

# Gravitation Without Curved Space-time

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## Abstract

A quantum-mechanical theory of gravitation is presented, where the motion of particles is based on the optics of de Broglie waves. Here the large-scale geometry of the universe is inherently flat, and its age is not constrained to  $< 13$  Gyr. While this theory agrees with the standard experimental tests of Einstein's general relativity, it predicts a different second-order deflection of light, and measurement of the Lense-Thirring effect in the upcoming NASA experiment Gravity Probe B.

# 1 Introduction

Modern physics has two different representations of gravitation: in addition to the geometric one of Einstein's general relativity, there is also the quantum-mechanical description. According to general relativity's weak equivalence principle, the motion of a test particle in a gravitational field is independent of its mass. However, in quantum mechanics, the motion depends intimately on particle mass. The mathematical structures of the two representations "seem utterly incompatible," in the words of Francis Everitt.

Weinberg [1] suggests that the prevailing geometric model of gravitation "has driven a wedge between general relativity and the theory of elementary particles." He also points out that this approach is unnecessary:

Einstein and his successors have regarded the effects of a gravitational field as producing a change in the geometry of space and time. At one time it was even hoped that the rest of physics could be brought into a geometric formulation, but this hope has met with disappointment, and the geometric interpretation of the theory of gravitation has dwindled to a mere analogy, which lingers in our language in terms like "metric," "affine connection," and "curvature," but is not otherwise very useful. The important thing is to be able to make predictions about images on the astronomers' photographic plates, frequencies of spectral lines, and so on, and it simply doesn't matter whether we ascribe these predictions to the physical effect of gravitational fields on the motion of planets and photons or to a curvature of space and time.

It's often held that, beyond describing gravitation, curved space-time explains it. On this basis, Einstein's theory is taken to be superior to others based solely on potentials. But it does not explain how mass-energy results in such curvature, so one unknown is only replaced by another. And, despite heroic efforts by Einstein and others, no geometric basis has been found for electromagnetism. We are left with inconsistent representations of these phenomena.

Gravity and electromagnetism are more closely related in the theory introduced here. *It is assumed the effects of gravitational potentials do not come indirectly, via space-time curvature, but from their direct influence on quantum-mechanical waves.* Beyond its immediate compatibility with quantum mechanics, the mathematical description of gravity obtained is simpler and more precise than the present one. As shown below, this theory agrees equally well with the usual experimental tests of general relativity. Also, it makes new predictions for future experiments.

The Hubble redshift has been taken by many as the ultimate vindication of Einstein's general relativity. (See Misner, Thorne and Wheeler [2].) In the associated "standard" Big Bang model, the redshift is attributed to a curved, expanding space-time. That model is contradicted now by various observations.

Those include measurements of the distribution of galaxies, which reveal no discernible large-scale curvature [3]. According to Linde [4], the discrepancy is approximately *sixty* orders of magnitude. (Toward a flatter geometry, hypotheses

of inflation, strange dark matter, the cosmological constant, and now strange dark energy have been introduced *post hoc*. But there remains no explanation why space-time is not curved by the quantum zero-point energy of the vacuum.)

Also, from the redshifts and distances of nearby spiral galaxies estimated by Mould *et al.* [5], the “standard” Big Bang model puts the maximum age of the universe at 13 Gyr. (Direct measurement of the distance to the galaxy NGC4258 by Herrnstein *et al.* [6] now indicates that age needs to be revised downward [7].) Tsujimoto, Miamoto and Yoshi [8] find that substantially less than the ages of stars in globular clusters.

This new theory leads instead to an evolutionary cosmology, in which the Hubble redshift can be attributed to gradual change in basic properties of the universe and atomic spectra. This gives a universe older than its stars, with an inherently flat geometry.

## 2 Gravitational Potential

At the most fundamental level, electromagnetism is described in terms of the effects of potentials on the phases of quantum-mechanical waves. The phase shift  $\Delta S$  of de Broglie waves associated with a charged particle is given by

$$\Delta S = \frac{q}{h} \int \Phi dt - \frac{q}{hc} \int \mathbf{A} \cdot ds \quad (1)$$

where  $\Delta S$  is measured in cycles, and the integrals are taken over a possible trajectory,  $s$ . Using the Gaussian system of units,  $\Phi$  and  $\mathbf{A}$  are the scalar and vector electromagnetic potentials,  $q$  is charge,  $h$  Planck’s constant, and  $c$  the speed of light. As pointed out by Aharanov and Bohm [9] (and reiterated by Feynman [10]), this subsumes the familiar Lorentz equation for the force on a charged particle,

$$\mathbf{F} = q \left( \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \quad (2)$$

Colella, Overhauser and Werner [11], demonstrated in 1975 that de Broglie waves are influenced similarly by gravitational potentials. That experiment measured the gravitational phase shift of neutron waves. The result was consistent with

$$\Delta S = \frac{m}{h} \int \Phi_g dt \quad (3)$$

where  $m$  is the neutron mass and the subscript  $g$  indicates a gravitational potential.

Such effects are taken as the basis of gravitation here. While not fully described by this equation, within experimental accuracy, the same phase shift is predicted for the Colella-Overhauser-Werner experiment. As in existing quantum mechanics, the motion of particles will be found by applying Huygens’ principle to de Broglie waves, without introducing an additional geodesic principle.

In rectangular coordinates, the scalar electromagnetic potential for a particle at the origin, moving in the  $x$  direction, is

$$\Phi = \frac{q}{\sqrt{x^2 + (y^2 + z^2)(1 - v^2/c^2)}} \quad (4)$$

where  $v$  is the velocity. And there is a vector potential,

$$\mathbf{A} = \frac{\mathbf{v}}{c} \Phi \quad (5)$$

Unlike the two potentials in electromagnetism, or *ten* in Einstein's gravity, there (currently) is only one gravitational potential in this theory. We'll take it to have the same relativistic form as the electromagnetic scalar. With the role analogous to charge played by its inertial rest mass  $m_0$ , the gravitational potential due to a small mass element is

$$\Phi_g = \frac{-Gm_0}{\sqrt{x^2 + (y^2 + z^2)(1 - v^2/c^2)}} \quad (6)$$

where  $G$  is the gravitational constant. (The inertial rest mass includes the usual relativistic contributions of its moving particles.) The equipotential surfaces then have the same shape and arrangement for the three potentials  $\Phi$ ,  $\mathbf{A}$ ,  $\Phi_g$ . (Consequently, they may be attributable to a unified source [12].)

While these potentials themselves behave similarly and superpose in the same linear fashion, the gravitational one will be seen to differ in its nonlinear effects. As described in Section 10, *despite the absence of a gravitational vector potential, there are velocity-dependent effects on moving bodies*. Those sometimes resemble magnetism. In the case of the lunar orbit, there is an equivalent of the gravitomagnetic effect in Einstein's theory, as shown in Section 11.

Electromagnetic potentials are governed by the wave equations

$$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -4\pi\rho \quad (7)$$

and

$$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi\mathbf{j}}{c} \quad (8)$$

To satisfy special relativity, the gravitational potential is taken to obey an equation of the same form. From the corresponding quantities in Eqs. (4) and (6), that gives

$$\nabla^2 \Phi_g - \frac{1}{c^2} \frac{\partial^2 \Phi_g}{\partial t^2} = 4\pi G\rho_m \quad (9)$$

where  $\rho_m$  refers to the density of the mass appearing in Eq. (6).

The gravitational waves this describes have the same velocity  $c$  as in general relativity. For a single gravitational potential, they are simple longitudinal waves like those in acoustics. Because  $c$  will depend on  $\Phi_g$ , they are also nonlinear. As in the acoustic wave approximation, the waves can be treated as linear for small amplitudes such as those found in the solar system.

In his search for the basis of general relativity's geometry, "pre-geometry," Wheeler has called attention to a proposal by Sakharov [13] that gravitation ultimately arises from variation in the quantum zero-point energy of the vacuum.

(See Puthoff [14].) Sakharov’s conjecture is that space-time curvature is determined by the distribution of vacuum energy. We’ll adopt a similar hypothesis: that gravitational potentials correspond to regions of diminished vacuum energy, which determines the velocity of quantum-mechanical waves. Like the speed of sound in a gas, the velocity is less where the energy density is lower [12].

### 3 Wave Velocities and Transformations

In 1911, before arriving at the current theory of general relativity, Einstein [15] proposed that the effect of a gravitational potential is a decrease in the speed of light. While accurately describing the gravitational redshift and the behavior of clocks, that theory predicted only half the measured value for the deflection of starlight by the Sun.

In this theory, a reduction of the velocities of *all* quantum-mechanical waves, including light, is taken as the fundamental effect of gravitational potentials. The amount of reduction, and the way this is manifested differ substantially from Einstein’s initial scheme. Here, the speed of light is given by

$$c = c_0 e^{2\Phi_g/c_0^2} \quad (10)$$

where  $c_0$  is the value in the absence of a gravitational potential.

As shown in Appendix A, this function can be derived directly from general principles. One is an extended principle of relativity, holding that the observed laws of physics are also unchanged for reference frames in uniform gravitational potentials. (However, there is no assumption of the equivalence principle, in any of its various forms. Nor is there any assumption of a geodesic principle, general covariance, or Mach’s principle.) Eq. (10) has also been derived by Puthoff [16] from a model of the vacuum.

From the extended principle of relativity, the velocity  $V$  of de Broglie waves slows in proportion to light in a gravitational potential. Hence we can also write

$$V = V_0 e^{2\Phi_g/c_0^2} \quad (11)$$

where the 0 subscript again indicates the corresponding quantity without a potential. While these wave velocities are defined with respect to a preferred reference frame (such as that identified by the cosmic microwave background), the system nevertheless obeys special relativity where the gravitational potential is uniform. The special relativity adopted here is that advocated by Lorentz and Poincaré, and more recently by Bell [17].

(Bell saw this preferred-frame relativity as a likely prerequisite for a causal quantum mechanics. An advantage is its provision of a preferred direction for time and entropy. For that purpose, parameterized quantum mechanics introduces a time parameter  $\tau$  into Minkowski space-time, as a function of a Newtonian time  $t$ . Hartle [18] has noted it may be impossible to incorporate such a parameter into general relativity. However, in Lorentz-Poincaré relativity,  $t = \tau$  and no added parameter is needed.)

At the quantum-mechanical level, the frequencies of de Broglie waves determine the rates of clocks, and their wavelengths the sizes of atoms and measuring rods. In special relativity, the frequencies of constituent particles in moving atoms, and their wavelengths in the direction of motion, diminish by the same factor. Here a similar effect exists in gravitational potentials.

The de Broglie wavelength  $\lambda$  and frequency  $\nu$  are related to wave velocity by

$$V = \lambda \nu \quad (12)$$

As derived in Appendix A,  $\lambda$  and  $\nu$  diminish identically as

$$\nu = \nu_0 e^{\Phi_g/c_0^2} \quad (13)$$

and

$$\lambda = \lambda_0 e^{\Phi_g/c_0^2} \quad (14)$$

where the subscript 0 has the same meaning as before. (In this case, the change in  $\lambda$  is isotropic, and doesn't occur in just one dimension.)

It follows the rates of clocks and lengths of measuring rods change in gravitational potentials by the factor  $e^{\Phi_g/c_0^2}$ . Consequently, the locally measured speed of light remains constant. These represent changes in *objects themselves* rather than the geometry of space-time.

“Relativity” was Poincaré’s term, and to illustrate the principle [19] he asked: What if you went to bed one night, and when you awoke the next day everything in the world was a thousand times bigger? Would you notice anything? He pointed out such effects aren’t observed locally, since measuring devices change with the objects they measure. As Einstein noted [20], he also insisted the true geometry of the universe is Euclidean. Objects behave here as in a Poincaré world.

We can also derive transformations for energy and mass. In accord with the extended principle of relativity, Planck’s constant  $h$  is taken to be invariant in a gravitational potential. From the relation  $E = h\nu$  and Eq. (13), a particle’s rest energy  $E_0$  varies with its de Broglie frequency as

$$E_0 = E_{00} e^{\Phi_g/c_0^2} \quad (15)$$

where  $E_{00}$  represents the energy for both a zero velocity and zero gravitational potential. Since its energy is the sum of its particles’, this also holds for a macroscopic body.

From Einstein’s relation, the last equation can be expressed as

$$m_0 c^2 = m_{00} c_0^2 e^{\Phi_g/c_0^2} \quad (16)$$

where similarly  $m_0$  and  $m_{00}$  are the rest mass and rest mass without a gravitational potential. Substituting for  $c$  from Eq. (10) gives the transformation of rest mass in a gravitational potential,

$$m_0 = m_{00} e^{-3\Phi_g/c_0^2} \quad (17)$$

This is also the mass responsible for gravitational potentials appearing in Eq. (6).

## 4 Optics of Light and Matter Waves

To calculate trajectories, the basic method we'll use dates (remarkably) to Johann Bernoulli (1667-1748). Bernoulli discovered the motion of a body in a gravitational field can be treated as an optics problem, through assuming a fictitious refractive index, proportional to the square root of the difference between its kinetic and potential energies [21]. The same deep analogy between mechanics and optics underlies the Hamiltonian representation of classical mechanics. Hamilton based that on hypothetical surfaces of constant “action” orthogonal to the trajectories of bodies.

The “optical-mechanical analogy” also had a central role in the development of wave mechanics by de Broglie and Schrödinger [22]. There the analogy becomes more direct: instead of fixed surfaces of constant action, there are moving de Broglie wavefronts. Like light rays, the trajectories of particles are orthogonal to those. Consequently, when diffraction and interference effects can be neglected, the trajectories of particles or bodies can be found by the methods of geometrical optics.

To arrive at matter waves, de Broglie began by treating massive particles as Planck oscillators [23]. Equating the Planck and Einstein energies for a particle

$$E = h\nu = mc^2 \tag{18}$$

its frequency  $\nu$  is  $mc^2/h$ . Next, from the Lorentz transform for a moving oscillator, de Broglie showed the velocity  $V$  for waves matching a moving particle's phase is

$$V = \frac{c^2}{v} \tag{19}$$

where  $v$  is the particle's velocity. Its wavelength  $\lambda$  then is  $V/\nu$  or

$$\lambda = \frac{h}{mv} \tag{20}$$

Note the relationship between  $V$  and  $v$  in Eq. (19) doesn't depend on mass. In this gravity theory, it obviates the usual assumption of the weak equivalence principle, which dictates that a test particle's motion is mass independent.

Interference in beams of atoms and molecules shows these equations hold beyond elementary particles. In principle, as emphasized by Wheeler [24], large bodies also have de Broglie wavelengths – although from the last equation they become too short to observe. In that case diffraction and interference are negligible, and geometrical optics can be used to describe the waves.

The variational principle in geometrical optics is Fermat's principle of least time. From Eq. (19), matter particles and their waves have different velocities. What's minimized is the travel time for *wavefronts* between two points, with respect to other trajectories. (At a more fundamental level, Fermat's can be viewed as a consequence of Huygens' principle. Unlike general relativity, we'll introduce no additional variational principle beyond those already in quantum mechanics.)

Fermat's principle can be expressed as

$$\delta \int_{s_0}^s n ds = 0 \quad (21)$$

where  $s$  refers to distance over some path connecting two fixed points and  $n$  is the refractive index (inverse of wave velocity) as a function of  $s$ .

As Marchand [25] shows, for any spherically symmetric index gradient, the resulting path in polar coordinates is

$$\theta = \theta_0 + k \int_{r_0}^r \frac{dr}{r \sqrt{r^2 n^2 - k^2}} \quad (22)$$

where  $\theta_0$  and  $r_0$  are the values at an arbitrary starting point. The quantity  $k$  is a constant given by

$$k = \pm r n \sin \psi \quad (23)$$

where, for any point on the ray trajectory,  $\psi$  is the angle between the trajectory and a radial line connecting the point and origin.

The last two equations can be used to describe both light rays and the trajectories of bodies in central gravitational fields. We'll introduce the notation

$$\mu = \frac{GM}{c_0^2} \quad (24)$$

where  $G$  is the gravitational constant and  $M$  the mass of a gravitational source. For a spherical source body at rest, the dimensionless potential becomes

$$\frac{\Phi_g}{c_0^2} = -\frac{GM}{c_0^2 r} = -\frac{\mu}{r} \quad (25)$$

The speed of light from Eq. (10) is then

$$c = c_0 e^{-2\mu/r} \quad (26)$$

giving the spherically symmetric refractive index

$$n = \frac{c_0}{c} = e^{2\mu/r} \quad (27)$$

Putting this expression for  $n$  into Eq. (22) gives the path of a light ray near a massive body.

As done by de Broglie for charged particles [26], we can construct a similar index of refraction for matter waves

$$n = \frac{c_0}{V} \quad (28)$$

(Again, this is a dimensionless number inversely proportional to the wave velocity. Only its relative variation matters, so its scale is arbitrary.) We'll use it here to find the trajectory of a test body in a central gravitational field. For that we'll need an expression for  $V$  as a function of the body's radial position.



To put  $V$  in terms of energy, we can use the relativistic transform

$$E = \frac{E_0}{\sqrt{1 - v^2/c^2}} \quad (29)$$

where  $E_0$  again is the body's rest energy. Solving for  $v$  gives

$$v = c \sqrt{1 - E_0^2/E^2} \quad (30)$$

Inserting this into Eq. (19), we get

$$V = \frac{c}{\sqrt{1 - E_0^2/E^2}} \quad (31)$$

While the principle of energy conservation says  $E$  is constant for a freely orbiting body,  $E_0$  depends on the gravitational potential. From Eqs. (15) and (25),  $E_0$  is

$$E_0 = E_{00} e^{-\mu/r} \quad (32)$$

And from this and Eq. (26) for  $c$ , we can rewrite the previous equation as

$$V = \frac{c_0 e^{-2\mu/r}}{\sqrt{1 - \frac{E_{00}^2}{E^2} e^{-2\mu/r}}} \quad (33)$$

in terms of orbital constants and the single variable  $r$ . Eq. (28) becomes

$$n = \sqrt{e^{4\mu/r} - \frac{E_{00}^2}{E^2} e^{2\mu/r}} \quad (34)$$

Putting this  $n$  into Eq. (22) gives an exact equation for the orbital trajectory

$$\theta = k \int \frac{dr}{r \sqrt{r^2 \left( e^{4\mu/r} - \frac{E_{00}^2}{E^2} e^{2\mu/r} \right) - k^2}} \quad (35)$$

(As the velocity of a particle or body approaches  $c$ , the quantity  $E_{00}^2/E^2$  vanishes and this becomes the light ray trajectory found previously.) Again, from the derivation of Eq. (22),  $k$  is a constant. Here it plays essentially the same role as conserved angular momentum in Newtonian mechanics.

To describe  $k$ , we can express  $n$  as a function of the orbital velocity  $v$ . Rewriting Eq. (19) in terms of  $c_0$  via Eq. (26), then substituting the resulting expression for  $V$  into Eq. (28) we get

$$n = \frac{v e^{4\mu/r}}{c_0} \quad (36)$$

Inserting this  $n$  into Eq. (23),  $k$  is given by

$$k = \frac{r v e^{4\mu/r} \sin \psi}{c_0} \quad (37)$$

where  $\psi$  is the angle between the orbital velocity vector and radial position vector.

To illustrate the observer problem in quantum mechanics, both Einstein and Wheeler have asked whether the Moon is there when no one looks. Regardless of the interpretation of quantum mechanics adopted, its usual rules apply here to large bodies. That includes de Broglie's requirement that orbital lengths correspond to integral numbers of wavelengths. For a body following the Moon's approximate orbit, the possible states are effectively infinite in number and indistinguishable. But *in principle* they're discrete, like an atomic electron's.

As in quantum optics, Huygens' principle could be used instead of Fermat's for calculating wave amplitudes and the probability a body goes from point A to point B in a gravitational field. Section 9 outlines how such amplitudes can be calculated using the basic methods of quantum electrodynamics, from the phases of bodies over various possible paths. (Preferably small bodies like neutrons.)

## 5 Predicted Frequency Shift

For metric theories of gravity, the PPN formalism of Will and Nordtvedt [27] provides a convenient way of checking them against an array of experimental tests, in terms of an isotropic space-time. Metric theories have come to be defined as any obeying the equivalence principle. However, even under that broad definition, this isn't a metric theory and the extent to which the PPN formalism can be applied isn't clear.

The alternative pursued in this paper is to compare the theory's predictions directly against each existing test of general relativity. While we'll use isotropic coordinates, in this case no conversion from curved to isotropic space-time is needed. (Of course measurements still need to be corrected for the condition of the measuring devices, to match those of observers removed from gravitational potentials.)

Although preferred-frame effects are possible in this theory, it can be shown they are small for existing Solar System experiments. For simplicity, this paper treats the Solar and Earth-Moon systems as being at rest in the preferred frame of the universe. (Corrections for their additional motions will be given in a subsequent paper.)

Where  $\nu$  represents the rate of a clock near a massive spherical body at rest, from Eqs. (13) and (25), the series expansion for the exponential gives

$$\nu = \nu_0 e^{-\mu/r} = \nu_0 \left( 1 - \frac{GM}{c_0^2 r} + \frac{1}{2} \left( \frac{GM}{c_0^2 r} \right)^2 + \dots \right) \quad (38)$$

Unlike this theory, gravitational potentials in general relativity don't vary simply as  $1/r$  in isotropic coordinates. (And are not analogous to electromagnetic potentials in this respect.) The rates of clocks also differ from Eq. (13) as functions of those potentials. *However, these differences effectively cancel, and in terms of the isotropic radius, the clock rates predicted by Einstein's final theory are the same to the second order* [28]. Both theories agree well with the best direct measurement of the gravitational frequency shift to date, by NASA's Gravity Probe A [29], which is only accurate to the first order in  $GM/(c^2 r)$ .

## 6 Precession of Mercury's Orbit

Mercury's orbit is given by Eq. (35). To solve that for the orbital precession, we can take just the first three terms of the series expansions for  $e^{4\mu/r}$  and  $e^{2\mu/r}$ , since higher powers of  $\mu/r$  are vanishing in the Solar System. The result is

$$\theta = k \int \frac{dr}{r \sqrt{r^2 \left(1 + \frac{4\mu}{r} + \frac{8\mu^2}{r^2} - \frac{E_{00}^2}{E^2} - \frac{2\mu E_{00}^2}{rE^2} - \frac{2\mu^2 E_{00}^2}{r^2 E^2}\right) - k^2}} \quad (39)$$

Multiplying terms by  $r^2$  inside the square root gives a quadratic there,

$$\theta = k \int \frac{dr}{r \sqrt{\left(1 - \frac{E_{00}^2}{E^2}\right) r^2 + \left(4\mu - 2\mu \frac{E_{00}^2}{E^2}\right) r + \left(8\mu^2 - 2\mu \frac{E_{00}^2}{E^2} - k^2\right)}} \quad (40)$$

We'll use these notations for the quadratic coefficients:

$$A = 1 - E_{00}^2/E^2 \quad (41)$$

$$B = 4\mu - 2\mu E_{00}^2/E^2 \quad (42)$$

$$C = 8\mu^2 - 2\mu^2 E_{00}^2/E^2 - k^2 \quad (43)$$

From these and a table of integrals, we get

$$\begin{aligned} \theta &= k \int \frac{dr}{r \sqrt{Ar^2 + Br + C}} \\ &= \frac{k}{\sqrt{-C}} \sin^{-1} \left( \frac{Br + 2C}{r \sqrt{B^2 - 4AC}} \right) \end{aligned} \quad (44)$$

Solving for  $r$  gives the time-independent equation for the orbit:

$$r = \frac{-2C/B}{1 - \sqrt{1 - 4AC/B^2} \sin \left( \theta \sqrt{-C/k^2} \right)} \quad (45)$$

This has the basic form of a polar equation of an ellipse,

$$r = \frac{a(1 - \epsilon^2)}{1 - \epsilon \sin \theta} \quad (46)$$

where  $a$  is the semi-major axis and  $\epsilon$  is the eccentricity. (Eq. (45) describes hyperbolic trajectories also.) In Eq. (45), the quantity corresponding to  $a(1 - \epsilon^2)$  (called the *parameter*, or  $p$ , in orbital mechanics) is given by

$$\begin{aligned} a(1 - \epsilon^2) &= -2C/B \\ &= \frac{-2(8\mu^2 - 2\mu^2 E_{00}^2/E^2 - k^2)}{4\mu - 2\mu E_{00}^2/E^2} \end{aligned} \quad (47)$$

Since Mercury's orbital velocity is effectively nonrelativistic,  $E_{00}^2/E^2$  is extremely close to one. Also, the value of  $\mu^2$  is minuscule compared to  $k^2$ . In this case, the last equation reduces to

$$a(1 - \epsilon^2) = k^2/\mu \quad (48)$$

Calling the total change in  $\theta$  from one minimum of  $r$  (perihelion) to the next  $\Delta\theta$ , it follows from Eq. (45) that

$$\Delta\theta\sqrt{-C/k^2} = 2\pi \quad (49)$$

Again, since the value of  $E_{00}^2/E^2$  is very nearly one, Eq. (43) becomes

$$C = 6\mu^2 - k^2 \quad (50)$$

Rearranging and making this substitution for  $C$  gives

$$\Delta\theta = 2\pi/\sqrt{1 - 6\mu^2/k^2} \quad (51)$$

We can take just the first two terms of the binomial expansion for the inverse square root, since higher powers of  $\mu^2/k^2$  are vanishing. This gives

$$\Delta\theta = 2\pi\left(1 + 3\mu^2/k^2\right) \quad (52)$$

Then substituting for  $k^2$  via Eq. (48), we arrive at

$$\Delta\theta = 2\pi + \frac{6\pi\mu}{a(1 - \epsilon^2)} \quad (53)$$

From the last term, the perihelion is shifted in the direction of orbital motion. This corresponds to Einstein's equation for the orbital precession [30], and agrees closely with the 43'' per century value observed for Mercury.

## 7 Deflection of Light

The gravitational frequency shift in Einstein's 1911 variable-speed-of-light theory [15] was

$$\nu = \nu_0\left(1 + \frac{\Phi_g}{c^2}\right) \quad (54)$$

which agrees with Eq. (13) to the first order. But there was no effect on  $\lambda$ , or the dimensions of measuring rods, corresponding to Eq. (14). Consequently, the speed of light in a gravitational potential was

$$c = c_0\left(1 + \frac{\Phi_g}{c^2}\right) \quad (55)$$

Einstein used a wave approach in his 1911 paper to derive the deflection of starlight by the Sun and Jupiter. From Huygens' principle and this expression for  $c$ , he obtained the first-order approximation

$$\alpha = \frac{2GM}{c^2 r} \quad (56)$$

where  $\alpha$  is the angular deflection in radians and  $r$  is the radial distance to the light ray at its closest point.

(This deflection agreed with Einstein's original equivalence principle, which held that the observed laws of physics in a uniform gravitational *field* are the same as those in an extended, accelerating reference frame. Later, he restricted this to infinitesimal regions of space, discarding the spatial framework. And in terms of isotropic coordinates, the gravitational bending of light is no longer equivalent to that in an accelerating reference frame.)

In this theory, from the series expansion for  $c$

$$c = c_0 \left( 1 + \frac{2\Phi_g}{c_0^2} + \frac{2\Phi_g^2}{c_0^4} + \dots \right) \quad (57)$$

Omitting the terms above first-order and using the same method, this  $c$  gives a deflection twice as large,

$$\alpha_1 = \frac{4GM}{c_0^2 r} \quad (58)$$

where  $\alpha_1$  represents the first-order deflection in radians. This is the first-order effect predicted by general relativity, about 1.''75 for a star near the Sun's limb.

Eqs. (22) and (27) give the exact path of a light ray near a massive body. Illustrating the close analogy between light and matter, the second-order deflection of light is given by the solution of Eq. (35) obtained for Mercury's orbit. For the constant  $k$ , we set  $r$  in Eq. (37) equal to the light ray's closest radial distance. We also set  $\sin \psi$  to 1 and the particle or body's velocity  $v$  to the speed of light given by Eq. (26).

With that  $k$ , Eqs. (41)-(44) give the hyperbolic ray trajectory in polar coordinates. (For light, the terms involving  $E_{00}^2/E^2$  disappear.) Setting  $r$  to infinity in Eq. (44),  $\theta$  is then the deflection for half the ray trajectory. The resulting first-order deflection for the complete trajectory again is that in Eq. (58). Taking the sign of  $\alpha_1$  as positive, the second-order term  $\alpha_2$  can be expressed as

$$\alpha_2 = -\frac{1}{2} \alpha_1^2 \quad (59)$$

when  $\alpha_1$  and  $\alpha_2$  are in radians. (Since  $\Phi_g$  is negative, the same relationship between first and second-order terms is seen in Eq. (57).) For a star near the Sun's limb,  $\alpha_2$  represents a decrease of about 7.4  $\mu$ arcseconds.

In contrast, general relativity predicts a second-order decrease of about 3.5  $\mu$ arcseconds [31]. It also predicts an additional deflection due to the Sun's rotation and dragging of space-time [32]. That would depend on the Sun's angular

momentum, which is uncertain, but the effect would be at least  $+0.7$  and  $-0.7$   $\mu$ arcseconds on opposite sides of the Sun. Such an effect is absent in this theory.

The proposed NASA/ESA experiment LATOR (Laser Astrometric Test Of Relativity) [33] would measure the Sun's gravitational deflection of light to high accuracy. Using triangular laser ranging, between two probes on the far side of the Sun and a laser transponder on the International Space Station, the expected accuracy is 1 part in  $10^3$  of general relativity's second-order light deflection – far more than needed to choose between these theories.

## 8 The Lunar Orbit

In many alternative theories of gravity, such as the Brans-Dicke theory, the predicted accelerations of bodies depend on their masses and binding energies. Nordtvedt pointed out in 1968 that if the accelerations of the Earth and Moon toward the Sun differed, there would be an additional oscillation in the Moon's orbit which could be measured by lunar laser ranging [34]. The oscillation amplitude exceeds a meter in some cases, and such theories have been ruled out by the range measurements, which are accurate now to the order of a few centimeters.

Einstein's theory also predicts a small difference in the accelerations, corresponding to a 3 cm oscillation [35], which is consistent with the lunar ranging data. This is due to the Moon's orbital motion around Earth, and without that, the post-Newtonian approximation of Einstein's theory predicts identical solar accelerations for the Earth and Moon in isotropic coordinates. That would be the case if the two bodies orbited the Sun separately at the same radius, and we will compare this theory's predictions for that example next.

To include the effects of a body's own mass in the equations of motion, we introduce a new variable  $P$ . This will represent the potential due to its mass, divided by  $c_0^2$  to make it dimensionless. We take  $P$  to describe the average potential seen by the body's various mass elements, treating it as a whole.

To find the Sun's effect on  $P$ , we express its value of  $\Phi_g/c_0^2$  again as  $-\mu/r$ . Putting that into Eq. (17), the rest mass of a nearby body is increased by the factor  $e^{3\mu/r}$ . Also, from Eq. (14), the body's radius decreases by  $e^{-\mu/r}$ . Since the potential contributed by each of its own mass elements is given by  $-Gm_0/r$ , the rest case for  $P$  is described by

$$P_0 = P_{00}e^{4\mu/r} \quad (60)$$

where  $P_{00}$  is the value when the body's velocity and Sun's potential are both zero. Then from the relativistic transformation of a scalar potential,

$$P = \frac{P_0}{\sqrt{1 - v^2/c^2}} = \frac{P_{00}e^{4\mu/r}}{\sqrt{1 - v^2/c^2}} \quad (61)$$

With the value of  $\Phi_g/c_0^2$  given now by  $P - \mu/r$ , we'll determine the velocity and acceleration of a body following Earth's orbit. The orbit has little eccentricity, and as done by Nordtvedt, we will treat its shape as circular. As in Section 6, we can

use conservation of energy to find the velocity, although the relationship between the total and rest energies will be slightly different.

In this case,  $E_{00}$  will represent the body's rest energy in the absence of the Sun's potential, where its own potential is still present. To put the general rest energy  $E_0$  in terms of  $E_{00}$ , we need to account for the difference in its own binding potential, in addition to that from the Sun. Eq. (32) then becomes

$$E_0 = E_{00} e^{(P_0 - P_{00} - \mu/r)} \quad (62)$$

Also, with the contributions of both the Sun and orbiting body to the total potential, Eq. (26) for the speed of light becomes

$$c = c_0 e^{2(P - \mu/r)} \quad (63)$$

Substituting these expressions for  $E_0$  and  $c$  into Eq. (30) gives the velocity

$$v = c_0 e^{2(P - \mu/r)} \left( 1 - \frac{E_{00}^2}{E^2} e^{2(P_0 - P_{00} - \mu/r)} \right)^{1/2} \quad (64)$$

From de Broglie's  $V = c^2/v$ , the corresponding wave velocity is

$$V = c_0 e^{2(P - \mu/r)} \left( 1 - \frac{E_{00}^2}{E^2} e^{2(P_0 - P_{00} - \mu/r)} \right)^{-1/2} \quad (65)$$

Again, the wavefronts are orthogonal to a trajectory, and are arranged radially around the Sun for a circular orbit.  $V$  increases exactly in proportion to  $r$ , and the condition for a circular orbit is

$$\frac{d}{dr} \left( \frac{V}{r} \right) = 0 \quad (66)$$

Taking the derivative, solving for  $E_{00}^2/E^2$ , then substituting for  $E_{00}^2/E^2$  in Eq. (64), the resulting velocity is

$$v = c_0 e^{2(P - \mu/r)} \left( \frac{\mu/r + r(dP_0/dr)}{1 - \mu/r + r(dP_0/dr) - 2r(dP/dr)} \right)^{1/2} \quad (67)$$

We will need an estimate for  $P$ , which depends in part on  $v$ . The result of the above equation is strongly influenced by the term  $r(dP_0/dr)$  in the numerator and the precise variation of  $P_0$ , but this is not the case for  $P$ . And since  $v$  for the Earth or Moon is almost the same as the velocity of a body with a negligible gravitational potential, we can use that to estimate  $P$ . In that case, Eq. (67) gives

$$v = c_0 e^{-2\mu/r} \left( \frac{\mu/r}{1 - \mu/r} \right)^{1/2} \quad (68)$$

Also,  $c = c_0 e^{-2\mu/r}$ . Replacing  $v$  and  $c$  in Eq. (61), we find

$$P \cong P_{00} e^{4\mu/r} \left( \frac{1 - \mu/r}{1 - 2\mu/r} \right)^{1/2} \cong P_{00} e^{9\mu/(2r)} \quad (69)$$

Hence

$$\frac{dP}{dr} \cong -\frac{9\mu P_{00}}{2r^2} e^{9\mu/(2r)} \quad (70)$$

From Eq. (60), the exact derivative of  $P_0$  is

$$\frac{dP_0}{dr} = -\frac{4\mu P_{00}}{r^2} e^{4\mu/r} \quad (71)$$

Putting these expressions for  $P$ ,  $dP/dr$ , and  $dP_0/dr$  into Eq. (67) gives  $v$  entirely in terms of  $c_0$ ,  $\mu$ ,  $r$  and  $P_{00}$ . Since these equations are already in isotropic coordinates and the orbit is circular, the acceleration then is just  $a = v^2/r$ .  $P_{00}$  is approximately twice the ratio of a body's gravitational binding energy to its total relativistic energy. Using  $-9.2 \times 10^{-10}$  and  $-0.4 \times 10^{-10}$  for the Earth and Moon respectively, these values of  $P_{00}$  give effectively identical accelerations at Earth's orbital radius, differing by only 1.9 parts in  $10^{17}$ .

From the changed speed of light alone, appearing as the term  $c_0 e^{2(P-\mu/r)}$  in Eq. (67),  $P$  would introduce an acceleration difference more than *eight orders of magnitude greater*. But that is canceled almost perfectly by the effect of change in the binding potential. (A similar cancelation occurs in the post-Newtonian approximation of Einstein's theory.) The residual difference in the accelerations translates [36] to an unobservable  $0.6 \mu\text{m}$  oscillation in the lunar orbit. An observable perturbation of the orbit is described in Section 11.

## 9 Lagrangian and Quantum Mechanics

In the tradition of metric theories of gravity, we could have begun by simply writing the Lagrangians for matter and light, and using those for calculations. But we also want to show their physical basis and the connection to quantum mechanics.

Compared to general relativity, the gravitational Lagrangian in this theory is more closely related to that in electromagnetism. Referring to Eq. (1), substituting  $\mathbf{v}dt$  for  $ds$ , the fundamental equation of electromagnetism can be rewritten as

$$\Delta S = \frac{q}{h} \int (\Phi - \mathbf{v} \cdot \mathbf{A} / c) dt \quad (72)$$

Again, over a possible path of a charged particle, this integral gives the cumulative shift in the de Broglie phase due to electromagnetic potentials. As described by Feynmann [10], summing the resulting phase amplitudes over all possible paths from point A to point B gives the total amplitude and probability of finding it at B.

We'll show that effect corresponds to the Lagrangian for a charged particle

$$L = -m_0 c^2 \sqrt{1 - v^2/c^2} - q(\Phi - \mathbf{v} \cdot \mathbf{A} / c) \quad (73)$$

Note this equation lacks the property of manifest covariance advocated by Einstein. (Einstein took the opposite position during one period [37], writing that the mathematical representation of gravity "cannot possibly be *generally* covariant.") Nevertheless, the expression is properly relativistic, and as Goldstein [38] has noted, no manifestly covariant formalism has been found for electrodynamics.



Again, de Broglie waves match a particle's phase at its moving position. Where  $v$  is the particle's velocity, its frequency there is

$$\nu_v = \nu' \sqrt{1 - v^2/c^2} \quad (74)$$

with  $\nu'$  the de Broglie frequency in the inertial frame where the particle is at rest. Using the Planck and Einstein relations, and the relativistic transformation for the potential, the particle's energy in the primed frame can be expressed as

$$\begin{aligned} h\nu' &= m_0c_0^2 + q\Phi' \\ &= m_0c_0^2 + q(\Phi - \mathbf{v} \cdot \mathbf{A} / c) / \sqrt{1 - v^2/c^2} \end{aligned} \quad (75)$$

Dividing both sides by  $h$ , then substituting for  $\nu'$  in the previous equation, we get

$$\nu_v = \frac{m_0c_0^2}{h} \sqrt{1 - v^2/c^2} + \frac{q}{h} (\Phi - \mathbf{v} \cdot \mathbf{A} / c) \quad (76)$$

corresponding to the phase shift  $\Delta S$  in Eq. (72).

Differing only by the constant factor  $-h$ , this and the Lagrangian in Eq. (73) represent *the same quantity*, expressed in frequency or energy terms. Like the integral of the Lagrangian (action), the variation of the de Broglie wave phase  $S$  near a particle trajectory is zero. (Corresponding to orthogonal wavefronts.) As noted by Lanczos [21], either gives the same trajectory between two fixed points in space.

From Eq. (13), in a gravitational potential, a body's de Broglie frequency at its moving position becomes

$$\nu_v = \left( \frac{m_{00}c_0^2}{h} \sqrt{1 - v^2/c^2} + \frac{q_0}{h} (\Phi_0 - \mathbf{v} \cdot \mathbf{A}_0 / c) \right) e^{\Phi_g/c_0^2} \quad (77)$$

where  $q_0$ ,  $\Phi_0$  and  $\mathbf{A}_0$  are the values of  $q$ ,  $\Phi$  and  $\mathbf{A}$  for a zero gravitational potential. Without electromagnetic potentials, the corresponding gravitational phase shift is

$$\Delta S = \frac{m_{00}c_0^2}{h} \int \sqrt{1 - v^2/c^2} (e^{\Phi_g/c_0^2} - 1) dt \quad (78)$$

which can be used to calculate probability amplitudes. *For weak fields and slow velocities, this reduces to Eq. (3), which describes the neutron interference experiment of Colella, Overhauser and Werner [11].*

Multiplying the frequency of Eq. (77) by  $-h$  gives the Lagrangian for this theory

$$L = \left( -m_{00}c_0^2 \sqrt{1 - v^2/c^2} - q_0 (\Phi_0 - \mathbf{v} \cdot \mathbf{A}_0 / c) \right) e^{\Phi_g/c_0^2} \quad (79)$$

In the absence of electromagnetic potentials, this is the same Lagrangian found by Puthoff [16]. Here the full Lagrangian has the property that, for any system of particles in a uniform gravitational potential, all particles experience the same

relative effects. And special relativity is recovered in the limit where gravitational fields go to zero, as in general relativity.

The Euler-Lagrange equations of motion are given, as in electrodynamics, by

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) = 0 \quad (80)$$

where (rather than charge)  $q_i$  represents a generalized coordinate in isotropic space. Appendix B shows these give the same orbital equations obtained previously from de Broglie wave optics.

## 10 Acceleration Equation

With no electromagnetic potentials, the gravitational Lagrangian can be written as

$$L = -E_{00} e^{\Phi_g/c_0^2} \sqrt{1 - (v^2/c_0^2)} e^{-4\Phi_g/c_0^2} \quad (81)$$

expressing the term  $m_{00}c_0^2$  as  $E_{00}$ , and substituting for  $c$  from Eq. (10). As usual, the canonical momentum  $\mathbf{p}$  is the partial derivative with respect to velocity (see Duffey [39])

$$\mathbf{p} = \frac{\partial L}{\partial \mathbf{v}} = \frac{\mathbf{v} E_{00} e^{-3\Phi_g/c_0^2}}{c_0^2 \sqrt{1 - (v^2/c_0^2)} e^{-4\Phi_g/c_0^2}} \quad (82)$$

From Eqs. (29) and (32),

$$E_{00} = E e^{-\Phi_g/c_0^2} \sqrt{1 - (v^2/c_0^2)} e^{-4\Phi_g/c_0^2} \quad (83)$$

Putting this into the preceding equation gives

$$\mathbf{p} = \frac{\mathbf{v} E e^{-4\Phi_g/c_0^2}}{c_0^2} \quad (84)$$

$E$  is a constant for conservative systems, and the time derivative of  $\mathbf{p}$  is

$$\frac{d\mathbf{p}}{dt} = \frac{E e^{-4\Phi_g/c_0^2}}{c_0^2} \left( \frac{d\mathbf{v}}{dt} - \frac{4\mathbf{v} d\Phi_g}{c_0^2 dt} \right) \quad (85)$$

To find the gradient of a Lagrangian,  $v$  is held constant. Taking the gradient of Eq. (81) and then substituting for  $E_{00}$  from Eq. (83) we get

$$\nabla L = \frac{E e^{-4\Phi_g/c_0^2}}{c_0^2} \left( -\nabla\Phi_g e^{4\Phi_g/c_0^2} - \frac{v^2}{c_0^2} \nabla\Phi_g \right) \quad (86)$$

From the relation  $d\mathbf{p}/dt = \nabla L$  [39], we can equate the right sides of the last two equations, eliminating the conserved  $E$ . Then solving for  $d\mathbf{v}/dt$  gives

$$\mathbf{a} = -\nabla\Phi_g \left( e^{4\Phi_g/c_0^2} + \frac{v^2}{c_0^2} \right) + \frac{4\mathbf{v}}{c_0^2} \left( \frac{d\Phi_g}{dt} \right) \quad (87)$$

where  $dv/dt$  is written as the acceleration  $\mathbf{a}$ . This is the gravitational counterpart of the Lorentz force equation in electromagnetism, describing the relativistic acceleration of bodies in gravitational fields. (Multiplying both sides by the same mass would give the force.) Note there are effects here that depend on a body's relative velocity, as in electromagnetism.

To describe many body systems in general relativity, it's customary to use a many-body Lagrangian [40]. Here, this acceleration equation can be used instead. *Like the Lorentz force equation, it can be employed to describe systems of any type, including those that don't conserve energy.*

It's easily shown the equation agrees with Eq. (68) for a body in a circular orbit. We have  $\mathbf{a} = -v^2/r$ ,  $\nabla\Phi_g = c_0^2\mu/r^2$ ,  $\Phi_g/c_0^2 = -\mu/r$  and  $d\Phi_g/dt = 0$ . Eq. (87) becomes

$$-\frac{v^2}{r} = -\frac{c_0^2\mu}{r^2} \left( e^{-4\mu/r} + \frac{v^2}{c_0^2} \right) \quad (88)$$

and solving for  $v$  gives Eq. (68). It can also be shown to agree with the general equations of motion for orbiting test bodies given in Appendix B.

## 11 Gravitomagnetism

Shahid-Saless [35] has shown that, for a Fermi (geocentric) reference frame, general relativity predicts a gravitomagnetic interaction between the Moon and Sun. Such an effect appears to have been detected the lunar ranging experiment, and we'll investigate this theory's prediction for that experiment next.

The ranging experiment gauges the lunar distance by the time-of-flight of laser pulses originating from an Earth observatory and returning from one of four reflectors on the Moon (positioned by Apollo astronauts and the Russian Lunakhod II spacecraft). It provides a comparison of the distances at the new and full Moons. We'll estimate the predicted distances at those points in the lunar orbit. To compare our estimates with the ranging experiment, we'll also need to account for the effect of the Sun's gravity on  $c$  and a laser pulse's time-of-flight.

Newtonian mechanics provides a remarkably accurate description of the lunar orbit, and here we want to describe lunar distances with respect to their corresponding Newtonian values. Like Shahid-Saless, we'll use a geocentric inertial frame, where the Sun is in relative motion. As done by Nordtvedt, we'll treat the lunar orbit as lying in the plane of the ecliptic.

For simplicity, we'll also take the Moon's velocity component in the Sun's direction to be zero at the new and full Moons. The term  $d\Phi_g/dt$  in Eq. (87) vanishes and, for those points in the lunar orbit, the Moon's acceleration due to the Sun's potential alone can be written as

$$a_s = \frac{GM}{R^2} \left( e^{-4GM/(c_0^2R)} + \frac{v^2}{c_0^2} \right) \quad (89)$$

where  $M$  is the solar mass,  $R$  is the Sun-Moon (not Sun-Earth) distance,  $v$  is the lunar velocity, and acceleration toward the Sun is defined as positive. (This uses

the slow-motion approximation for the Sun's potential, which will not affect our results.)

At the full Moon, the gradient of Earth's potential is in the same direction as the Sun's, and when the effects of Earth's potential are included we obtain

$$a_{s+e} = \left( \frac{GM}{R^2} + \frac{Gm}{r^2} \right) \left( e^{-\frac{4GM}{c_0^2 R} - \frac{4Gm}{c_0^2 r}} + \frac{v^2}{c_0^2} \right) \quad (90)$$

where  $m$  is Earth's mass, and  $r$  is the Earth-Moon distance. After expanding the exponentials in the last two equations, the ratio of the lunar accelerations with and without Earth's potential can be expressed accurately as

$$\frac{a_{s+e}}{a_s} \cong \frac{\frac{GM}{R^2} + \frac{Gm}{r^2}}{\frac{GM}{R^2}} \left( 1 - \frac{4Gm}{c_0^2 r} \right) \quad (91)$$

This equation differs from the corresponding Newtonian ratio only in the last term. We'll assume that, for a given  $R$ , when this ratio of accelerations equals the Newtonian value, the observed curvature of the lunar orbit is the same. Then, where  $r_0$  is the Newtonian Earth-Moon distance, we ask what value of  $r$  gives the same curved trajectory relative to the Sun's position. We'll write

$$\frac{GM}{R^2} + \frac{Gm}{r_0^2} = \left( \frac{GM}{R^2} + \frac{Gm}{r^2} \right) \left( 1 - \frac{4Gm}{c_0^2 r} \right) \quad (92)$$

where we have put the Newtonian acceleration ratio on the left side, Eq. (91) on the right, and multiplied both sides by  $GM/R^2$ . In the small last term of this equation,  $r_0$  can be substituted for  $r$ , since they are nearly identical. Then solving for  $r$  gives

$$r \cong r_0 - \frac{2GM r_0^2}{c_0^2 R^2} - \frac{2Gm}{c_0^2} \quad (93)$$

At the new Moon, the gradient of Earth's potential has the opposite sign and Eq. (92) becomes

$$\frac{GM}{R^2} - \frac{Gm}{r_0^2} = \left( \frac{GM}{R^2} - \frac{Gm}{r^2} \right) \left( 1 - \frac{4Gm}{c_0^2 r} \right) \quad (94)$$

where the solution for  $r$  is now

$$r \cong r_0 + \frac{2GM r_0^2}{c_0^2 R^2} - \frac{2Gm}{c_0^2} \quad (95)$$

Alternatively, the same results can be obtained using the lunar acceleration with and without the Sun's potential, for a given lunar trajectory around Earth. Eq. (91) becomes  $a_{s+e}/a_e$ , the Newtonian Sun-Moon distance  $R_0$  and  $r$  are given, and  $R$  is solved for. Translating the difference found in  $R$  to one in  $r$  having an equal effect on the lunar acceleration, Eqs. (93) and (95) are arrived at again.

In Eq. (93) for the full Moon, we'll take  $r_0$  equal to the mean Earth-Moon distance, and  $R$  to the mean Sun-Earth distance plus  $r_0$ . Keeping the terms separate, the radius of the lunar orbit in centimeters is then

$$r \cong r_0 - 1.9 - 0.9 \quad (96)$$

In Eq. (95) for the new Moon,  $R$  becomes the mean Sun-Earth distance minus  $r_0$ , and we get

$$r \cong r_0 + 2.0 - 0.9 \quad (97)$$

From the middle terms, the Moon is 1.9 cm closer to Earth than its Newtonian distance when full, and 2.0 cm more remote at the new Moon, corresponding to a shift (polarization) of the lunar orbit of about 2 cm toward the Sun. (The final  $-0.9$  cm terms only decrease the size of the orbit.)

The ranging experiment also sees an *apparent* shift in the lunar orbit, due to variation in the speed of light over the paths of the reflecting laser pulses. At the full Moon's location, the Sun's potential is weaker by approximately  $GMr/R^2$  than at Earth's position, and a pulse's propagation speed is increased, in accord with Eq. (10). Averaging the relative speed of light over the beam path and translating that to a decrease in the apparent lunar distance, the result is

$$\Delta r_a \cong -\frac{GMr^2}{c_0^2 R^2} \cong -1.0 \quad (98)$$

The effect at the new Moon is the opposite, giving a 1.0 cm increase in the apparent distance, and the equivalent of an additional 1 cm shift of the lunar orbit toward the Sun.

For Einstein's theory, Nortvedt [41] obtains an identical "effective range perturbation" due to the Sun's influence on light propagation. As he notes, in terms of isotropic coordinates, the actual effects of that influence are changes in the pulse propagation times. The delay of a laser pulse reflected at the new Moon represents the well-known "Shapiro effect," and is indistinguishable here from that in general relativity.

Combining the actual and apparent perturbations of the Newtonian lunar orbit, Shahid-Saless [35] and Nortvedt [42] obtain a total shift of 3 cm for Einstein's theory. The same result is obtained here. Using almost identical sets of ranging data, Williams, Newhall and Dickey [43] and Müller and Nordtvedt [36] find that agrees with the observed orbital shift to  $-0.8 \pm 1.3$  cm and  $+1.1 \pm 1.1$  cm respectively.

For gravitomagnetism affecting the Moon's trajectory, the significant motions of the interacting bodies are effectively in a single plane. That isn't so for a satellite in a polar orbit near the rotating Earth, and there the predictions of Einstein's theory and this one diverge.

According to Einstein, gravitational potentials don't act directly on particles or their de Broglie waves, but on the space-time they reside in. In addition to the curvature caused by mass-energy, gravitomagnetism is assumed to result from the dragging of space-time by mass-energy currents. As first described by Lense and

Thirring [44], this would involve a local rotation of space-time near a rotating, massive body.

In the low-speed, weak-field limit of general relativity, the effect of space-time rotation on a test body's trajectory can also be described in terms of a gravitational vector potential and a corresponding gravitomagnetic field [45]. For an Earth satellite in a polar orbit, the field induces a varying acceleration perpendicular to the satellite's orbital plane, and a precession of that plane in the direction of Earth's rotation.

This theory (currently) has no gravitational vector potential, and no full equivalent of the gravitomagnetic field. Looking at the effects of Earth's potential given by Eq. (87), one component of a satellite's acceleration is directed along the potential gradient – toward Earth's center. The other, containing  $\mathbf{v}$ , is in the direction of the satellite's motion. For a perfectly spherical Earth, that equation gives no accelerations perpendicular to the orbital plane, or orbital precession due to Earth's rotation.

A controversial analysis of the LAGEOS and LAGEOS II satellite orbits by Ciufolini and Pavlis [46] finds a Lense-Thirring precession 99% of general relativity's, with  $\pm 10\%$  error. (The effect is tiny compared to others such as those of Earth and ocean tides, which must be estimated with great accuracy.) Iorio [47] finds their calculations flawed and that no reliable measurement was made. A definitive measurement of Lense-Thirring precession is expected from NASA's Gravity Probe B, launched in April 2004. That experiment reads the orientation of a sensitive gyroscope carried by a satellite in polar orbit, with respect to a selected star [45].

Gravity Probe B can be regarded as a test of Mach's principle – a pillar of Einstein's theory but not this. According to the former, the gyroscope will rotate with its local inertial frame, and is predicted to undergo a Lense-Thirring precession of .042 arcseconds/year. In this theory, since there are no rotating inertial frames, a null measurement of the Lense-Thirring effect is predicted. (The probe will also measure de Sitter precession, where the gyroscope's direction within the satellite's orbital plane changes [45]. There the predictions of the two theories are identical.)

## 12 Gravitational Radiation

The strongest evidence of gravitational waves at present is the orbital decay of the binary pulsar, PSR 1913+16. This star orbits its companion with a short period of 7.75 hours, in a highly eccentric orbit ( $\epsilon = 0.617$ ). Since pulsars have the regularity of atomic clocks, the orbital period and other parameters of the system can be determined very accurately from pulse arrival times. As shown by Weisberg and Taylor [48], the observed orbital period shift agrees closely with the gravitational radiation damping predicted by general relativity.

It's assumed the stars suffer no tidal effects and the surrounding space is free of matter, so the loss of orbital energy is due entirely to gravitational waves. The loss is calculated from an equation derived by Peters and Mathews [49]. That gives the average power radiated by point masses in Keplerian orbits, based on the weak-field, slow-motion approximation of Einstein's gravitation. Acknowledging dissen-

sion on this point, Peters and Mathews assume the energy carried by conventional gravitational waves is real and positive, citing the analogy to electromagnetism.

Several parameters needed for the calculation, the stellar masses and projected axis of the pulsar orbit, are not directly available from the pulsar data. Taylor and Weisberg obtain the missing parameters by solving three simultaneous equations, including one for the periastron advance, and another for the combined transverse Doppler and gravitational frequency shifts. (The equations for those effects are the same here.) The resulting energy loss has been found to agree with the observed orbital period shift to an accuracy of about 0.4% [50].

Feynman [51] has pointed out that, even in classical electromagnetism, there is ambiguity in the usual derivation of radiated energy. The change in field energy for an infinitesimal volume of space is described by

$$-\frac{\partial u}{\partial t} = \nabla \cdot \mathbf{S} + \mathbf{E} \cdot \mathbf{j} \quad (99)$$

where  $u$  is the field energy density,  $\mathbf{S}$  is the field energy flow normal to the volume's surface,  $\mathbf{E}$  the electric field and  $\mathbf{j}$  is the electric current density.

As done originally by Poynting, when  $u$  and  $\mathbf{S}$  are defined as

$$u \equiv \frac{\epsilon_0}{2} \mathbf{E} \cdot \mathbf{E} + \frac{\epsilon_0 c^2}{2} \mathbf{B} \cdot \mathbf{B} \quad (100)$$

and

$$\mathbf{S} \equiv \epsilon_0 c^2 \mathbf{E} \times \mathbf{B} \quad (101)$$

Maxwell's equations can be used to show these expressions agree with Eq. (99). However, Feynman notes that the latter is also satisfied by other combinations of  $u$  and  $\mathbf{S}$ , infinite in number, and we have no way of proving which is correct. Eqs. (100) and (101) are believed correct because they are the simplest pair of expressions in agreement with experiment.

There is no ambiguity in Eq. (99). From energy conservation, the work done by an electric field on a stream of charged particles with constant velocity must be radiated accordingly. (See Feynman [51].) To arrive at the gravitational counterpart of that equation, we'll consider the work done on a stream of particles with constant velocity in a gravitational field.

From Eq. (83), the energy (Hamiltonian) for a particle in a gravitational potential is

$$E = \frac{m_{00}c_0^2}{\sqrt{1 - v^2/c^2}} e^{\Phi_g/c^2} \quad (102)$$

For weak potentials and low velocities, that can be approximated as

$$E \cong \frac{m_{00}c_0^2}{\sqrt{1 - v^2/c^2}} + m_{00}\Phi_g \quad (103)$$

Then, for a single particle entering a gravitational potential at a constant velocity, the energy lost to radiation is given by

$$\Delta E \cong m_{00}\Phi_g \quad (104)$$

In addition to the particle's energy, we'll need to account for energy associated with its surrounding potential. Again, a gravitational potential represents a region of diminished vacuum energy. (From the extended principle of relativity, vacuum energy diminishes along with that of ordinary particles. If not, a measurement of the Casimir effect wouldn't give the same result in a uniform gravitational potential.)

In electromagnetism, the potential due to a uniformly moving charge doesn't change when it enters a different external potential. But there *is* such a change for a particle entering a gravitational potential. From Eq. (17), its rest mass  $m_0$  increases as  $e^{-3\Phi_g/c_0^2}$ . And from Eq. (6), its own potential strengthens proportionately. (Section 8 showed this effect also influences the lunar orbit.)

The resulting loss of vacuum energy is likewise proportional to the increase in the particle's rest mass. Where  $\Delta E_\Phi$  is the change in vacuum energy due to the particle's gravitational potential, and the factor of proportionality between mass and energy is the usual  $c^2$ , that be expressed as

$$\Delta E_\Phi = -(\Delta m_0)c^2 \quad (105)$$

For weak potentials, the transformation of rest mass then gives

$$\Delta E_\Phi \cong 3m_{00}\Phi_g \quad (106)$$

According to the gravitational wave equation, Eq. (9), as a particle's rest mass increases, the change in its potential propagates outward from the particle at the local speed of light. Where the gravitational potential strengthens and vacuum energy diminishes, the principle of energy conservation requires a corresponding energy flow out of the region. And from the last equation, the amount is three times the flow of energy lost by the particle itself to radiation.

From the combined effects described in Eqs. (104) and (106), the radiation for a stream of particles with constant velocity in a weak gravitational field is given by

$$-\frac{\partial u}{\partial t} \cong \nabla \cdot \mathbf{S} - 4\nabla\Phi_g \cdot \mathbf{j}_m \quad (107)$$

where  $\mathbf{j}_m$  is the mass current density. The value here is four times that suggested by the electromagnetic analogue, Eq. (99), when charge is replaced by rest mass in the current density and the gradient of the potential is opposite in sign. The factor of four difference arises because the equivalent of charge is not conserved.

General relativity predicts no dipole gravitational radiation, and Misner, Thorne and Wheeler [52] note this would also be the case for a gravity theory analogous to electromagnetism. There the reason is that the center of mass of a gravitationally bound star system doesn't accelerate. Consequently, the dipole contributions of its individual stars cancel.

However, dipole radiation is possible in scalar-tensor gravity theories, such as Brans-Dicke, which violate the strong equivalence principle. In those, a body's acceleration depends partly on its gravitational binding energy. If the stars in a binary system have different binding energies, its inertial and gravitational centers



of mass will lie at slightly different points. And its gravitational center will circulate as the stars orbit, resulting in dipole radiation. Arzoumanian [53] finds that, for a pulsar-white dwarf binary, where the binding energies differ markedly, the effects of such dipole radiation may be observable. He doesn't find that to be the case for PSR 1913+16, whose companion is another neutron star.

Unlike the Brans-Dicke theory, this one effectively obeys the strong equivalence principle – at least for slow motion and weak potentials. (As shown in Section 8, the solar accelerations of the Moon and Earth are effectively identical, despite the difference in their binding energies.) Hence measurable dipole radiation seems unlikely, even for a pulsar-white dwarf pair.

Misner, Thorne and Wheeler [52] note the quadrupole gravitational radiation predicted by general relativity is four times stronger than suggested by the analogy to electromagnetism. Since the same factor of four difference exists in this theory, there appears to be similar agreement with the gravitational radiation damping observed in PSR 1913+16.

### 13 A Different Cosmology

Is the Riemann geometry of Einstein's general relativity necessarily real? Again, Poincaré held that nature singles out the simplest of geometries, the Euclidean [20].

A Riemann geometry like that conceived by Einstein can also be used to describe ordinary optical systems [54]. Similarly, the speed of light is treated as absolute, while the "optical path distance" varies according to the refractive index. Although it's possible to solve optics problems this way, of course measurements with meter sticks show the true geometry of ordinary optical systems is Euclidean. Measurements of the distribution of galaxies [3] appear to be saying the same for the geometry of the universe.

While the current geometric interpretation of general relativity rests on an absolute speed of light in vacuo, that isn't the case here. Besides gravitation without space-time curvature, this permits an alternate basis for the Hubble redshift: a decreasing value of  $c$ . Since the frequencies and wavelengths of de Broglie waves depend on it, the wavelengths of atomic spectra would also be shifted.

In accord with Eq. (10), the diminishing speed of light is taken to represent a gradual strengthening of the universe's overall gravitational potential. (This effect may be attributable in part to a transfer of energy between different scales in the universe [12].) The extension of Poincaré's relativity principle to uniform gravitational potentials is also taken to hold again. Consequently, while  $c$  is evolving, the locally observed value and laws of physics don't change.

From Eqs. (13) and (14), the de Broglie frequencies and wavelengths of particles diminish over time. I.e., the rates of clocks are slowing, and lengths of meter sticks are contracting. For local observers, who see no change in their units of length, the result is an apparent expansion of the universe.

Particle rest masses also increase, from Eq. (17). That brings a cosmological instability, not unlike the one in Einstein's theory which once compelled him to introduce a cosmological constant. While the increasing magnitude of the universe's

overall gravitational potential drives the masses of its particles higher, that in turn drives the potential's magnitude up.

Suppose this cosmological potential has the value zero when light leaves a distant galaxy, and  $\Phi_g$  when it reaches Earth. Then  $c$  diminishes by the factor  $e^{2\Phi_g/c_0^2}$  over that time. However, no difference arises in the relative speeds of two successive wavefronts, so there is no shift in the light's absolute wavelength. An observer on Earth sees an apparent redshift, however, because the wavelength of a spectral reference shortens by the factor  $e^{\Phi_g/c_0^2}$ . To the first order in  $\Phi_g/c_0^2$ , the relative wavelength observed is

$$\lambda \cong \lambda_0 (1 - \Phi_g/c_0^2) \quad (108)$$

where  $\lambda_0$  is the wavelength of a local spectral reference.

For low redshifts, the empirical relationship is

$$\lambda \cong \lambda_0 (1 + Hd/c) \quad (109)$$

where  $H$  is the Hubble constant and  $d$  a galaxy's distance. From measurements using the Hubble Space Telescope [5],  $H$  has been estimated at 71 km/sec/Mpc. Equating the final terms of the last two equations gives

$$Hd/c = Ht = -\Phi_g/c_0^2 \quad (110)$$

where  $t$  is the time light travels from a galaxy to Earth. And the resulting rate of change for the cosmological potential is

$$\frac{d}{dt} \left( \frac{\Phi_g}{c_0^2} \right) = -H \quad (111)$$

(In principle, this cosmological redshift could be measured in a two-beam interferometer with arms of unequal lengths. However, if the interferometer's components are physically connected, the arms will contract as the cosmological potential strengthens. And the resulting displacement of its mirrors will cause a Doppler blueshift which cancels the redshift.)

Because the slowing of light signals is about twice that of clocks, an apparent time dilation is also seen in distant objects. Like the Big Bang model, this agrees with the observed lifetimes of type Ia supernovae, proportional to their redshifted wavelengths [55]. No attempt has been made yet to model the cosmic microwave background. (Since this is not a steady-state cosmology, this will involve inferring conditions in the remote past, and at distances where individual objects are not resolved.) However we know at least that the sky between resolvable objects would be filled with longer-wavelength radiation from more distant ones.

As indicated by Eq. (26), here the speed of light near a massive body is always positive in isotropic coordinates, and light rays can always escape in the radial direction. Hence the event horizons and black holes predicted by current general relativity don't occur (Einstein denied their existence), and there is no problem of information loss. (For supermassive bodies such as galactic nuclei, a subsequent paper will show the fraction of emitted light escaping is close to zero.)

A survey of galactic black hole candidates by Robertson [56] finds a strong resemblance to neutron stars in all cases (with the exception of the galactic nucleus), and no evidence of event horizons. He also finds the fluctuating X-ray spectra of galactic candidates agree well with a model where infalling matter meets a hard surface, rather than falling through an event horizon.

This gravity theory bears some mathematical resemblance to the exponential metric theory of Yimaz [57], where likewise there are no event horizons. (The Yilmaz theory is also renormalizable [58].) That was the basis of an early quasar model by Clapp [59], in which quasars are represented as massive star-like objects. Those avoid the collapse predicted by the Schwarzschild metric, and would show high gravitational redshifts. Cumulative observations by Arp [60], of quasars associated with disturbed, low-redshift galaxies, appear to support such a model.

## 14 Conclusions

John Wheeler has urged general relativity be “battle tested” against fundamentally different theories of gravity. But it seems many believe this is no longer necessary. Measurements of general relativistic effects, such as gravitational bending of starlight, are often cited as proof of space-time curvature. The implicit assumption is that no alternative is possible. The theory described here suggests this assumption is unjustified.

As in electromagnetism, we can attribute gravitation to the direct influence of potentials on quantum-mechanical waves. Unlike the “standard” Big Bang model based on Einstein’s theory, this one agrees with the flat large-scale geometry of the universe observed and permits stars with ages well above 13 Gyr. And, unlike general relativity, this theory is immediately compatible with quantum mechanics. This calls into question the need for a curved space-time, its great mathematical complexity, and many degrees of freedom.

In the upcoming results from the satellite experiment Gravity Probe B, general relativity calls for a Lense-Thirring precession of the gyroscope, while a null effect is predicted here. Measurement of the second-order solar deflection of light in the LATOR experiment could also distinguish clearly between the two theories.

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## Appendix A: Derivation of c

In accord with Einstein's view that the theory of gravity should be derived from a set of general principles, here we'll derive Eq. (10) for the speed of light. These are the principles adopted:

- 1) Absolute space and time (as in the preferred-frame relativity of Lorentz and Poincaré [17]).
- 2) Superposition of gravitational potentials.
- 3) Relativity also holds for uniform gravitational potentials; i.e., the observed laws of physics remain unchanged.

We'll use an Einstein-style *Gedanken* experiment: Imagine a clock enclosed in a spherical shell of matter, where there is a uniform gravitational potential. The clock can be seen from outside through a small window, and an observer with a second clock sits at a distant location, where the potential due to the shell is negligible. We want to know how the observed clock rates compare.

The rate of any clock is determined by the de Broglie frequencies of its constituent particles; hence, we can use the frequency of a representative particle to gauge the relative rates. From quantum mechanics, the energy of a particle at rest in a *weak* gravitational potential is described by

$$h\nu = m_{00}c_0^2 + m_{00}\Phi_g \quad (112)$$

where  $\nu$  is the frequency and  $m_{00}$  refers to the mass when both the velocity and gravitational potential are zero. Dividing the particle energy at the inner clock's position by that at the outer clock (where  $\Phi_g$  vanishes) we find the relative rates are given by

$$\frac{\nu}{\nu_0} = 1 + \frac{\Phi_g}{c_0^2} \quad (113)$$

when the gravitational potential is weak.

Next, we enclose everything in a second mass shell, positioning another observer and clock outside that, where *its* potential is vanishing. We also insure that the contribution of the second shell to the cumulative potential is equal. By our third principle, the frequency ratio measured by the inner observer is unchanged, and those measured by the two observers are identical. Numbering our clocks from the outside, the subscript 0 will indicate the outermost, and 2 the inner one. We have

$$\frac{\nu_1}{\nu_0} = \frac{\nu_2}{\nu_1} = 1 + \frac{1}{2} \frac{\Phi_g}{c_0^2} \quad (114)$$

where  $\Phi_g$  represents the cumulative potential from both shells. Hence the frequency ratio for the innermost vs. the outermost clock is

$$\frac{\nu_2}{\nu_0} = \left(1 + \frac{1}{2} \frac{\Phi_g}{c_0^2}\right)^2 \quad (115)$$

The process can be repeated indefinitely, and for  $n$  shells, we get

$$\frac{\nu_n}{\nu_0} = \left(1 + \frac{1}{n} \frac{\Phi_g}{c_0^2}\right)^n \quad (116)$$

The effect of a *strong* gravitational potential can be represented as that of an infinite series of mass shells, each producing a weak contribution to the total potential. Taking the limit as  $n$  goes to infinity, we find

$$\frac{\nu}{\nu_0} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \frac{\Phi_g}{c_0^2}\right)^n = e^{\Phi_g/c_0^2} \quad (117)$$

As shown by the Collela-Overhauser-Werner experiment, de Broglie wavelengths also diminish in a weak gravitational potential as

$$\frac{\lambda}{\lambda_0} = 1 + \frac{\Phi_g}{c_0^2} \quad (118)$$

For a strong potential, a similar *Gedanken* experiment measuring wavelengths gives

$$\frac{\lambda}{\lambda_0} = e^{\Phi_g/c_0^2} \quad (119)$$

The general velocity  $V$  of quantum-mechanical waves is given by  $V = \lambda \nu$ . And from the extended principle of relativity,  $c$  and  $V$  change identically. Hence

$$\frac{c}{c_0} = \frac{V}{V_0} = e^{2\Phi_g/c_0^2} \quad (120)$$

Einstein: “When the answer is simple, God is talking.”

## Appendix B: Equations of Motion

Here we’ll illustrate the use of the Lagrangian introduced in Section 9, by deriving the equations of motion for a body in a central gravitational field. And we’ll show they agree with the equations for Mercury’s orbit found earlier by the de Broglie wave method.

Again, the inertial frame will be that where the Sun or central body is at rest. Putting Eq. (81) in terms of  $\mu$  and polar coordinates, the Lagrangian becomes

$$L = -E_{00} e^{-\mu/r} \sqrt{1 - e^{4\mu/r} (\dot{r}^2 + r^2 \dot{\theta}^2)} / c_0^2 \quad (121)$$

where the dots indicate time derivatives. Taking  $\theta$  as the generalized coordinate, the Euler-Lagrange equation of motion is

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) = 0 \quad (122)$$

For  $\partial L/\partial \dot{\theta}$ , we obtain

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{r^2 \dot{\theta} E_{00} e^{3\mu/r}}{c_0^2 \sqrt{1 - e^{4\mu/r} (\dot{r}^2 + r^2 \dot{\theta}^2)}} / c_0^2 \quad (123)$$

The square root in the denominator represents the quantity  $\sqrt{1 - v^2/c^2}$ , and from Eqs. (29) and (32)), that can be expressed as

$$\sqrt{1 - e^{4\mu/r} (\dot{r}^2 + r^2 \dot{\theta}^2)} / c_0^2 = \frac{E_{00} e^{-\mu/r}}{E} \quad (124)$$

Since the term  $\partial L/\partial \theta$  in Eq. (122) is zero, we also have

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) = 0 \quad (125)$$

From the last two equations, Eq. (123) can be written as

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{r^2 \dot{\theta} E e^{4\mu/r}}{c_0^2} = C \quad (126)$$

where  $C$  is a constant.

Again, from the principle of energy conservation,  $E$  is a constant for a freely orbiting body. And dividing by  $E/c_0$  gives

$$\frac{r^2 \dot{\theta} e^{4\mu/r}}{c_0} = k \quad (127)$$

where  $k$  represents another constant. This is our earlier Eq. (37), in polar coordinates. Rearranging, the time derivative of  $\theta$  is

$$\dot{\theta} = \frac{k c_0 e^{-4\mu/r}}{r^2} \quad (128)$$

Putting this expression for  $\dot{\theta}$  into Eq. (124) and solving for  $\dot{r}$ , we also obtain

$$\dot{r} = \frac{c_0 e^{-4\mu/r} \sqrt{r^2 \left( e^{4\mu/r} - \frac{E_{00}^2}{E^2} e^{2\mu/r} \right) - k^2}}{r} \quad (129)$$

Taking the ratio  $\dot{r}/\dot{\theta}$  eliminates their time dependence, and gives the first-order differential equation of the orbit:

$$\frac{dr}{d\theta} = \frac{r \sqrt{r^2 \left( e^{4\mu/r} - \frac{E_{00}^2}{E^2} e^{2\mu/r} \right) - k^2}}{k} \quad (130)$$

The second-order differential equation of the orbit can be obtained by differentiating this one. After substituting for square root terms via Eq. (130), the result can be arranged as

$$\frac{d^2 r}{d\theta^2} = \frac{1}{r} \left( \frac{dr}{d\theta} \right)^2 + \frac{r^3 \left( e^{4\mu/r} - \frac{E_{00}^2}{E^2} e^{2\mu/r} \right) - \mu r^2 e^{4\mu/r} - \mu r^2 \left( e^{4\mu/r} - \frac{E_{00}^2}{E^2} e^{2\mu/r} \right)}{k^2} \quad (131)$$

Also, from Eq. (130),

$$r^2 \left( e^{4\mu/r} - \frac{E_{00}^2}{E^2} e^{2\mu/r} \right) = \frac{k^2}{r^2} \left( \frac{dr}{d\theta} \right)^2 + k^2 \quad (132)$$

After this substitution, the previous equation reduces to

$$\frac{d^2r}{d\theta^2} = \frac{2 - \mu/r}{r} \left( \frac{dr}{d\theta} \right)^2 + r - \mu - \frac{r^2 \mu e^{4\mu/r}}{k^2} \quad (133)$$

which can be compared directly to the Newtonian case.

Differentiating both sides of Eq. (35) to remove the integral and rearranging, the result is Eq. (130). This shows the solution of these differential equations is the same obtained by the de Broglie wave method, which is in agreement with Mercury's orbit. And we have the possibility that matter waves are the basis of *all* mechanics.

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