Constants of nature, scientific revolutions, and simplicity

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Abstract: Since Isaac Newton, many physicists have conveyed the idea of the true laws of nature being governed by "simplicity," a notion that has rarely been properly defined. When analyzing the history of fundamental physics until 1930, the number of constants of nature appears to be a useful measure for the complexity of theories, as opposed to the notion of simplicity. It can be observed that paradigm-shifting progress is often related to explanations of physical constants, thereby reducing their total number. Thus, it is argued that scientific revolutions are usually characterized by a pattern consisting of (1) a conceptual idea, (2) a mathematical formalism, and (3) a reduction of the number of independent constants of nature. This leads to a better understanding of the long-term impact of physical theories and may help to evaluate the current state of fundamental physics.
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Résumé: Depuis Isaac Newton, de nombreux physiciens ont avancé l'idée que les véritables lois de la nature étaient régies par la 'simplicité', une notion qui a rarement été correctement définie. En analysant l'histoire de la physique fondamentale jusqu'en 1930, le nombre de constantes de la nature semble être une mesure utile de la complexité des théories, par opposition à la notion de simplicité. On peut observer que les progrès en matière de changement de paradigme sont souvent liés à l'explication des constantes physiques, réduisant ainsi le nombre total de ces dernières. Ainsi, il est avancé que les révolutions scientifiques sont généralement caractérisées par un schéma consistant en 1) une idée conceptuelle, 2) un formalisme mathématique et 3) une réduction du nombre de constantes indépendantes de la nature. Cela permet de mieux comprendre l'effet à long terme des théories physiques et peut aider à évaluer l'état actuel de la physique fondamentale.

Key words: Constants of Nature; Simplicity; Epistemology; History of Physics; Fundamental Physics.

I. INTRODUCTION

A. Why study constants?

The ancient philosopher Leucippus contended that "Nothing is created by coincidence; rather there is reason and necessity for everything." In the same vein, Isaac Newton said, "Truth, if ever, is found in simplicity—not in the multiplicity and confusion of things." This statement certainly reflects a deep insight, but transforming it into a formal argument seems difficult. Einstein held a similar point of view:[1] "I'd like to state a law of Nature which is based upon nothing more than the belief in simplicity; that means [the] comprehensibility of Nature." He linked the question of simplicity to numerical parameters, continuing, "There are no arbitrary constants" […] "I cannot imagine a rational theory that explicitly contains a number that a whim of the Creator could just have chosen differently." It is, thus, worthwhile to investigate physical constants from an epistemological perspective.

The role of constants of nature in physics, sometimes called fundamental constants, is not entirely clear. Some have approached the topic from the perspective of metrology, problematizing even the use of different units such as inches and meters. While those definitions are obviously arbitrary, fundamental constants are linked to measuring scales. This became clear as early as 1900, when Max Planck introduced the length, time, and mass scales

$$\mathrm{l_{pl}} = \sqrt{\frac{Gh}{c^3}}, \qquad \mathrm{t_{pl}} = \sqrt{\frac{Gh}{c^5}}\mathrm{l}, \quad \mathrm{m_{pl}} = \sqrt{\frac{hc}{G}} \tag{1}$$

named after him. While there have been extensive debates about whether these scales have a fundamental meaning, there is no need to discuss this for the purpose of this paper. However, Planck's units demonstrate that the physical units of elementary concepts, such as length, time, and mass, can be obtained by reference to constants of nature (h, c, G), avoiding arbitrary definitions such as the Earth's parameters or the kilogram prototype. However, it is too narrow to reduce the purpose of constants to defining physical units. There are much more constants than units, as it is also obvious from the CODATA list of fundamental constants.

Yet, in a kind of semantic confusion, the issue of constants and units has led some researchers to question the importance of those constants, as if they were arbitrary as well. While their numerical expressions, expressed in earthly units, are obviously irrelevant, there physical content is not. The strength of gravity, expressed by the quantity G, is an important message of nature, as it should be obvious not only to experimenters, who have dedicated years of sophisticated work to determine the value of G, since Henry Cavendish in 1798 first realized his ingenious apparatus. Also,

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cosmologists, who for decades have pondered the coincidence

$$G = \frac{c^2 R_u}{M_u}, \tag{2}$$

modernly called "flatness," would find their models in turmoil if the value of G was, say, ten times bigger or smaller. Even in the case of c, where, due to the current definition $c = 299\ 792\ 458$ m/s, the meter is determined by the definition of the second, the very existence of the speed of light, being also a limiting velocity for matter, remains a remarkable property of nature.

B. Controversies

It is, therefore, quite astonishing that distinguishing the notion of "dimensionful" and "dimensionless" constants has attracted so much attention, culminating in the claim that only the latter ones bear physical significance.[2] While this statement has been challenged by many theoreticians[3] and is certainly foreign to observers who dedicate their carrier, for example, to $\dot{G}/G$ measurements, it is still found in several texts. From the historical perspective which is taken here, the question arises how Kirchhoff and Weber would have discovered the relation

$$\varepsilon_0 \mu_0 = \frac{1}{c^2}, \tag{3}$$

if they had considered ε_0 and μ_0 irrelevant due to their physical dimensions. This example also proves that one should be careful with conventions that set physical constants equal to unity. While this is certainly of some practical use for theoretical calculations, it has led to some misleading interpretations. The very fact that it can always be done means that there are no deep insights to be expected from this procedure.

Nevertheless, many physicists regard constants as mere conversion factors futile to ponder over. Some have even claimed that all existing constants can be eliminated, clearing the scene for mathematical constructions that are believed to be the source of further insight. Here, it is argued that such an approach is premature. Analyzing the role of constants may offer unique methodological value. At least from a historical perspective, consideration of constants and their units has led to progress, most notably, perhaps, when the above coincidence Eq. (3) was vindicated by the discovery of electromagnetic waves by Heinrich Hertz in 1886, proving Maxwell's theory. The example of simplification (reducing 3 constants to 2) observed in this formula can be, indeed, formalized and extended to a systematic counting of fundamental constants.

Thus, the premise of this paper is that physical constants do tell something about nature, and they are a useful object to study when taking a bird's eye view of the development of scientific theories in the past centuries. Progress in understanding has never been linear in time. Usually, only with the benefit of hindsight, is one able to judge whether a physical theory was successful. On the other hand, misconceptions, such as the geocentric dogma in astronomy, have stalled progress in ages, without the intellectual elite being aware of that. Investigating the role of physical constants in history might, therefore, be useful either for establishing confidence in existing paradigms or as an early red flag for physical theories going astray.

In this novel approach, I emphasize the crucial role of physical constants during the key moments in natural science, although a complete account of the history of physics cannot be given.

Rather, it should establish the hitherto uncharted connections between physical constants and key insights that led to scientific revolutions. Thus, the following might be called a sketch of the history of physical constants that promises some epistemological insight about physical theories.

C. A tentative definition

Whether "constants" of nature deserve their name at all because they are truly invariable in time, is a matter of ongoing research[4] which is however tangential to our considerations. Despite its widespread use in physics, the term constant of nature seems to lack a precise definition. There have been attempts to classify constants according to their importance,[5] without establishing consensus.[6] Much debate has deliberated whether certain constants are "fundamental" or which ones can be regarded as a physical constant at all. Such controversy is not a good starting point for an analysis that uses constants of nature as a key marker of scientific revolutions.

Fortunately, however, we can limit ourselves to a simple operational definition of "constant of nature": every quantifiable measurement of physical properties of the universe that is not patently random; any data that elicits the question "why this value and not another?" Obvious examples of constants are numbers such as the proton-electron mass ratio 1836.15…, but the concept, as explained above, is not limited to dimensionless quantities. The numerical value of the gravitational constant $G = 6.674 \times 10^{-11}$ m^3/s^2 kg is arbitrary, yet the strength of G is not, as we have seen. The numerical value of the speed of light c is even a matter of pure definition, but the very existence of c is by no means a trivial or self-evident fact. At the end of the analysis that follows, certain constants will turn out to be more relevant than others but, for this systematic approach, it is reasonable to begin by accepting any data as constants. For example, observations such as the spectral lines of hydrogen considered by Johann Jakob Balmer in 1885 can be seen as "messages" from nature that call for an explanation. A less respectful name for a constant of nature, yet equivalent for our purposes so far, is "free parameter." Free parameters are often used to group observational data, which itself may be seen as the first step toward simplification.

Sometimes, the appearance of a new constant of nature is considered an important discovery, and rightly so. However, it is also worth noting that the discovery of such a constant of nature or free parameter is often an anomaly in the sense that Thomas Kuhn meant, indicating a problem in the existing theory. It turns out, however, that sound progress in understanding is almost always linked to an elimination of constants.

D. The general pattern

Now let us consider the three-step pattern conceptual idea-mathematical formalism-elimination of constants in more detail. The first element, the bold and often visionary idea, is usually underestimated. It requires more intuitive than formal thinking, creativity, and the ability to recognize the encompassing structure among many insignificant details. Such an idea can often be characterized as "unifying," the textbook example being the unification of celestial and mundane gravity by Newton's theory, even if the key idea probably dates back to Robert Hooke. In history, such a bold thought is often not properly recognized at the time and is dismissed as "speculation" or "numerology." Yet such leaps of imagination are the seeds without which our best scientific theories would have never emerged.

The second element, the mathematical formulation, shows a broad variety of sophistication, ranging from an almost trivial link of physical quantities into an equation to very difficult concepts in differential geometry or function spaces. It requires technical skills in the first place, often only mastered by a few geniuses of the time. Sometimes, as in the case of differential calculus or Delta distributions, the math even had to be developed. The mathematical formulation of a physical theory is often the intellectually most challenging part of the scientific revolution and usually the most recognized one in the scientific community.

The third element, the reduction of the number of constants on which the current analysis is focused, is the often overlooked, subtle consequence of a theory. There is no additional intellectual hurdle to overcome when reducing the number of constants; sometimes it was even included in the conceptual idea. However, the lower number of constants or free parameters is what counts for fundamental physics in the long term. The equation that eliminates the then superfluous constants is just a glimpse of the mathematical theory but contains the essence of the unification. In the above example, $\varepsilon_0 \mu_0 = \frac{1}{c^2}$ [Eq. (3)], the equation links the electromagnetic quantities to the speed of light, but of course, it is only justified by the entire mathematical apparatus of Maxwell's theory of electromagnetism. Nonetheless, it reflects the most important unification for our civilization, electromagnetics and optics. In this case, considering the physical units of a quantity has proved to be effective—although such intuitive approaches are less appreciated in the current scientific tradition. Two more remarks should be given. First, a reduction of the number of constants can *not* be obtained by conventions such as $c = h = \varepsilon_0 = 1$ as it is sometimes practiced for technical reasons. Real progress always requires an equation with physical content. Second, it is the equation that acts as a "concept synthesizer," not, as claimed by Levy-Leblond,[5] the fundamental constant itself. While constants can sometimes associated with a theory (such as quantum mechanics with h), it is the equation that introduces the unifying character.

E. Elements of scientific revolutions

Observational confirmation and general acceptance of a theory in the scientific community are an important part of what is generally considered a scientific revolution. While these sociological questions are certainly interesting, the focus is here on the epistemological aspects that led us to consider physical constants.

The reduction of the number of constants allows one to quantify the notion of simplification and thereby simplicity, which, according to many prominent physicists, characterizes good physical theories. Albert Einstein even postulated a meta-law of nature: "Nature is so constructed that it is possible logically to lay down such strongly determined laws which only contain logically deduced constants." It seems that Einstein called for reducing the number of unexplained constants to zero; certainly, an ambitious goal.

Several aspects need to be mentioned before investigating the history of physics to identify the above pattern. It is rather the exception than the rule that one single researcher completes all three steps of such a scientific revolution since the different abilities (creative, intuitive versus formal, mathematical thinking) are rarely present in one individual.

Scientific revolutions may occur quite instantly or cover a significant period. For example, the development of atomic theory in the early 20th century can be seen as a completion of the visionary idea of the Greek philosopher, Democritus (400 B.C.), who was influenced by Leucippus. There are many instances where different scientific revolutions are entangled and a subsequent one started before the first one was completed. In some cases, partial revolutions occur in a field before they are encompassed in a grand picture that completes the process. And sometimes, a sequence of revolutions takes physics to a corresponding higher level of abstraction and understanding.

On the other hand, the math can be almost trivial while the idea is not, such as in the case of the German physician, Robert Mayer, who associated temperature and kinetic energy, thus setting the basis for the unification of mechanics and thermodynamics.

F. Exceptions to the rule

There are also huge differences in how many free parameters a theory can eliminate. At the time of the development, there might be just a few, such as the four spectral lines from which Balmer drew his conclusions. The impact of such a revolution is, however, better estimated by the potential of simplification. Indeed, thousands of spectral lines can be explained by (the generalization of) Balmer's formula with just one Rydberg constant R—a significant reduction in the number of free parameters. In this particular case, the conceptual idea of the revolution consisted of little more than suspecting that a mathematical structure was hidden in the spectral lines; nevertheless, it required a leap of creativity to pursue that idea.

Very often, a scientific revolution eliminates existing constants of nature and proceeds to a new level of understanding by explaining what was considered an anomaly. However, in some cases, a revolutionary theory was developed before the observational anomaly could even emerge. Einstein's special theory of relativity explained phenomena such as time dilatation and mass increase prior to their discovery. Had particle accelerators been built before relativity, the anomalous behavior surely would have been described by several free parameters. The reduction of the number of fundamental constants was, therefore, invisible in 1905 because the unexplained numbers had not had time to show up. Another example is Foucault's pendulum, publicly demonstrated in 1851. Had it not been accepted since Galileo that the earth was rotating, the phenomenon of a pendulum changing its plane of oscillation probably would have been considered anomalous and described by a free parameter.

In the epistemological sense, not every discovery is a revolution. For example, the constant of nature h may proudly look back on an extraordinary "career." For an entire century, it has led not only to exciting discoveries but also to groundbreaking technology: lasers, digital cameras, and computer technology. This is probably why its methodological downside—namely, the role of h as an anomaly—has gone practically unnoticed. In fact, the occurrence of h was characterized precisely by phenomena that could not be understood within the realm of existing knowledge.

In the historic analysis that follows, it is almost impossible to follow a chronological order when discussing the essential steps of this pattern; we would lose the connections that the various revolutions have in common. There are also few exceptions in which the application of the pattern conceptual idea-mathematical formalism-less constants seems to be overstretched, such as in the case of continental drift. Regarding the continents as moving objects on the earth's crust was certainly a bold idea of Alfred Wegener, but there is neither much math involved—nor is a constant of nature eliminated. At best, its paradigm-shifting insight can be viewed as a simplification that is, however, difficult to quantify. Nevertheless, we should not forget that the motion of continents on one small planet is not really a fundamental property of the universe.

I now review the most important scientific revolutions in physics using the above three-step pattern. Rather than a strict timeline, it is important to show the systematics of the often entangled revolutions.

II. SCIENTIFIC REVOLUTIONS

A. The Copernican revolution

1. Kepler

The obvious example to begin our discussion is the Copernican revolution. The revolutionary thought was that the earth, rather than being the center of the universe, orbits the sun. This central idea long preceded the development of the formalism; it is also clear that without being backed by an appropriate mathematical theory, the idea would never have prevailed. A closer examination allows partial "revolutions" to come to light. One might even say that Johannes Kepler, who discovered the mathematical formalism, contributed also a part of the idea by overcoming the prejudice that orbits must be "divine" circles. On the other hand, when he realized that the orbit of Mars indeed matched an ellipse with the sun placed at one of its two foci, Kepler was already in the process of developing a mathematical theory of planetary motion. There is no doubt that mastering the mathematics of an ellipse was the achievement that required the most elaborated technical skills.

If we consider this partial revolution by Kepler, it is quite interesting to analyze quantitatively how his insights simplified astronomy: according to our definition, this requires a reduction of the number of free parameters. In the fully developed geocentric system of Ptolemy, there were many of them: the radius of the orbit approximating the distance to the celestial body, another radius of the epicycle that accounted for the retrograde motion, the *equant* that describes off-center displacements of the small circle, and the *deferent* that accounted for the nonuniformity of the motion first approximation. Undoubtedly, if the Ptolemaic model had persisted, additional free parameters would have been postulated.

Kepler's ellipse, instead, is an object with just two parameters: the major and minor axis a and b (which might be expressed by other albeit not independent parameters). One might argue that a and b do not account for the planet's motion, and indeed, Kepler's second law, stating that, seen from the sun, the planet sweeps out equal areas in equal times, is necessary, although it introduces the orbital period T as an additional parameter. Still, Kepler's system would require three parameters for each planet. This reveals the importance of Kepler's third ("harmonic") law: he showed that the cubes of the major axes are proportional to the squares of T: $a^3/T^2 = \text{const.}$, a formula that eliminates constants and greatly reduces the amount of astronomical data. Given the distance from the sun, the orbital period of all five planets known at the time could be calculated.

Kepler's laws were then further vindicated by Galileo Galilei's observations of the moons of Jupiter. Their orbits also followed Kepler's third law, albeit with a different quotient a^3/T^2. That quotient, also called Kepler's constant, may still be seen as a characteristic number of each planet, or constant of nature. Despite all revolutionary insights and the considerable simplification achieved by Kepler, the planetary system still contained a variety of unexplained numbers.

2. Newton

Kepler's laws and Newton's additional insights are a good example of a sequence of partial revolutions that ultimately form a grand simplified picture. Another conceptual idea was required, namely, the force on the moon exerted by the earth had the same origin as all-day gravity or in a more general manner, mundane and celestial objects followed the same law of gravitation, certainly a great leap of imagination in the 17th century. Newton's mathematical theory of gravity was by far the greatest intellectual achievement of mankind at the time. Not only discovering the inverse square law but also deriving Kepler's ellipses as solutions of this particular radial dependence required extraordinary skills.

Before we discuss the number of constants of nature, one should mention that the development of mechanics alone, without any reference to astronomy, can be seen as a scientific revolution. Practically speaking, any trajectory of a freely falling body contained data that one could call constants of nature. Newton's mechanics (with important precursory work done by Galilei) instead reduced all this seemingly complicated behavior to the action of one single constant, the local acceleration g. This was a tremendous achievement and a dramatic reduction in the data generated by falling bodies.

We return to astronomy and discover that a consequence of Newton's inverse square law was that the various Kepler constants of the planets were related to their mass by means of

$$\frac{a^3}{T^2} = \quad K = \frac{\mathrm{GM}}{4\pi^2}. \tag{4}$$

Since the mass of a celestial body is nothing to be explained but is a random configuration of matter, this formula contains a dramatic reduction in the number of constants. Without such a simplification, every Kepler constant would be a peculiar feature of nature, quantified by an unexplained number. Alternatively, one could count the respective local gravity g on each planet as a constant of nature, though measurements were impossible at the time. The fact that the general gravitational constant "big G" was measured no earlier than 1798 by Cavendish does not change any aspect of our discussion here; it just means that completing a scientific revolution may take a long time. To evaluate the impact of the Copernican revolution completed with the measurement of big G, one must not only count the number of free parameters in the medieval epicycle model but also the potential simplification when taking into account all modern data. There are more than 90 000 celestial objects in the solar system with known trajectories. Consequently, the number of free parameters to describe their motion would be of the same order of magnitude had modern science not been developed. Apart from patently random parameters, all these potentially unexplained numbers have been condensed in one single fundamental constant G, using the relation

$$g = \frac{GM}{r_E^2}. \tag{5}$$

Thus, not only from the historical point of view but also in quantitative terms, the Copernican revolution is probably the most significant one.

B. Cosmology

A long time was to pass before the picture of the cosmos was enlarged by many orders of magnitude. Edwin Hubble's discovery in 1923 that Andromeda was indeed a galaxy certainly constituted a scientific revolution. In this case, the conceptual idea dates back to the German philosopher Immanuel Kant, who, by associating the visible star band called the Milky Way to the form of distant nebulae, suspected as early as 1755 that we may indeed live inside a huge disk-shaped object. Nevertheless, the arguments exchanged by Harlow Shapely and Heber Curtis in the "Great Debate" in 1920 showed that the issue was not settled. The mathematical formalism is almost absent here unless one wants to invoke modern models of galaxy dynamics that may explain the form of galaxies (actually, they have difficulties to do so[7]). Realizing, however, that what he observed in Andromeda was indeed a well-known type of star of variable brightness greatly simplified matters, or rather avoided a hugely complicated description of the data generated by competing hypotheses. Thus, from an information-theoretical perspective, the element of simplification is quite obvious, yet hard to quantify. The absence of a mathematical formalism in this revolution was possible because in this special case, the question to be decided was of a binary nature: yes, a galaxy, or not. There was no competing prior mathematical formulation to be overcome. Yet, given the amount of cosmological data, it is clear that the conceptual idea led to a simplification.

The discovery of the cosmological redshift, Hubble's second great accomplishment, is essential for modern cosmology, and it also constitutes a scientific revolution. Again, there is an obvious simplification in assuming that the light of distant galaxies is redshifted, though, until today, there is no explanation from first principles for what appears to be the expansion of the universe. The constant of nature (or free parameter) linked to this observation is the Hubble constant H_0, still presents an anomaly in the Kuhnian sense. While it is again obvious that the discovery of the redshift led to a much simpler description of the observational data, we may discuss later whether the constant H_0 is a hint that the revolution regarding cosmology is not yet complete.

C. Relativity

1. General relativity

For the many scientists who try to establish a link between the cosmological observations and Einstein's general theory of relativity, it would be not appropriate to regard the latter as the mathematical formalism for Hubble's discovery. Since there are practically no direct tests of general relativity on the cosmological scale,[b)] the current models do not distinguish between Newton's and Einstein's gravitational theory, though the latter is believed to be true. The conceptual idea of general relativity, instead, is rather abstract, but it had been clearly formulated by its founder: the equivalence principle. While the mathematical apparatus of general relativity, in its differential geometric formulation, was of unprecedented difficulty, the simplification by eliminating fundamental constants is difficult to see. The perihelion shift of the planet Mercury, however, known since 1859 and amounting to 43 arc sec per century, can be seen as a then superfluous message of nature, even if it was never called a constant. Mostly, the simplification obtained by general relativity consisted of explaining anomalies that were not yet discovered. The deflection of the starlight during the eclipse in 1919 is a famous example (one of the four

[b)]It has been claimed that general relativity "predicted" the expansion of the cosmos, since the Einstein's equations do not allow for stable solutions. However, this remains a disputable interpretation.

"classical tests"), as is the slower time-lapse in gravitational fields, demonstrated by the Hafele-Keating experiment in 1972. One can only speculate about how many unexplained numbers would have shown up if technology had advanced without general relativity. Certainly, GPS developers would have used free parameters to quantify how atomic clocks and satellites run slightly faster than their copies on earth. Fortunately, scientists were spared such a tedious development due to Einstein's ingenuity.

2. Special relativity

The same holds for special relativity formulated by Einstein in 1905. The bold idea, in this case, was the constancy of the speed of light with respect to moving observers, while the mathematical tools are much less demanding than those required for general relativity. Henri Poincaré and Hendrik and Lorentz had prepared much groundwork, but it is remarkable that both in special and general relativity, Einstein developed both the conceptual idea and the mathematical formalism. Again, his leap of genius was so ahead of its time that anomalous free parameters had no time to show up. The phenomena of time dilation, length contraction, and mass increase are all described by the same formula

$$\frac{t'}{t} = \frac{l'}{l} = \frac{m_0}{m} = \sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}}. \tag{6}$$

Certainly, the phenomenon would have created a plethora of constants, whose elimination by a later-in-history Einstein would have been a tremendous simplification. The famous formula $E = mc^2$, also part of special relativity, had an even greater impact. The entirety of nuclear reactions conducted from 1932 onwards can only be understood by that relation. All energy releases due to mass defects, usually observed as the wavelength of gamma rays, confirm Einstein's formula. Again, if all these data were collected and regarded as raw messages of nature, an incredibly complicated "theory" of nuclear reactions would have been the consequence. Special relativity thus eliminates thousands of otherwise unexplainable constants of nature, leading to a tremendous simplification.

Then, the completion of nuclear physics, unfortunately, accompanied by the development of the atomic bomb, is an example of long-lasting and interwoven scientific revolutions. Einstein's $E = mc^2$ is nothing else than the finalization of atomic theory, or, more generally, the theory of matter consisting of elementary building blocks originated by the Greek philosopher Democritus. In this case, the revolution initiated by the bold conceptual idea needed more than 2000 years to be completed. While atomism can be seen as one big scientific revolution, it is worthwhile to more closely examine the many partial revolutions that contributed to its eventual success.

D. Atomic theory of matter

The chemists Joseph Proust ("law of definite proportions"), Antoine Lavoisier, and John Dalton were the pioneers that deserve credit for having quantified atomic weights. Similar to Johann Jakob Balmer in the atomic spectra, they searched for small integers in the fractional mass ratios and ultimately discovered regularities. While the element of mathematization in these findings may be considered quite primitive, the heuristic obviously led to simplification by ordering the various results in a reasonable manner. Though the molecular hypothesis, an important step, had been formulated by Amedeo Avogadro already in 1811, it took until 1860 for the breakthrough when Italian chemist Stanislao Cannizzaro pointed out the difference between atomic and molecular weight during a conference in Karlsruhe (Germany).

Lothar Meyer and Dmitri Mendeleev were attentive listeners in the audience, and they subsequently developed the periodic table of chemical elements. The visionary idea of this partial revolution in 1869 indicated a connection between atomic mass and chemical properties, while the classification into eight groups—a primitive form of mathematization—was at that time justified only phenomenologically, by considering chemical similarities.[c)] The atomic masses of the various elements—undoubtedly constants of nature in our sense—were not yet explained, but an important step in that direction had been taken. Unfortunately, precision was much hampered by nature's whim to create the same chemical elements with different atomic masses, the so-called isotopes.

Soon thereafter, it was established that the nuclei of a given chemical element contained the same number of protons but a different number of uncharged neutrons (which have approximately the same mass).[d)] By classifying atomic nuclei in this way, Democritus' vision already shone through: their masses were approximate multiples of the atomic mass unit u, named after the pioneer John Dalton. When the first nuclear reactions were conducted in the 1930s, Einstein's special relativity led to a precise confirmation of the results. The masses of all known chemical elements were thus found to be multiples of the atomic mass unit $u = 1.66055 \times 10^{-27}$ kg, approximately the proton mass, which, as long as we consider mass, practically remains the only constant of nature. Overall, the results of atomism represent a brilliant confirmation of Democritus' old idea and constitute an essential part of mankind's knowledge.

E. Electromagnetism, optics, heat, and radiation

1. Electric charge, electricity, and magnetism

In the above discussion, we have so far bypassed the electric charge of atomic nuclei, which played an important role in the history of atomism. In 1923, experiments conducted by the American physicist Andrew Millikan had proved the astounding fact that nature allows the electric charge to occur only in certain quanta (namely, the charge of the electron), even if no deeper explanation for this mystery

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c) Only the Schrödinger equation found in 1925 and its solutions justified this classification and thus completed the mathematical formulation of the periodic sytem. Of course, this is another a great achievement of quantum theory.

d) Before the discovery of the neutron in 1932, this fact was formulated by postulating electrons residing in the nucleus.

is known to this day. Soddy was awarded the Nobel Prize for Chemistry in 1922 for the discovery that chemical properties were determined not by the mass of the atomic nucleus (since there are different isotopes anyway), but by its charge. Before Soddy, this had already been suggested by Niels Bohr.

It is worthwhile to pause and contemplate the tremendous progress in understanding that was achieved by the collective efforts of physicists and chemists over many centuries: almost one hundred stable chemical elements, starting with hydrogen, helium, lithium, etc., can readily be explained by 1, 2, 3… protons in the nucleus! Again, the enormous simplification of natural phenomena needs no further comment. It is reflected in the existence of the elementary charge $e = 1.602 \times 10^{-19}$ C,[e)] an important constant of nature.

Of course, all this would have been impossible without prior revolutions. Charles-Augustin de Coulomb developed the theories of electrostatics and magnetostatics, which were, of course, inspired by the analogous form of Newton's law of gravitation. Obviously, this led to a unified and simplified description of phenomena, though it would be quite tedious to quantify it through natural constants. Another important unification took place with electricity and magnetism at the beginning of the 19th century. In 1820, the Danish physicist Hans Christian Ørstedt demonstrated that a magnetic needle oriented itself perpendicular to a current flowing through an electric conductor, though the same experiment had already been published in 1802 by the Italian physicist Gian Domenico Romagnosi. When the effect became known in Central Europe, the preeminent British scholar Michael Faraday, more than anyone else, performed systematic investigations on the subject, while the French physicist André-Marie Ampère succeeded in formulating the relevant mathematical laws.

Though the first results of this revolution were not yet expressed quantitatively, it is obvious that the unification of formerly separated electric and magnetic phenomena was a simplification that reduced the number of arbitrary constants. In modern notation, this is reflected by the convention that the magnetic field constant μ_0 is no longer a "real" constant of nature, but has been incorporated into the definition of electric current. Consequently, the exact[f)] value $4\pi \times 10^{-7}$ Vs/Am has been assigned to μ_0.

2. Electromagnetism and optics

Since the Dutch physicist, Christiaan Huygens, had developed the concept of diffraction in the mid-17th century, it was assumed that light had wave properties. Indeed, it had long been possible to measure the wavelength precisely. However, the nature of light remained unclear until James Clark Maxwell formulated his theory of electrodynamics around 1864. In this theory, separately measurable constants ε_0 and μ_0 appear that quantify the respective strength of the electrical and magnetic interaction. A surprising consequence of Maxwell's equations was that electric and magnetic fields could propagate in empty space, without nearby electric charges. According to the theory, these waves should propagate at a certain speed.

The German physicists Wilhelm Weber and Rudolf Kohlrausch had already deduced this velocity from their experiments in 1855/56, and it was probably Weber or Kirchhoff who had the visionary idea that light was an electromagnetic wave.[8] When Heinrich Hertz produced electromagnetic waves in the laboratory for the first time in 1886, he discovered that they actually spread at the speed of light, spectacularly confirming the bold hypothesis. To bring all this into a mathematically consistent form, the entire Maxwellian theory is required,[g)] but the revolutionary idea is already contained in the above-mentioned formula [Eq. (3)], which reduces the number of constants of nature by one. Instead of three independent constants c, ε_0, and μ_0, only two are left.[h)] This is a particularly striking example of how scientific revolutions are characterized by simplification.

3. Thermodynamics

In the scientifically highly productive 19th century, several parallel revolutions were taking place that prepared the ground for later developments. One of them dealt with heat. Surprisingly, long after the invention of the thermometer, the origin of heat was still unknown. The conceptual idea that heat is nothing more than motion on a molecular level had been formulated by the German physician Robert Julius Mayer in 1842. The courage required to develop such thoughts is hardly appreciated nowadays because things can be formulated so easily in retrospect. It is remarkable how Mayer struggled with the math when developing his theory. In the formula

$$\frac{1}{2} m v^2 = \frac{3}{2} kT, \tag{7}$$

which related the mean kinetic energy of a particle to the absolute temperature T, Mayer had initially forgotten the factor $1/2$. His thoughts were subsequently completed by James Prescott Joule. Also the ideal gas law, $PV = NkT$, reflects the same fact. If we recall the definition of simplification by reducing the number of constants, then Mayer's formula does precisely that. Before his discovery, temperature measurements were messages of nature displaying unexplained numerical values such as $-273.16\,^\circ$C for the absolute zero. It was only Mayer's insight that established a connection between the (arbitrary) temperature scale and microscopic kinetic energy. Consequently, the constant k is now no longer considered fundamental but as a definition of temperature. This is correct but herein lies exactly the achievement of Mayer and Joule: one constant less.

[e)] The unit of charge.

[f)] In the 2019 update of SI units, this century-old convention was overturned again, yet this has no relevance for our discussion here.

[g)] Weber made very significant contributions to this theory, as is clear from Maxwell's treatise that mentions Weber numerous times.

[h)] In view of the definition of μ_0, one might also say the number was reduced from 2 to 1.

4. Heat and radiation

At the end of the 19th century, blackbody radiation at the center of intense research and precise measurements were carried out to determine how much radiation a body of a given temperature emitted within a range of wavelengths. Two empirical models emerged: the Rayleigh–Jeans law, which describes large wavelengths only, and Wien's displacement law, which also has a limited range of application. Both models contained only one free parameter, respectively, which was obtained by fitting the data to the model. However, the fact that all materials contain electrically charged particles that move when exposed to heat led Max Planck to the visionary idea that the radiation characteristics of all bodies followed only one law.[i] His mathematical capabilities allowed him to identify the above empirical laws as limiting cases of a more general formula, today known as Planck's law of thermal radiation

$$I(\lambda)d\lambda = \frac{8\pi hf^3}{c^3}\frac{1}{e^{\frac{hf}{kT}}-1}. \quad (8)$$

The emission increases with temperature, while the maximum shifts toward smaller wavelengths. It was only much later that Planck, who had initially guessed the correct formula, provided a theoretical justification—the mathematical formalism, so to speak. Regarding the number of constants, the gain is subtle but consequential: Instead of two free parameters in the laws of Wien and Rayleigh–Jeans, Planck managed to use only one, h, which he over-carefully called "auxiliary quantity without any physical meaning."[j] According to our criterion of simplicity, Planck's radiation law reduced the number of constants of nature by one.

As a by-product, Planck's law of radiation led to another simplification. The total radiation per area emitted by a blackbody is proportional to the fourth power of the absolute temperature T. Independently of Planck, this was already known as the Stefan-Boltzmann law: $w=\sigma T^4$. The constant σ ("sigma") can be determined empirically and represents a generic case of a free parameter, in other words, a constant of nature. With Planck's law, it became possible to deconstruct σ—i.e., to calculate it using other constants of nature

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2}. \quad (9)$$

Once again, the number of constants of nature had decreased by one.

F. The quantum revolution

1. Recognizing the role of h

The great importance of h as a constant of nature, however, was recognized by Einstein. At the beginning of the 20th century, the strange results of the photoelectric effect (emission of electrons when light hits a metal) had puzzled many physicists. This effect led Einstein in 1905 to the formula $E=hf$, which describes the energy of "light quanta"—a bold assumption that started the quantum era. As a mathematical formula, the equation $E=hf$ is almost trivial, and it is not immediately obvious which constants of nature it eliminates. However, any other interpretation of the photoelectric effect would have produced poorly understood parameters, while Einstein's formula elegantly solved everything. Thus, here, too, the quantum of action h led to a reduction in unexplained numbers in nature, apart from the fact that the concept of light quanta had revolutionary consequences. Although named after Planck, it was Einstein who breathed life into the "auxiliary constant," providing an interpretation that Planck, of all people, was reluctant to accept. As late as 1913, he publicly mocked Einstein, saying that he had "overshot the mark with his speculations." As a consequence of Einstein's formula $E=hf$, h thus has the interesting physical unit energy (Nm) per frequency (1/s), thus Nms, which is also called *action*.

Due to the identity $N=\text{kg m/s}^2$, the units of h may be rewritten as kg m^2/s, and Niels Bohr realized that this was also the unit angular momentum,[k] which led him to the ingenious conceptual idea that related h to the angular momentum of an electron orbiting the atomic nucleus.

2. Atoms as little solar systems

However, an even more general concept is present: The idea that electrons orbit the atomic nucleus, just as planets orbit the Sun. The possibility that the solar system has a microscopic counterpart in the atom evoked an enormous fascination among physicists at that time. While the first idea dates back to Wilhelm Weber in the 19th century,[10] around 1904, the Japanese physicist Hantaro Nagaoka independently invented the model, and Niels Bohr integrated it into a grand new picture of the atom. Even if that model turned out to be incomplete in parts, it certainly constituted the central conceptual idea during the scientific revolution in atomic theory. Bohr's hypothesis about h being the angular momentum filled that visionary idea with life. However, the greatest achievement of the model was to explain why the electron's motion around the nucleus was constrained to certain distances from it. While planets, in principle, can be found at any distance from the sun (and this distance determines the orbital period in terms of Kepler's third law), the same does not apply in the case of electrons: Bohr realized that they can orbit the nucleus only if their angular momentum is a multiple of the constant $h/2\pi$, which is commonly abbreviated as $\hbar$ ("h bar"). Accordingly, the electrons' trajectories are characterized by $L=\hbar, 2\ \hbar, 3\ \hbar \ldots$, which also correspond to different energy levels. Of course, Bohr's model required the entire mathematical apparatus of quantum theory to be justified, later developed by Heisenberg and Schrödinger.

[i] Planck's law of radiation is in fact one of the most important findings of modern physics, even if it is often incorrectly applied to gases that cannot emit blackbody radiation. The law is another example of a scientific breakthrough leading to a reduction in the number of constants of nature.

[j] The German term for auxiliary ("Hilfs-") was the eponym for h.

[k] For the sake of historical accuracy, the English mathematician John William Nicholson, who also considered angular momentum[9] must be mentioned. However, this does not diminish Bohr's accomplishment of having discovered a coherent picture of the atom.

3. Johann Jakob Balmer, the Kepler of the atoms

Just as Newton relied on Kepler, Bohr's model of the atom had an important precursor—the Swiss math teacher Johann Jakob Balmer. Like Kepler, Balmer closely examined the observations and revealed a mysterious relation, though he could not yet explain its true origin. In 1885, Balmer studied the spectrum of the hydrogen atom, a sequence of wavelengths 656, 486, 434, and 410 nm. According to our technical definition, these values present constants of nature. Balmer noted[11] that $\frac{410{,}2}{656{,}3} \approx \frac{5}{8}$ and $\frac{486{,}1}{656{,}3} \approx \frac{20}{27}$, and years of pondering over these relations led him to the eventual discovery of the formulae

$$\begin{aligned}\frac{1}{656,3\ \text{nm}} &= R\left(\frac{1}{2^2}-\frac{1}{3^2}\right),\\ \frac{1}{486,1\ \text{nm}} &= R\left(\frac{1}{2^2}-\frac{1}{4^2}\right),\\ \frac{1}{434,0\ \text{nm}} &= R\left(\frac{1}{2^2}-\frac{1}{5^2}\right),\\ \frac{1}{410,2\ \text{nm}} &= R\left(\frac{1}{2^2}-\frac{1}{6^2}\right),\end{aligned} \tag{10}$$

while introducing the constant $R = 1.0973731 \times 10^7\,\text{m}^{-1}$ that was later named after the Swedish physicist Rydberg.[l)]

Two more things need to be noted here. First, the appearance of squares in the denominator actually reflects the similarity between Newton's law of gravitation and the corresponding electrical law of Coulomb, a resemblance that led to the idea of atoms as tiny solar systems. However, the way Balmer arrived at his conclusions is also remarkable from a methodological point of view. Like Kepler, Balmer looked out for mathematical coincidences without it being clear that they existed at all, an approach that nowadays is often dismissed as "numerology." But Balmer's success, as in Kepler's case, relied precisely on that: finding a relationship that was obviously correct, as further measurements of spectral lines would soon confirm. By doing so, Balmer dramatically simplified the description of nature's messages, providing a glimpse of the emerging mathematical framework. As in Kepler's case, the conceptual idea here is just to search for such mathematical patterns.

Balmer's discovery drastically reduced the number of constants of nature, which is our definition of simplification. All spectral lines of the hydrogen atom (in principle, this applies to all atoms) are described by Balmer's formula and its generalization;[m)] the only remaining constant R.

4. The simplification in quantum physics

Although Bohr was unable to provide a deeper reason for the postulate of angular momentum to be multiples of $\hbar$, his model led to a spectacular success. When jumping from an outer to an inner shell, electrons had to release energy that corresponded to a frequency or wavelength according to Einstein's formula $E = hf$. Bohr showed that the possible jumps exactly matched the wavelengths of the light found by Johann Jakob Balmer. The numbers labeling the respective orbits, later called shells, turned out to be the squared integers that had appeared in the denominator of Balmer's formula. Thus, Rydberg's constant R, which Balmer had measured but not explained, was no longer a mystery but could be derived from a short calculation. The analog of Newton's law of gravitation needed for the atom is Coulomb's inverse-square law of electric force. Combining it with Bohr's postulate for the angular momentum yields the energy levels that can be compared with Balmer's formula, resulting in

$$R = \frac{m_e e^4}{8c\varepsilon_0^2 h^3}. \tag{11}$$

Consequently, Bohr's model of the atom made it possible for the unexplained number[n)] R found by Balmer to be expressed by other known constants of nature. By doing so, the number of fundamental constants was reduced by one, and this is *the* epistemological progress provided by atomic and quantum theory. Having explained just one of several constants of nature may not seem significant at first. Rydberg's constant R, however, was distilled from so many single measurements of atomic spectra that one must consider Bohr's achievement based on Balmer as similar to what Kepler and Newton had accomplished—altogether a dramatic simplification.

5. Other quantum phenomena

The discovery of the wave nature of electrons was also a decisive step toward understanding atoms. In his doctoral thesis in 1923, the French physicist Louis Victor de Broglie argued that electrons, like any elementary particle, could also display a wave nature. If, according to Einstein's quantum interpretation of light, a wave sometimes behaved like a particle, then according to de Broglie's reasoning, an electron could also behave like a wave. De Broglie developed a model[12] in which the wavelength of the electron was given by

$$\lambda = \frac{h}{mv}, \tag{12}$$

where m is the mass of the electron and v its velocity. This proposition was vindicated by diffraction experiments of electron beams on crystals by Davisson and Germer in 1927. How does this, certainly revolutionary, insight fit into the above scheme of discoveries? Again, the simplification took place before any unexplained numbers could even emerge. If diffraction experiments with electrons had been conducted earlier, a poor understanding would have led to a phenomenological description of the outcome. That would certainly

[l)]To be precise, Balmer discovered the constant R_H valid for the hydrogen atom. R_H also accounts for the motion of the nucleus, as was discovered later. Therefore, a minute, though calculable difference between R_H and R (for heavier atoms) remains, which is irrelevant for our methodological arguments.

[m)]In modern terms, Balmer's formula describes only the second atomic shell, because it is only this shell that emits light visible to the eye. With the discovery of ultraviolet spectral lines of the first shell (Lyman series) and infrared light from higher shell transitions (Paschen series, etc.), the generalization of Balmer's formula was obvious.

[n)]Technically, R differs from R_H (for hydrogen) by a well-known factor involving m_e and m_p.

TABLE I. Scientific revolutions in physics.

Players	Year	Visionary idea	Formula	Obsolete
Copernicus, Kepler	1521, 1600, 1610	Sun at the center Elliptic orbit	Kepler's laws	Epicycles
Newton	1687	Earthly and celestial gravity	$g = \frac{GM}{r^2}$	$g, K_J, K_M, \ldots$
Balmer	1885	Mathematics in atomic spectra	$1/\lambda = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right), \ldots$	$\lambda_1, \lambda_2, \ldots$
Weber, Nagaoka, Bohr	1879, 1904, 1913	Atoms as Solar System h as angular momentum	$R = \frac{m_e e^4}{8c\ \varepsilon_0^2 h^3}$	Rydberg constant
Planck	1900	Unifying law of radiation	$I(\sigma, \lambda) = \ldots$	Wien, Rayleigh–Jeans
Maxwell, Weber, Ampère	1864	Unification electricity magnetism	Maxwell's equations	μ_0
Hertz, Weber	1886	Light is an electro-magnetic wave	$1/c^2 = \varepsilon_0\mu_0$	ε_0
Mayer, Joule	1842	Heat is kinetic energy	$\frac{1}{2}\, mv^2 = kT$	k
Democritus, Avogadro, Dalton, Mendeleev, Einstein	500 BC-1930	Matter from elementary building blocks	Schrödinger equation, Periodic table, etc.	Atomic masses
Einstein	1905	Constancy of c	$E = mc^2$, $t'/t = \ldots$	…
Kant, Hubble	1923	Galaxies as "island universes"	—	

have been done in terms of free parameters, which would have been regarded as properties of the electron. Instead, with his visionary idea, de Broglie had already explained these potentially mysterious numbers in advance.

The wave nature of the electrons resolved a dilemma inherent in Bohr's atomic model. Being electrically charged particles, orbiting electrons would inevitably radiate energy, the energy loss in turn leading them to crash into the atomic nucleus soon—a fatal inconsistency of the "little solar systems." If, however, electrons in a given orbit are regarded as stationary waves, this catastrophe is avoided. At the same time, the quantization of the angular momentum is explained by the fact that only an integer number of waves "fit" into the electron orbit. If we examine the development of atomic physics as a whole, many interrelated breakthroughs were needed to arrive at a convincing picture.

6. Formalisms of quantum physics

One more aspect must be mentioned. A primarily mathematical achievement led to a consistent form of the atomic model and made wavelike electrons possible in the first place. This important justification dates back to Werner Heisenberg and Erwin Schrödinger, who achieved it by using quite different approaches in 1925 and 1926. Their equivalence was shown later by Paul A.M. Dirac.

The mathematically consistent formulation of a scientific revolution is often the most difficult and intellectually challenging part, as in the case of Newton's derivation of the elliptic orbits proposed by Kepler. Without such a solid foundation, however, a new scientific theory would never prevail. Nevertheless, the role of the conceptual idea must not be underestimated. It requires more creative than technical skills and, in the case of quantum theory, was achieved primarily by Einstein and Bohr.

7. Further developments

If one tries to summarize fundamental progress, the element of simplification remains on the balance sheet, even if it sometimes appears as a by-product of brilliant math. Condensing a multitude of unexplained numbers into one single constant R, found by Balmer, and the subsequent calculation of R by Bohr that anticipated the explanation of so many later experiments are the central achievements of quantum mechanics.

The quantum h continued to play an important role. In 1970, the British physicist Bryan Josephson discovered surprising currents between different metals. The strength of these currents in these so-called Josephson junctions is related to h.[o)] In 1985, German physicist Klaus von Klitzing discovered that the electrical resistance in a so-called Hall sensor was always a multiple of h/e^2, in other words, quantized. Another discovery was the fractional quantum Hall effect. In all these cases, h anticipated free parameters that would have been used if h were not known.

III. SUMMARY AND CONSEQUENCES

We may summarize the findings so far in Table I.

Several questions arise in the aftermath of the historical account on constants. Can the number of constants be determined and, if yes, how many independent constants do we observe? Is there any measure of ranking or quality of a constant that can be applied? Why does it make sense to stop counting the number of constants around 1930? Is there any justification for this and any insight to be gained? Moreover, apart from historical curiosity, what useful consequence can be drawn for the rest of physics? I will attempt to answer these questions.

A. Counting the constants of nature

From our initial definition, every measuring value could be seen as a constant of nature. A series of discoveries, some of them called revolutions, led to a tremendous simplification, namely, the reduction of the number of constants. Once we try to count the remaining fundamental constants, it is obvious that different choices can be made. For example, one could consider either the Hubble constant H_0 as a

[o)] When applying a DC voltage U, an alternating current of frequency $f=2\ e\ U/h$ is observed.

fundamental constant, or the age of the universe (approximately $T_u = 1/H_0$), or even the radius of the universe $R_u = c\,T_u$. This might depend on a model, but whatever choice one adopts, the number of independent constants does not change.

To begin with, G is considered a fundamental constant responsible for the strength of gravity. Then, evidently, the speed of light c is a fundamental property of spacetime, and so is h. When considering electrodynamics, there are not many constants left. Since μ_0 defines the unit Ampere of the electric current (and thereby the electric charge $1\,C = 1$ As), $\varepsilon_0 = \frac{1}{c^2 \mu_0}$ is not an independent constant either. Thus, the only constant that remains to account for the strength of the electromagnetic interaction is the elementary charge e. However, this property is usually expressed by another quantity e, the so-called fine-structure constant

$$\alpha = \frac{e^2}{2hc\varepsilon_0} \approx \frac{1}{137}, \tag{13}$$

which is actually dimensionless, leading to the obvious question: why this value and not another? The same applies to the ratio of the proton and the electron mass, $m_p/m_e = 1836.15$, a number that Paul Dirac has been pondering over for decades. To complete our collection so far, we have to consider two additional aspects: first, cosmological measurements are messages of nature, though in most cases their accuracy leaves much to be desired, even today. Second, the mass and size of elementary particles are unexplained quantities. While these values are not given much importance by the current paradigm, they must undoubtedly count as constants of nature. It is interesting to hear Einstein's point of view:

> *The real laws of nature are much more restrictive than the ones we know. For instance, would it not violate our known laws if we found electrons of any size or iron of any specific weight? Nature, however, only realizes electrons of a particular size and iron of very specific weight.*

While there has been no evidence for a measurable size of an electron so far, the proton's radius has been determined since Rutherford's scattering experiments and has the actual[13] value 8.4×10^{-16} m, while the proton mass, which we consider instead of the atomic mass unit u, is 1.6726×10^{-27} kg. In cosmology, the most direct measurement is the Hubble constant H_0, though we might as well consider the size scale $R_u = c/H_0$ as a fundamental constant. Cosmological measurements can even determine the total mass of the universe M_u contained within the visible horizon, amounting to approximately 10^{53} kg. The cosmos is in a state of evolution and, thus, it is clear that these constants do vary slowly in time; nevertheless, they represent messages of nature that we try to make sense of. Around 1930, there were a total of nine[p)] independent constants: G, c, h, 137, 1836, m_p, r_p, M_u, R_u.

[p)]As a matter of principle, also half-lifes of radioactive particles are messages of nature that ought to be explained. However, in 1930, not even the neutron was discovered.

B. Quality of constants

To avoid misunderstandings, it should be clarified that the physical units of meters, seconds, and kilograms are arbitrary choices. Therefore, the numerical values of most of the constants are unremarkable. It would, however, be grossly misleading to discard their importance for that reason. In a similar vein, one could complain about the fine structure constant written in the decimal system. While the decimal system is arbitrary, as are the definitions of meters and seconds, the existence of constants of nature is not. The strength of gravity does tell something about nature, and so does the fact that there is a limiting velocity for all material bodies that cannot be exceeded, however arbitrary the units may be in which this law is stated.

Judging the quality of fundamental constants is a difficult task. Being on the above shortlist already means that these constants have resisted any attempt to explain them over the centuries. A ranking could be done with regard to relative accuracy, but it would be distorted by the fact that some constants have been defined by a fixed value, already incorporating the consequences of past scientific revolutions. Nevertheless, those constants that are open to electronic measurements, such as the fine structure constant, would fare much better than the gravitational constant, which has shown unsettling discrepancies over the past decades. Still less secure is the value of the Hubble constant and the mass of the universe deduced therefrom; both can be determined as an order of magnitude, at best. However, this does not mean that those constants with poor accuracy do not tell us a great deal. From an epistemological point of view, the Hubble constant encompasses the data of billions of galaxies.

Therefore, it might be better to consider the potential of simplification if you talk about the quality of constants. As already mentioned, G is responsible for hundreds of thousands of celestial bodies, while h, but also α, as contained in an almost unlimited number of atomic spectra (e.g., the NIST database). Due to the mutual dependencies, a quantitative measure of the importance of constants would be, nonetheless, difficult to state.

C. Why 1930?

Now let us return to the question of why the situation around 1930 should be of particular importance—and not earlier or later. It is simply because, in this period, there is a minimum of the number of constants. Evidently, important discoveries, such as the neutron, which in turn led to nuclear fission, were about to happen. Thus, at least from the technological perspective, important developments seem to be missing. Regarding theory, however, the state of relative simplicity around 1930 is unique and has never been reached again in the history of physics. One might argue that the concept of quarks, established in the 1960s, greatly reduced the number of free parameters by describing a plethora of elementary particles discovered until then. However, the sheer number of constants never went down to less than a dozen.

On the other hand, one might also ask whether that minimum number had not been reached earlier. In the 19th century, for example, the quantum of action h had not been discovered. However, spectroscopy already had revealed several spectral lines that easily exceeded even today's number of constants. The early 1930s were indeed an era in which all available observations of nature could be described by a remarkably small set of numbers. Even if cosmological data suffer from poor accuracy, the entirety of galactic observations would have to be considered an almost unlimited number of messages of nature, had their origin not been understood by the observations of Edwin Hubble. Thus, it is essential to include those cosmological findings, apart from the fact that the human picture of reality would be ridiculously incomplete without an idea of the size of the universe. In all due modesty, it can be said that around 1930, humans had discovered the external reality to a degree that makes it unlikely that a similar extension occurs. This does not mean, however, that our description of reality might not be completely overturned in the future.

Investigating the further history is certainly worthwhile, but 1930 may also be seen as a point of reflection on the status of fundamental physics. Not coincidentally, the Solvay conference in 1927 for the first time showed enormous dissent among the leading physicists about how to interpret the observational evidence at the time. The fact that around 1930, so many questions were discovered that have not been solved to this day, and the increasing number of fundamental constants from that time, is an additional hint that we might regard it as a turning point in history.

We will be considered the generation that left behind unsolved such essential problems as the electron self-energy.—Wolfgang Pauli, 1945 Nobel Laureate

D. Unifications

Some more remarks on this era regarding unifications and applications should be given. In many, albeit not all cases, scientific revolutions can be seen as a unification of existing theories. Visionaries have suspected a common origin in natural phenomena so far believed to be unrelated. Incidentally, such unifications can be traced back to the elimination of constants. The fact that local gravity g (measured by weight only) can act as an acceleration (measured dynamically) can be seen as a unification of statics and dynamics. The unification of earthly and celestial gravity (the former including dynamics) is contained in $g = \mathrm{GM}/r_e^2$ [Eq. (5)], expressing g by means of G. Mechanics and thermodynamics are unified by the equation $\frac{1}{2}mv^2 = \frac{3}{2}kT$ [Eq. (7)], which downgraded the constant k to a definition of temperature. The unification of electricity and magnetism is now included in the definition of μ_0 by forces acting on parallel wires. Electromagnetism and optics, as already mentioned, are linked by the formula $\varepsilon_0\mu_0 = \frac{1}{c^2}$ [Eq. (3)]. Planck's law of radiation then established a link between thermodynamics and optics, thereby reducing two constants to only one, the quantum of action h. Moreover, dynamics was linked to optics by Einstein's special relativity, while general relativity can be regarded as a unification of optics (constant c!) with gravity. Then Einstein's interpretation $E = hf$ created the field of quantum physics, which was linked by Niels Bohr, who saw h as an angular momentum, to atomic physics. In the same period, atomism itself was completed by demonstrating that all atomic weights were multiples of the atomic mass unit u, the "details" being solved by $E = mc^2$.

One might argue now that quantum electrodynamics, as the name suggests, is a unification of electrodynamics and quantum physics. It certainly led to simplification by explaining both the anomalous magnetic moment of the electron and the Lamb shift in the spectrum of hydrogen. However, it does not explain the value of the fine structure constant α, although it extensively makes use of it. A complete unification would certainly require a calculation that yields α, or, if one phrases it in another way: if people succeed in calculating α, would not quantum electrodynamics be an appropriate name for such a theory?

Such a calculation would undoubtedly be a revolution, reducing the above number of constants by one. Unfortunately, there are few conceptual ideas on how to tackle the problem, although Richard Feynman, more than 30 years ago, had claimed that "all good theoretical physicists put this number up on their wall and worry about it."

Another scholar who worried about fundamental problems regarding the existing constants was Paul Dirac. In 1937, he was the first to point out a possible connection between the physics of the cosmos and the physics of atoms,[14] called Large Number Hypothesis. Dirac's thoughts are not at the focus of current activities, but from an epistemological point of view, they could be the starting point for a future unification. Also, Mach's principle is such a possible visionary idea: while the original form just qualitatively stated that distant masses in the universe might be the origin of inertia and, therefore, gravity, it was later vindicated as a remarkable coincidence relating G, c, and the mass and size of the universe

$$G \approx c^2 \frac{R_u}{M_u}, \tag{14}$$

also called flatness.

IV. CONCLUSION

The guiding hypothesis throughout this paper was that the number of independent constants does tell something about the overall epistemological status of an empiric science. It has been shown that, due to scientific revolutions being rewarded by eliminating constants, the number of constants can be meaningfully defined and, indeed, reached a minimum around 1930 after an exceptionally productive period in fundamental physics. Even if the term simplicity has often been ambiguous and prone to subjectiveness, it appears that the number of constants is a useful measure of simplicity to evaluate the long-term impact of physical theories. There is no doubt that the intuitive judgment of all the great physicists who have expressed themselves about the matter, favored simplicity over complexity. The approach presented here allows for a quantitative definition and is a step toward rigor in the epistemology of science.

From the above considerations, it is clear that the number of constants around 1930, nine, is not significant. If progress is, indeed, linked to simplification, it will probably be obtained by a further reduction of that number. Apparently, Einstein's ideal was a physical theory doing without any arbitrary constants.[15] Regardless of whether this goal is realistic, it shows that physics still has a long way to go toward an ultimate understanding of reality. Given that the entire quantum revolution, considered the major development of the 20th century, essentially led to the elimination of one constant (Rydberg's R), it is clear that enormous efforts, if ever, can advance physics. The question is whether this will occur under the prevailing paradigm.

While going into detail is beyond the scope of this analysis, the current number of constants (in the above definition) can roughly be estimated. The number of free parameters in the standard model of particle physics is usually quoted[q)] as about 20, while the concordance model of cosmology requires just a few less.[16] However, that record does not take into account all particle masses, the inclusion of which would bring the number to more than 50; when lifetimes are considered, probably more than 100. If there is any value in the appreciation of simplicity by the founding fathers of modern physics, then the current situation must be called a crisis.

Incidentally, this diagnosis coincides with what had been identified by other scholars in recent years, for example, Lee Smolin[17] and Sabine Hossenfelder.[18] Both have convincingly argued that the heuristics "beauty" is a wholly inappropriate meta-criterion for judging the correctness of a theory or for finding new ones. While leading to the same conclusion, the criterion of simplicity, defined via natural constants, may be such a heuristic.

[q)]Lee Smolin comments: "The fact that there are that many freely specifiable constants in what is supposed to be a fundamental theory is a tremendous embarrassment" (Ref. 17, p. 13).

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