lambda} \tag{25}$$

and the equivalence of the principles of Fermat and Maupertuis.

This result is generally considered as the true great achievement of de Broglie and the corner stone of wave mechanics. But this is not the opinion of de Broglie himself. He sees a radical difference between his two resonings (the principle of phase harmony and the equivalence of the two principles of minimum) and he prefers the first. He considers, indeed, that the latter is strictly confined to the geometrical optics limit and classical mechanics, whereas the law of phase harmony has a general meaning and covers not only the classical approximation, but the whole of wave mechanics, including all the features of wave propagation.

Is this opinion correct or not ? We are to this day unable to answer this question, which remains open for the future. But we may consider that the scientific testament of Louis de Broglie consists essentially in the hope that, one day, somebody will explain the profound nature of this strange link between waves and particles, which he discovered 60 years ago.

3. DE BROGLIE'S THEORY OF MEASUREMENT

In this part, we shall briefly explain some consequences on the theory of measurement of de Broglie's conceptions on wave-particle dualism. These consequences were developed by de Broglie and his co-workers during the last 30 years.

We shall essentially speak about three questions :

The first will be the special role played in the process of measurement by the localisation of particles, i.e. the preeminence of the observation of the position among all other physical quantities which may be measured in microphysics.

The second problem, strongly linked to the first, will be the definition of three kinds of probabilities in quantum mechanics : the so called present, predicted and hidden probabilities.

Lastly, I shall say a few words about the controversial (and even explosive) problem of Bell's inequality.

First let us rapidly recall some well known points of quantum mechanics concerning the measurement of physical quantities. We know the principles suggested by Max Born at the very beginning of quantum mechanics : a) when a normalized function is given, we obtain directly the probability $|\psi|^2 dv$ of finding the particle in any elementary volume of space.

b) Secondly, if a physical quantity A is represented by an operator A in a Hilbert space, the proper values a_i of A will be the possible values obtained by a measurement of the quantity A. Now if we expand ψ on the normalized proper functions ϕ_i of A :

$$\psi = \sum_i c_i \phi_i , \tag{26}$$

the square modulus c_i gives the probability of finding precisely the corresponding value a_i when A will be measured.

The most important point about the difference between the current interpretation of quantum mechanics and de Broglie's one concerns the theory of transformations.

Classically, it is admitted in quantum mechanics that a certain symmetry, an equality of meaning exists between all the possible representations of a physical system. For example, we can write the q and p representations, with :

$$\delta(q-q_0) \text{ proper function of } Q_{op} = q$$

$$e^{-\frac{i}{\hbar}pq} \quad \text{"} \quad \text{"} \quad \text{"} \quad P_{op} = i\hbar\frac{\partial}{\partial q}$$

and we can express a ψ function, equivalently, in the two representations as :

$$\psi(q) = \int \psi(q_0)\ \delta(q-q_0)dq_0 = \frac{1}{\sqrt{\hbar}}\int c(p)\ e^{-\frac{i}{\hbar}pq}dp \qquad (27)$$

In most text books, starting with the famous one by Dirac, the equivalence of all representations is considered both as an evidence and an elegance of the quantum theory, which is, of course, from a purely mathematical point of view, absolutely correct. But from a physical point of view, this is a fundamental aspect on which de Broglie disagrees with the current interpretation. Thirty years ago, he made this remark : among all the quantities which can actually be measured, there is in fact a strong dissymetry between the position, on one side, and the whole set of all possible variables on the other side. In general, the position is the sole physical quantity you are able to observe without any preparation of the physical system. You just have to register what happens when a wave falls, say on a wall, and using a photographic film, or any other registering device, you will know where, i.e. in what point, of the wave, the particle was localized.

Generally, one is unable to answer directly any other question about any quantity characterizing the system than the question of localization. We may even say more. If you intend to measure another physical quantity and if you thoroughly examine all the devices we are presently able to construct in order to do so, you will see that they are always founded on the same procedure : this consists in the observation of the position of a particle in conditions such that the sole registration of the particle in a certain region of space leads you to a univocal conclusion on the value of the physical quantity you wanted to measure.

The most general measurement process obeys the scheme given on Fig.2.

We see that the process consists of a spatial separation of the wave packets corresponding to the different proper states (and thus, proper values) of A, so that the registration of the particle in one of those packets (say a_k)?. The necessity of a spectral analyser (of such or such a kind, according to the type of experiment), which must be adapted to the physical quantity we want to measure and which cannot be adapted at once to all the quantities, is the true origin of the Heisenberg uncertainties. More generally it also lies at the basis of the mysterious strangeness of "quantum probabilities" which has constituted such a delectable subject for theoreticians during the last fifty years. This brings us to the problem of the distinction between two kinds of probabilities: present and predicted. What does it mean ?

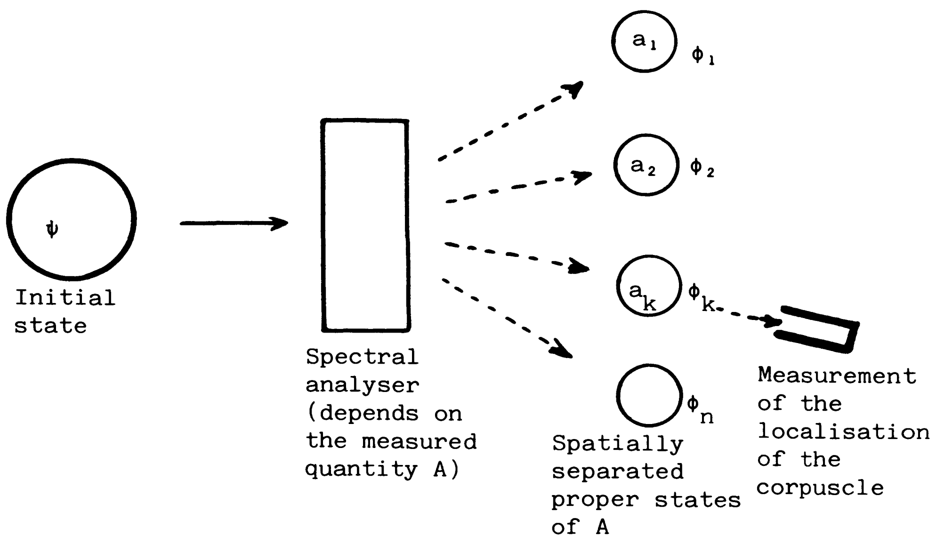

Fig.2 : General scheme of a measurement process.

In de Broglie's terminology, a present probability concerns an event which may be observed without any preparation. This will be the case for all "classical" probabilities, such as those which appear in the problem of games (dice game, heads or tails etc.). And it will also be the case for the probabilities concerning the measurement of the position in microphysics and, more generally, of any other physical quantity which may be measured by a simple inspection of realized events, i.e. for all the quantities for the measurement of which the preparation is already performed. In other words, the probabilities will be present for the values of a physical quantity after the system has passed through the spectral analyser.

But, on the contrary, it is not the case, in general, for most physical quantities, precisely because you need to prepare the system before the measurement, owing to an analyser which modifies the state of the system in such a way (as we have seen) that the measurement will be reduced to a localisation of the particle. This brings about a new type of probabilities : they are no longer present, but only <u>predicted</u>. Let us explain this important point more accurately. Let us suppose that we have two series of events, respectively numbered i and k. We shall write : P_i and P_k for the probabilities of i and k, $P_i^{(k)}$ and $P_k^{(i)}$ for the conditional probabilities of actualization of an event i (resp. k) when k (resp. i) is known, and P_{ik} for the joint-probability of realization of i <u>and</u> k. We know that the <u>classical scheme</u> of probability calculus is based on the formulae :

$$P_{ik} = P_i \, P_k^{(i)} = P_k \, P_i^{(k)} \tag{28_1}$$

$$P_i = \sum_k P_{ik} \quad ; \quad P_k = \sum_i P_{ik} \tag{28_2}$$

$$P_i = \sum_k P_k \, P_i^{(k)} \quad ; \quad P_k = \sum_i P_i \, P_k^{(i)} \tag{28_3}$$

(The third one is, of course, implied by the first two and let us note, also, that we shall confine the major part of our analysis to the case of <u>discrete</u> systems, which does not make our remarks less general).

The essential point is the existence of the joint probability P_{ik} which supposes the possibility of a <u>simultaneous</u> actualization of events i <u>and</u> k, without any interaction between the two of them ; the hypothesis stands thus : the fact that i (resp. k) is observed does not constitute an obstacle for the observation of k (resp. i), in the same conditions and with the same result as would have been obtained if i (resp. k) had not been observed.

Now, let us consider the quantum case of the observation, on a system in a state ψ, of two quantities represented by operators A and B, with respective eigen states ϕ_i and χ_k, and eigen values α_i and β_k.

Each vector ϕ or χ may be transformed into the other by a unitary transformation d :

$$\phi_i = \sum_k d_{ik} \, \chi_k \quad ; \quad \chi_k = \sum_i d^*_{ki} \, \phi_i \ , \tag{29}$$

and we may expand ψ in two ways :

$$\psi = \sum_i c_i \, \phi_i = \sum_{ik} c_i \, d_{ik} \, \chi_k . \tag{30}$$

We know that the probability of finding the system in a state ϕ_i, with the eigen value α_i for the quantity A, is :

$$P_i = |c_i|^2 \tag{31}$$

In the same way, the probability of finding the system in the state χ_k and the eigen value β_k for the quantity B, may be written, using (30)

$$P_k = |\sum_i c_i \, d_{ik}|^2 \tag{32}$$

Now we can infer from (29) the following conditional probabilities (the meaning is obvious) :

$$P_k^{(i)} = P_i^{(k)} = |d_{ik}|^2 \tag{33}$$

Thanks to (31), (32), (33), we can define two expressions :

$$P_{i,k} = P_i \, P_k^{(i)} = |c_i|^2 \, |d_{ik}|^2 \tag{34_1}$$

$$P_{k,i} = P_k \, P_i^{(k)} = |\sum_j c_j \, d_{jk}|^2 \, |d_{ik}|^2 \tag{34_2}$$

But, except in the case of commuting operators, we have :

$$P_{i,k} \neq P_{k,i} \tag{35}$$

Thus, <u>we cannot define a joint probability</u> P_{ik} ; (28_1) will be violated and so will (28_2) and <u>a fortiori</u> (28_3) because :

$$\sum_i P_i \, P_k^{(i)} = \sum_i |c_i|^2 \, |d_{ik}|^2 \neq P_k \tag{36_1}$$

$$\sum_k P_k \, P_i^{(k)} = \sum_{jk} |c_j \, d_{jk}|^2 \, |d_{ik}|^2 \neq P_i \tag{36_2}$$

We are confronted here with the central problem of quantum probabilities. As we know, the impossibility of defining a joint probability is equivalent to the existence of Heisenberg's uncertainties ; both are related to the presence of interference terms in (32) and these terms are nothing but the expression of the fact that on the measurement scheme given on Fig.2, it is <u>impossible to find a single spectral analyser</u> -and, therefore a single splitting of wave packets- which would allow us to measure simultaneously by <u>all</u> the physical quantities, when A,B etc. do not commute. Thus when A and B do not commute, one is not able to answer the two simultaneous questions : what is the value of A <u>and</u> what is the value of B, when the physical system is in a given system ψ. In such a case, we will equivalently get Heisenberg's uncertainties, different spectral analysers for the measurement of A and B on Fig.2 and the inequality (35) which destroys the classical probabilistic scheme.

Now, pay attention to the fact that we are speaking here about real waves propagating in <u>physical space</u> and not only about functions defined in an abstract space. The spectral analyser we considered above is, of course, a real physical device and not a Hamiltonian (although we obviously need such a Hamiltonian for a mathematical description of the device). But we are not speaking only about real waves but also about real particles and we have implicitly admitted the fundamental hypothesis that a particle is permanently localized in its wave. In other words, before being observed in a certain wave packet (on the right hand-side of Fig.2), it is supposed that the particle was already localized at every moment in some very small region of space : this means that we admit the existence of a hidden trajectory which went from a certain point of the initial wave packet ψ, through the analyser, up to the packet ϕ_k, where the particle was finally observed. This means that we suppose that the measuring apparatus (i.e. the counter, scintillating screen, photographic film etc.) set on the right side of Fig.2, only allows us to observe a pre-existing circumstance, namely the presence of the particle in such or such wave packet. Thus, it is supposed that the presence of the particle is only revealed by the observation, but in no way "created" or "brought on" by it.

Now we may -we must- ask this question : is this hypothesis compatible or not with the orthodox interpretation of quantum mechanics ? In other words, is the assertion of a preeminence of the position among all observable physical quantities compatible with the orthodox quantum theory of measurement ? However, we shall put this question aside : the problem has been treated in this same book by Francis Fer.

Up to now, we have only met with two of the three kinds of probabilities described by de Broglie : the present probabilities, i.e. those which correspond to a classical probabilistic set of observable events, and the predicted probabilities, i.e. those which concern events which are not observable by simple inspection, because they need a special preparation (spectral splitting) to be observed and, what is more, a preparation which cannot be the same for all the events. Now we shall meet with the third kind of probabilities : the hidden probabilities.

It is well known that in his famous book on the "Mathematical Foundations of Quantum Mechanics", von Neumann proved a theorem which claims that there are no pure states without statistical dispersions. This result is, in itself, intuitively obvious because the absence of dispersions in a pure state would mean that it would be possible to measure simultaneously the physical quantities attached to a system described by this pure state. But in fact we know that this is impossible for non commuting quantities. In this sense, the theorem is nothing but a consequence of Heisenberg's uncertainties. But from this theorem one may conclude that it is not possible to describe a pure state as a mixture of states without dispersions : this is also obvious, because such states do not exist ! This implies that one cannot consider a pure state of quantum mechanics as something like a probabilisable ensemble submitted to the classical scheme (28) of the probability theory : this is exactly what we were speaking about above. Now if you cannot do this, you are allowed to come to the conclusion that it is impossible to find a set of hidden parameters, obeying the laws of quantum mechanics and forming a statistical mixture with a classical probabilistic scheme (28), which would be able to give an account of the quantum probabilistic laws. The contradiction is made obvious by the preceding results ! But von Neumann goes further and claims that, in general, it is impossible to conceive any hidden parameters theory behind quantum mechanics.

De Broglie's answer consists essentially in asserting that if any hidden parameters do exist, they cannot obey quantum mechanics, because if you try to imagine hidden parameters, it is, of course, in order to restore the classical scheme of probabilities. Thus, since you cannot abandon this requirement, you are obliged, for the hidden motions of these parameters, to abandon the possibility of using quantum laws. Now, if you need a classical scheme of probabilities for the objective (but hidden) values of the physical quantities which are introduced in quality of hidden parameters, these probabilities cannot be the probabilities observed on the result of a measurement : simply because the observed probabilities do obey the quantum scheme and not the classical one !

So, de Broglie was led to the introduction of a third kind of probabilities : the so called hidden probabilities, that is, probabilities of a classical kind, which satisfy the scheme (28) but which concern the statistical distribution of hidden variables and which are hidden themselves because when you try to carry out a measurement, you must prepare the system, i.e. modify its state in a suitable manner so that the hidden classical statistics are broken and make way for quantum statistics.

To conclude, we must realize that if we intend to introduce hidden parameters into the quantum theory, the statistics which governs the distribution of these parameters must be hidden too and in no case can such a statistical distribution coïncide with the statistical distribution of the measurement results.

In our opinion, this is the basic point for every attempt to construct a hidden parameters theory. But unfortunately, this idea remains ignored and is never considered by the authors who work on hidden variables problems and, especially on related statistical problems.

For example, the correct conclusion to von Neumann's theorem should only be that all the probabilities of wave mechanics cannot be simultaneously present, which is, as we have said, a consequence of the existence of non commuting quantities. For this reason it is correct to assert that because these probabilities violate the classical scheme, they cannot be introduced into a hidden parameters theory with the aim of restoring the classical determinism : in this sense von Neumann's assertion was correct. But when he claimed that a hidden parameter theory is impossible, he was wrong because he neglected the fact that in such a theory the probabilities must be classical, but also need to be hidden (instead of being *present*) : his mistake consisted in attempting to assign to hidden parameters the *predicted* quantum probabilities instead of hidden classical ones.

Since the time when a refutation of von Neumann's theorem was published by de Broglie (for instance in his book on the *Theory of measurement in wave mechanics*), several other refutations have been published. All of them are based on arguments of logical incoherence of the theorem and some of them exhibit mathematical errors. But although the indication of a mathematical error is obviously sufficient to invalidate a theorem, de Broglie's refutation is farther reaching, because his reasoning is of a general physical character and forewarns us against other attempts of the same type. For instance, the refutation of Bell's inequality which I gave a few years ago (1976) was based on de Broglie's reasoning. It is really not difficult to show (see the following section) that Bell's inequality implies a classical probabilistic scheme for the statistical distribution of the measurement results and thus, contains a radical contradiction with a feature of the quantum theory which is notoriously confirmed by observed physical facts : namely quantum statistical previsions. It is not surprising that many experiments performed up to now (including that carried out by Aspect) have invalidated Bell's inequality. And I must add that this problem has *absolutely nothing to do with the problem of locality*, which is basically the aim of experiments undertaken in order to test the inequality.

4. A FEW WORDS ABOUT BELL'S INEQUALITY

When I showed that Bell's inequality necessarily implies a *classical* scheme for *measurement* statistics, in contradiction with quantum mechanics, I was surprised to see that some people strongly opposed this assertion and many others ignored it. I am glad to see that many authors have now rediscovered this pretty obvious fact and I wish to demonstrate it here, once again in two slightly different ways : first I would like to reintroduce the proof I presented in 1976 (in a more favorable atmosphere !) and then I shall give a refutation of a more recent proof of Bell's inequality suggested by d'Espagnat.

a) *My 1976 reasoning*. In the original proof (somewhat modified by Clauser, Horne, Shimony and Holt) of Bell's inequality, an integral is introduced :

$$P(\vec{a},\vec{b}) = \int d\lambda \ \rho(\lambda) \ A(\vec{a},\lambda)B(\vec{a},\lambda), \qquad (37)$$

which represents the correlation between the measurements of the spins, along two directions $\vec{a}$ and $\vec{b}$, of two particles which had previously interacted. $\underline{A}$ and $\underline{B}$ are the results of the two measurements ; λ is a set of hidden parameters (given in the initial state of the system) of which it is supposed that the measurements depend deterministically ; $\rho(\lambda)$ is the statistical distribution of these parameters, with the help of which are calculated all the mean values of the measurement results. $\rho(\lambda)$ is taken (like λ) in the initial state of the system, so that the statistics introduced in the measurement results do not depend on the measurement process itself. This is the very hypothesis that contradicts quantum mechanics, as we can easily see even in the case of a single physical quantity, for instance the spin of one of the two particles. With the help of this distribution $\rho(\lambda)$, we can easily define the probabiliy $P_{\vec{a}}(\alpha)$ for finding the value $A(\vec{a},\lambda) = \alpha(\alpha=\pm 1)$ as a result of the measurement of the spin of the particle a in the direction $\vec{a}$; and we can find a similar probability related to the measurement of the spin of the same particle in another direction $\vec{a}'$.

Indeed, if we define in the configuration space $E\{\lambda\}$ of the hidden parameters the following two subspaces :

$$
\begin{aligned}
E_\alpha &= E_{A(\vec{a},\lambda)=\alpha} = E\{\lambda \mid A(\vec{a},\lambda) = \alpha\}, && \alpha = \pm 1 \\
E_{\alpha'} &= E_{A(\vec{a}',\lambda)=\alpha'} = E\{\lambda \mid A(\vec{a}',\lambda) = \alpha'\}, && \alpha' = \pm 1
\end{aligned}
\tag{38}
$$

then we obtain the probabilities

$$
\begin{aligned}
P_{\vec{a}}(\alpha) &= \Pr\{A(\vec{a},\lambda) = \alpha\} = \int_{E_\alpha} \rho(\lambda)d\lambda \\
P_{\vec{a}'}(\alpha') &= \Pr\{A(\vec{a}',\lambda) = \alpha'\} = \int_{E_{\alpha'}} \rho(\lambda)d\lambda
\end{aligned}
\tag{39}
$$

Now, we shall also be able to define the quantity

$$
P_{\vec{a},\vec{a}'}(\alpha,\alpha') = \Pr\{A(\vec{a},\lambda) = \alpha, A(\vec{a}',\lambda) = \alpha'\} = \int_{E_\alpha \cap E_{\alpha'}} \rho(\lambda)d\lambda \tag{40}
$$

and this is nothing but the probability of finding value α for the spin measure of particle a in the direction $\vec{a}$ and value α' for the spin measure of the same particle in another direction $\vec{a}'$: it is the joint probability !

Since a measurement of a spin can only give the value ± 1 (reduced here to unity), we shall have

$$
E_{\alpha=-1} \cup E_{\alpha=1} = E\{\lambda\} \; ; \; E_{\alpha'=-1} \cup E_{\alpha'=1} = E\{\lambda\}. \tag{41}
$$

Therefore,

$$
\int_{E_\alpha \cap E_{\alpha'=1}} \rho(\lambda)d\lambda + \int_{E_\alpha \cap E_{\alpha'=-1}} \rho(\lambda)d\lambda = \int_{E_\alpha} \rho(\lambda)d\lambda \tag{42}
$$

whence, according to (39) and (40), we deduce

$$
P_{\vec{a}}(\alpha) = \sum_{\alpha'=\pm 1} P_{\vec{a},\vec{a}'}(\alpha,\alpha') \; ; \; P_{\vec{a}}(\alpha') = \sum_{\alpha=\pm 1} P_{\vec{a},\vec{a}'}(\alpha,\alpha'). \tag{43}
$$

This is the formula (28_2) and now we may also write (28_1), by introducing conditional probabilities :

$$P_{\vec{a}}^{(\vec{a}')}(\alpha,\alpha') = \frac{P_{\vec{a},\vec{a}'}(\alpha,\alpha')}{P_{\vec{a}'}(\alpha')} \; ; \; P_{\vec{a}'}^{(\vec{a})}(\alpha,\alpha') = \frac{P_{\vec{a},\vec{a}'}(\alpha,\alpha')}{P_{\vec{a}}(\alpha)} \qquad (44)$$

And lastly, using (43), we get :

$$P_{\vec{a}}(\alpha) = \sum_{\alpha'=\pm 1} P_{\vec{a}}^{(\vec{a}')}(\alpha,\alpha')P_{\vec{a}'}(\alpha') \; ; \; P_{\vec{a}'}(\alpha') = \sum_{\alpha=\pm 1} P_{\vec{a}'}^{(\vec{a})}(\alpha,\alpha')P_{\vec{a}}(\alpha), \qquad (45)$$

i.e. (28_3). Thus, we see that the hypothesis which is at the origin of Bell's inequalities implies a classical statistical scheme for measurement results, that is, implies (in contradiction with Heisenberg's uncertainties and with experimentation) the possibility of a simultaneous measure of two spin components of the same particle.

In other words Bell's mistake is opposite to von Neumann's one : the latter ascribed the quantum statistics of measurement results to hidden parameters whereas the former ascribed the classical statistics of hidden parameters to measurement results. They both neglect the difference pointed out by de Broglie between hidden classical probabilities and predicted quantum probabilities.

b) D'Espagnat's problem of the twin students (1979). In order to demonstrate the theorem in a simple general way, the author imagined an ingenuous example : he replaced spins by twins !

Let us consider with him a university where each student has a twin brother (a real one, of course) and where the only subjects taught and tested are three languages : Latin, Greek and Chinese.

First of all, it is easy to prove that in any case (without taking into account the "twin" hypothesis) we may divide the students into three "statistical samples" and assert that after the exams :

"The number of graduates in Latin and Greek in sample 1 will not be larger than the number of graduates in Latin and Chinese in sample 2 increased by the number of graduates in Greek who failed in Chinese in sample 3".

This is exactly Bell's inequality ! But one may object that these languages are difficult, that different exams undergone by a single student may perturb each other (these are perturbations caused by measurements !) and this is why d'Espagnat introduced the twins, so that one exam may be undergone by one brother and the second one by the second brother (without perturbing the first one), but both will be credited with the same mark since they are supposed to be perfect twins.

It seems that we will thus find Bell's inequality without supposing that the same student may undergo two examinations, that is without forgetting the measurement perturbations. This is wrong ! Let us try to give a real proof of the inequality.

In order to do so, let us begin by separating the twins so that we get two "twin sets" each of which will be divided into three samples (resp. : 1,2,3 and 1',2',3'). In each sample, we shall denote l, g, c the number of "direct" graduates (i.e. having undergone the exam themselves) in Latin, Greek, and Chinese, $\tilde{l}$, $\tilde{g}$, $\tilde{c}$ the number of those who "directly" failed and by $\underline{l}$, $\underline{g}$, $\underline{c}$ (resp. $\underline{\tilde{l}}$, $\underline{\tilde{g}}$, $\underline{\tilde{c}}$) the number of "indirect" graduates who passed (resp. failed) through an examination undergone by their twin brother. Of course, we have by hypothesis $l=\underline{l}$, $g=\underline{g}$, $c=\underline{c}$ etc.

Now consider the following figure :

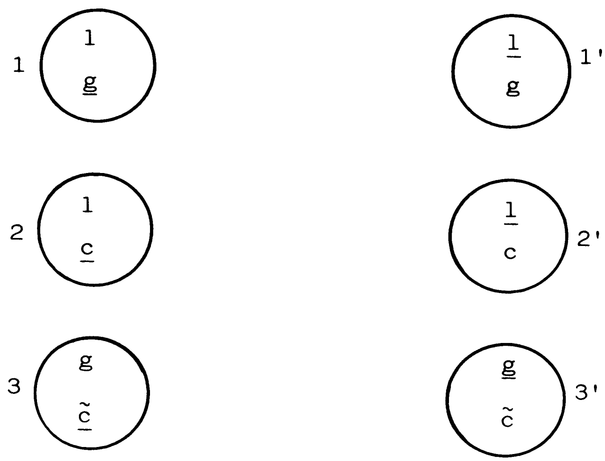

Fig.3

If we denote, for instance, $(l,\underline{g})_1$ the number of students in sample 1 who are "direct" graduates in Latin and "indirect" graduates in Greek and if in the same way we introduce analogous brackets, the previous expression of Bell's inequality will be written :

$$(l,\underline{g})_1 \leqslant (l,\underline{c})_2 + (g,\underline{\tilde{c}})_3. \tag{46}$$

Now, in order to prove this inequality, we can write, on sample 1, the trivial decomposition :

$$(l,\underline{g})_1 = (l,\underline{g},\underline{c})_1 + (l,\underline{g},\underline{\tilde{c}})_1 \tag{47}$$

But the three samples are statistically equivalent, whence :

$$(l,\underline{g})_1 = (l,\underline{g},\underline{c})_2 + (l,\underline{g},\underline{\tilde{c}})_3. \tag{48}$$

Now, in each sample, we have trivially :

$$(1,\underline{g},\underline{c})_2 \leqslant (1,\underline{c})_2 \ ; \ (1,\underline{g},\underline{\tilde{c}})_3 \leqslant (\underline{g},\underline{\tilde{c}})_3 \tag{49}$$

and introducing (49) in (48) we get (46).

Unfortunately, the decomposition we have introduced in (47) (and what else could we do in order to find an inequality ?) implies that it is physically meaningful to talk about students whose knowledge has been <u>directly</u> tested in two <u>languages</u> (for instance in Greek and Chinese). This means that finally, in order to prove the inequality, we have been obliged so to speak "to perform on the same object measurements of two quantities that we supposed were not simultaneously measurable". In other words we ended up supposing that the measurement statistics were <u>classical</u>.

5. IN SEARCH OF A NEW SOLUTION

One may pose the following question : is it really possible to build a model with hidden parameters obeying hidden classical probabilities which would nevertheless be in agreement with the quantum probabilistic scheme for measurement results ? In order to answer the question, we shall give a spin measurement model based on the ideas of de Broglie. Of course, it will just be sketched, not as a true theory but only as an example.

The main idea will be that the corpuscle is permanently localized in the wave (although we do not know exactly where) and that the spin is well defined at each point of the wave but cannot be measured without a special preparation because a measurement implies the following :

a) Detection of the particle.

b) Realization of a state such that a relatively rough determination of the position will lead to a univocal determination of the spin.

For the sake of simplicity, we shall consider the nonrelativistic case, so that the wave associated with the particle is represented by a spinor with two components obeying the Pauli equation. The hidden parameters which Bell denoted by λ are now the position coordinates $\vec{r}$ of the particle in the wave ; therefore the density $\rho(\lambda)$ will be written

$$\rho(\vec{r}) = \psi^*(\vec{r})\psi(\vec{r}) = |\psi_1(\vec{r})|^2 + |\psi_2(\vec{r})|^2 \tag{50}$$

with

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \qquad \int \rho(\vec{r})dv = 1 \tag{51}$$

In any state ψ of the particle, we shall define a spin

$$\vec{s}(\vec{r}) = \frac{\psi^*\vec{\sigma}\psi}{\psi^*\psi} \tag{52}$$

where $\vec{\sigma}$ denotes the set of the three Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{53}$$

Formula (52) means that if the particle is found at point $\vec{r}$ of the wave $\psi(\vec{r})$, it then has the spin $\vec{s}(\vec{r})$. But, for the time being, the position $\vec{r}$ of the particle remains unknown to us, and we know only the probability density $\rho(\vec{r})$ given by (50) ; the spin is thus a hidden variable.

Let us consider two different directions $\vec{a}$, $\vec{a}'$ in space. Before any measurement is made, if we suppose the particle to be localized, at a given instant, at a given point $\vec{r}$, in the wave, we will be led by the preceding formulae to attribute the following values to the spin :

$$\vec{s}(\vec{a},\vec{r}) = \frac{\psi^*\vec{\sigma}.\vec{a}\psi}{\psi^*\psi} \quad ; \quad \vec{s}(\vec{a}',\vec{r}) = \frac{\psi^*\vec{\sigma}.\vec{a}'\psi}{\psi^*\psi}, \tag{54}$$

in the directions $\vec{a}$ and $\vec{a}'$. But pay attention ! The two quantities $\vec{s}(\vec{a},\vec{r})$ and $\vec{s}(\vec{a}',\vec{r})$ are simultaneously defined but not simultaneously measurable : they are not to be mistaken, as in Bell's formula, with what he denotes by $A(\vec{a},\lambda)$ and $A(\vec{a}',\lambda)$, i.e. with the results of a measurement. At this stage, these quantities remain hidden quantities (we shall examine the question of measurement below) and they obey a classical statistics which is easy to construct, but which will be hidden too, and which is not to be confused with the measurement statistics.

In order to build the hidden statistics let us write more briefly r, s, s' instead of $\vec{r}$, $\vec{s}(\vec{a},\vec{r})$, $\vec{s}(\vec{a}',\vec{r})$, and let us denote as R, S, S' random variables whose possible values are respectively r, s, s'. First we will have the densities of conditional probabilities (δ = Dirac function) :

$$\rho_S^{(R)}(\alpha,r) = \delta(\alpha - s(r)), \quad \rho_{S'}^{(R)}(\alpha',r) = \delta(\alpha' - s(r')) \tag{55}$$

which simply means that if R takes the value r then, by hypothesis, S certain by takes the value s(r) and S' the value s'(r). The probability densities for S and S' will be :

$$\rho_S(\alpha) = \int \rho(r)\delta(\alpha - s(r))dr \ ; \ \rho_{S'} = \int \rho(r)\delta(\alpha - s'(r))dr \tag{56}$$

Here we see that the position r of the particle in the wave plays the role played by λ for Bell ; $\rho(r)$ is given by (50), and don't forget that this probability is not hidden, but present, as we said previously. Of course, we may also define the joint probability that S will take a value in $[\alpha, \alpha+d\alpha]$ and S' in $[\alpha',\alpha'+d\alpha]$:

$$\rho(\alpha,\alpha') = \int \rho(r)\delta(\alpha - s(r))\delta(\alpha' - s'(r))dr \tag{57}$$

and we can immediately verify that :

$$\rho_S(\alpha) = \int \rho(\alpha,\alpha')d\alpha' \qquad \rho_{S'}(\alpha') = \int \rho(\alpha,\alpha')d\alpha \tag{58}$$

Thus, we have the classical statitical scheme (28), from which Bell's inequalities may be deduced : but this scheme is hidden and so are Bell's inequalities, because if we try to measure a spin component, we cannot do so without disturbing the form of the wave and without changing the density ρ and thereby all the other probability distributions.

For us to be sure, let us first consider the measurement of a spin component parallel to a certain direction $\vec{a}$. This can be done by letting the particle enter an inhomogeneous magnetic field oriented along direction $\vec{a}$. This field will split the wave into two distinct wave trains which shall be such that, if the particle is found to be in one of them, we may then be sure that $\vec{s}(\vec{a},\vec{r}) = 1$ and, if it is found in the other, then we know that $\vec{s}(\vec{a},\vec{r}) = -1$.

Let us then take the direction $\vec{a}$ as coinciding with the z axis. We shall have according to (50), (52) :

$$\vec{s}(\vec{a},\vec{r}) = s_3(\vec{r}) = \frac{\psi^*\sigma_3\psi}{\psi^*\psi} = \frac{\psi_1{}^*\psi_1 - \psi_2{}^*\psi_2}{\psi_1{}^*\psi_1 + \psi_2{}^*\psi_2} \tag{59}$$

The wave train for which $s_3 = 1$ will then be such that

$$\psi^+ = \binom{\psi_1}{0} \tag{60}$$

and the one for which $s_3 = -1$ will be

$$\psi^- = \binom{0}{\psi_2} \tag{61}$$

The total wave on leaving the magnetic field will be $\psi = \psi^+ + \psi^-$, but, since the two components are separated in space, filling two distinct regions R^+ and R^-, the probability of getting the value +1 of the component S_3 of the spin will be

$$P_3(+) = \int_{R^+} \psi^*\psi dv = \int |\psi_1|^2 dv \tag{62}$$

while the probability of getting the value -1 will be

$$P_3(-) = \int_{R^-} \psi^*\psi dv = \int |\psi_2|^2 dv \tag{63}$$

The normalization of the wave ensures that

$$P_3(+) + P_3(-) = 1 \tag{64}$$

It is understood that only the s_3 component of the spin has its values distributed according to the elementary probability law given by $P_3(+)$ and $P_3(-)$, and one also sees that such a law is different from the law which yielded the hidden distributions before the measurement.

Let us now assume that we wanted to measure the spin component along another direction distinct from $\vec{a}$: Let us denote it by $\vec{a}'$.

We should then set up another apparatus in all ways similar to the one above, except that its magnetic field would then be orientated along direction $\vec{a}'$, which would again split the initial wave into two wave trains (obviously different from those we had obtained above). Each of these wave trains would correspond to one of the values +1 or -1 of the component $S(\vec{a}',\vec{r})$. We could then get for these two values certain probabilities $P_3'(+)$ and $P_3'(-)$ with expressions similar to those we considered above ; but in order to do this, it is necessary to change the reference frame and take for the z axis no longer the direction $\vec{a}$ but the direction $\vec{a}'$; in Eqs.(62) and (63) we obviously have now new functions ψ_1' and ψ_2'.

It is thus seen that if one assumes that the hidden parameters are the coordinates of the particle, then the measurement of two different components of a spin requires two partitions of the initial wave train that are different and incompatible with each other. It clearly follows that the probability density $\rho(r)$, which pertains to the initial wave, will give rise, when resulting from the magnetic field, to two different densities $\rho(\vec{a},\vec{r})$ and $\rho(\vec{a}',\vec{r}')$, corresponding to the measurements of components $S(\vec{a})$ and $S(\vec{a}')$ of the spin.

We are clearly unable to write here any integral like (31) with A or B representing measurement results : THE VERY IDEA OF BELL'S INEQUALITY IS ABSOLUTELY EXCLUDED FROM SUCH A HIDDEN VARIABLES THEORY.

Some conclusions

Among de Broglie's physical ideas the main conception which must above all be underlined is his profound belief in the existence of a physical material wave, in the permanent localization of the corpuscle within this wave, and in the preeminent role he assigns to the relation between the frequency of the wave and the clock-frequency of the particle. This is, briefly summarized, de Broglie's scientific legacy.

Now, his theory of the double solution must only be considered as a possible model, imagined by him in order to construct a theory on the basis of these ideas. In fact, in spite of the existence of some attractive features (such as the possibility of a visualization of the wave particle dualism) and some remarkable results in the problems of propagating waves (such as the description of diffraction phenomena), the theory remains unfinished.

Nevertheless, even in its present state, the theory has given rise to two important conceptions, namely : the introduction (thirty years ago) of solitons in quantum mechanics, and a physical (and not only operational) quantum measurement theory, even if the latter is still confined to measurements of the first kind (i.e. without intervention of correlated particles) and only to special cases of measurements of the second kind (i.e. with correlations).

This measurement theory is based on a general idea concerning hidden parameters which is at once so important and so obstinately ignored by most physicists that I am not afraid of emphasizing it too much : I am speaking, of course, about the absolute necessity of a distinction between hidden, predicted and present probabilities, if we are to speak about hidden parameters. Given that I am sure that most of the contributions that will be devoted to the same problem in this book will diverge from my opinion I want to insist and to claim once more that I consider Bell's inequalities as being in radical contradiction with statistical statements of quantum mechanics which are certainly in agreement with physical facts in all the experiments carried out in order to test these inequalities. For this reason, the experimental violation of Bell's inequalities has nothing to do, in my opinion, with the so called "non locality" or "non separability". This violation simply signifies that quantum probabilities are not classical probabilities ! This is a fact which was discovered more than half a century ago, which it is nice to see confirmed again, but which perhaps does not need so many sophisticated and beautiful experiments in order to be accepted. The true interest of such experiments is, in my opinion, to confirm the exactness of the quantum statement that two particles (having previously interacted) may be considered as constituting a pure state : this statement was originally used for very small systems (such as molecules) but it was a priori not obvious at all that it should remain true for particles which are separated by more than ten meters. The fact that this is true constitutes a genuine progress. But as puzzling as this fact may be, in no way does it prevent the possible existence of hidden parameters, for the elementary reason that such an assertion is based on an inequality which supposes a statistical hypothesis which is false even for one particle, so that the inequality looses any physical meaning.

I cannot conclude these few remarks without drawing attention to the fact that the best one can do at a given stage of the history of science is to provide more or less good arguments in favour of such or such a theoretical hypothesis. One must be very cautious about the alleged final character of a scientific conclusion, because any scientific conclusion is always subject to revision. The claim of having definitively closed a door, in science, even if no objection can momentarily he opposed (which is not presently the case !) is a very dangerouse one : such an attitude does not belong to science but to ideology and it is rather apt to arouse the laughter of future generations. What I consider to be true for other people is also true, of course, for me : I never forget that it is quite possible that a new microphysics may be elaborated without any reference to hidden parameters, but there is a great difference between considering a possibility and accepting the extravagant pretention of a final scientific prohibition.

[1] From (6) and (8) we get immediately de Broglie's wave-length, for :

$$\lambda = V/\nu = (c^2/v)/(\frac{m_0c^2}{h}/\sqrt{1 - \frac{v^2}{c^2}}) = h/(m_0v/\sqrt{1 - \frac{v^2}{c^2}}) = h/p$$

[2] A classical objection is that this is true for devices like a prism, a mass-spectrograph, an apparatus of Stern-Gerlach etc, but not for measurements obtained, for instance, by a resonance in a molecular beam. This objection is not valid because the resonance in the molecular beam is in fact tested by the intensity of the emerging beam, i.e. the presence or the absence of a particle in a certain region of space.

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