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A look at the abandoned contributions to cosmology of Dirac, Sciama, and Dicke

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The separate contributions to cosmology of the above researchers are revisited and a cosmology encompassing their basic ideas is proposed. We study Dirac's article (1938) on the large number hypothesis, Sciama's proposal (1953) of realizing Mach's principle, and Dicke's considerations (1957) on a flat-space representation of general relativity with a variable speed of light (VSL). Dicke's tentative theory can be formulated in a way which is compatible with Sciama's hypothesis on the gravitational constant G . Additionally, such a cosmological model is shown to satisfy Dirac's *second* 'large number' hypothesis on the total number of particles in the universe being proportional to the square of the epoch. In the same context, Dirac's *first* hypothesis on an epoch-dependent G – contrary to his prediction – does not necessarily produce a visible time dependence of G . While Dicke's proposal reproduces the classical tests of GR in first approximation, the cosmological redshift is described by a shortening of measuring rods rather than an expansion of space. Since the temporal evolution of the horizon R is governed by $\dot{R}(t) = c(t)$, the flatness and horizon problems do not arise in the common form.

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1 Introduction

Cosmology as a modern observational science started in the 1930s, when Hubble's observations set an end to the 'great debate' whether andromeda is a nebula inside the milky way or an independent galaxy. Shortly after, with the first mass and distance estimates of the whole universe, fundamental questions regarding the interrelation of the universe with elementary particles were raised by Eddington and Dirac [1]. However, the main interest in Hubble's redshift-distance law was due to the fact that it paved the way for Friedmann-Lemaitre (FL) cosmology which naturally accompanied the spectacular success of general relativity developed just 15 years before. As if cosmology liked great debates, the discussion was then dominated by the rivalry between steady-state and big-bang models, which was decided in favor of the latter by the discovery of the cosmic microwave background (CMB). Notwithstanding such remarkable advance, there were reflective voices like Dirac's in 1968 [2]:

'One field of work in which there has been too much speculation is cosmology. There are very few hard facts to go on but theoretical workers have been busy constructing various models for the universe based on any assumptions that they fancy. These models are probably all wrong. It is usually assumed that the laws of nature have always been the same as they are now. There is no justification for this. The laws may be changing, and in particular, quantities which are considered to be constants of nature may be varying with cosmological time. Such variations would completely upset the model makers.'

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Fortunately, cosmology in the meantime has many observational facts which allow to do much more quantitative science than in the 1930's and even the 1960's. Dirac's prediction of a change of the gravitational constant G he had expressed in a manner slightly too self-assured in his 1938 paper ('A new basis for cosmology' [3]) has not been confirmed, GR has undergone an impressive series of confirmations, and FL-cosmology has won all battles so far. This success and the lack of alternatives however bears the danger of interpreting new data assuming a model we still should not forget to test. Though having won all battles, this was due to introducing considerable elasticity in the originally rigid theory by means of dark matter, dark energy, and further numbers we must call fitting parameters poorly understood so far. A review of the observational evidence for standard cosmology and its problems is given elsewhere [4].

The great debates and the enthusiasm about new data of the past decades brushed aside the interest in old unresolved problems and it is a somehow unfortunate development that Dirac's criticism has not been considered any more by theoreticians. Doubts on our usual assumptions of the time-independence of physical laws have been expressed by various researchers [5]:

'The question if there is a unique absolute standard of time which globally is defined by the inner geometry of the universe, is a big unresolved problem of cosmology.'¹

In particular, the idea of time as an invisible river that runs without relation to the universe ('time is what happens when nothing else does') may be just wrong [6]. A redefinition of time by means of parameters which govern the evolution of the universe should have profound consequences, though observational evidence may be minute. Relating our local physical laws which base on apparently constant quantities to global properties of the universe is the greatest challenge of cosmology.

It should be clear that dealing with any alternative approach to cosmology requires much patience, and a reinterpretation of all new data we are flooded with cannot be done immediately. Contrarily, we shall prepare ourselves to stay for a little while in the period in which the unresolved problems first arose and some old seminal papers were trying to understand them. A closer look to their content is still fascinating: Dirac's large number hypothesis, which consists of two independent, struggling coincidences is one of the most mysterious, unexplained phenomena in cosmology. He considered them as 'fundamental though as yet unexplained truths', which remain valid, even though the Hubble "constant" varies with the age of the universe' [7]. No theoretical approach besides Jordan's one [8] has taken them seriously so far. Sciama's efforts to link the value of G to the mass distribution of the universe are accompanied by profound insights and for the first time realized Mach's principle concretely. Though Mach's ideas have always been as fascinating and convincing from a conceptual point of view, the missing quantitative formulation had remained an unsatisfying aspect. One of the most interesting proposals in this context, full of speculative ideas, has been given by Dicke [9], though this is much less known than the later developed scalar-tensor-theory. We shall first have a closer but compressed look at the mentioned papers, and then give a technical description of the proposal that makes use of the basic ideas expressed there. First consequences are discussed in Sect. 4.

2 Review of the seminal papers

2.1 Dirac's large number hypothesis

Strictly speaking, Dirac's article [3], relating three large dimensionless numbers occurring in physics, expresses three different coincidences we shall name Dirac 0, I, and II. Eddington had already noticed the number

$$\frac{F_e}{F_g} = \frac{e^2}{4\pi\epsilon_0 G m_p m_e} \approx 10^{39} \quad (1)$$

¹ Retranslated from the German edition.

and wondered how such a huge number could come out from any reasonable mathematical theory. Dirac then observed that the age of the universe is about the same multiple of the time light needs to pass the proton radius, or equivalently

$$\frac{R_u}{r_p} \approx 10^{40} =: \epsilon, \quad (2)$$

thus escaping from the mathematical difficulty of producing ϵ and postulating epoch-dependent forces. It was the first time somebody tried to relate properties of the atomic scale to those of the universe as a whole. Not enough here, he noted the total number of baryons² being

$$\frac{M_u}{m_p} \approx N \approx 10^{78} \approx \epsilon^2. \quad (3)$$

As a consequence, for the gravitational constant, the relation

$$G \approx \frac{c^2 R_u}{M_u} \quad (4)$$

must hold, a coincidence that was previously noted by Eddington and much earlier (though lacking data, not in an explicit way) suggested by Ernst Mach, who insisted that the gravitational interaction must be related to the presence of all masses in the universe [10]. I shall call the coincidences (1+2) Dirac I and (2+3) Dirac II, while (4) should be named Dirac 0, to emphasize that Dirac's considerations go much further: (4) could be realized either with a different radius of elementary particles (not satisfying Dirac I) or with another number of baryons of different weight (and being in conflict with Dirac II).³ It seems that Dirac himself was more convinced of hypothesis I than of II. According to [8], he abandoned the latter after various critiques, e.g. by [11]. Indeed, while Dirac I had a great influence on physics with a huge amount of experimental tests (see [12] for an overview of the $\dot{G} \neq 0$ search) contesting the appreciation of the idea, Dirac II remained completely out of any theoretical approach so far. While Dirac I would be fairly compatible with standard FL cosmology, Dirac II is in explicit conflict with it. To be concrete, FL cosmology assumes for the epoch $\epsilon = 10^{25}$ (BBN, creation of light elements) a horizon containing 10^{64} baryons, while in the epoch $\epsilon = 10^{50}$ still 10^{78} (or, considering the accelerated expansion, even less [13]) baryons should be seen. Dirac instead argued that 'Such a coincidence we may presume is due to some deep connection in nature between cosmology and atomic theory.' ([3], p. 201). Jordan [8] in 1959 commented on the second hypothesis:

'As far as I can see, I am the only one who was ready to take seriously Dirac's model of the universe which was immediately abandoned by its creator. I have to confess that I consider Dirac's thought as one of the greatest insights of our epoch, whose further investigation is one of the big tasks.'

According to the predominant opinion among cosmologists however $N \sim \epsilon^2$ is just a coincidence invented by nature to fool today's physicists.

Dirac was aware that a cosmological theory of this type could require a change of time scales. In Sects. 3 and 5 of his paper [3], he considered an idealized time t representing the epoch and an observable time ' τ ' which were quadratically related and considered⁴ an evolution of the horizon $R \sim t^{\frac{1}{3}}$. Since time

² This argument is not changed by the variety of elementary particles discovered in the meantime, since it involves orders of magnitude only.

³ We shall not go into detail regarding the question how Dirac II can be related to $h \approx cm_p r_p$ and to the so-called Eddington-Weinberg number. The agreement of the Compton wavelength of the proton with its actual radius (determined by Rutherford) is however not trivial, as the comparison with the electron shows.

⁴ See [7], Eq. (16.4.6), for a comment on that.

measurements necessarily involve frequencies of atomic transitions and therefore the speed of light, it is strange that he maintained the postulate $c = 1$. This omission led him to the inviting but somewhat premature claim that the gravitational constant G had to vary inversely with the epoch. The amount of experimental research done due to that prediction [12] illuminates the great influence of Dirac's ideas on physicists. The so far (negative) outcome of the $\dot{G} \neq 0$ search has prevented theorists from taking that deep principles too seriously, without however having challenged the prediction as such from a theoretical point of view.

2.2 Sciama's implementation of Mach's principle

Contrarily to Dirac, Sciama [14] focussed on the question how to realize Mach's principle in a quantitative form, having noticed that in Newton's theory the value of G is an arbitrary element (p. 39 below). From considerations we skip here he derived a dependence of the gravitational constant⁵

$$G = \frac{c^2}{\sum_i \frac{m_i}{r_i}}, \quad (5)$$

whereby the sum is taken over all particles and r_i denoting the distance to particle i . This is much more concrete and quantitative than Mach's ideas or the speculations of Eddington and Dirac. It provides further a reasonable dependency on distance and alleviates the somewhat mysterious property that G should 'feel' the whole universe. Sciama commented the apparent constancy of G :

'... then, local phenomena are strongly coupled to the universe as a whole, but owing to the small effect of local irregularities this coupling is practically constant over the distances and times available to observation. Because of this constancy, local phenomena appear to be isolated from the rest of the universe. ...'

Sciama further considered the gravitational potential (Eq. (6) there)

$$\phi = -G \sum_i \frac{m_i}{r_i} = -c^2. \quad (6)$$

Despite the inspiring and insightful discussion in the following, astonishingly he did not consider a spatial variation of c , though it seems a reasonable consequence to relate c^2 to the gravitational potential. As we shall see below, a variable speed of light in combination with (5) leads to a differential equation that satisfies Dirac's second hypothesis. Since Sciama considered the coincidence (5) as approximate, we shall be able to modify it by a numerical factor.

2.3 Dicke's 'electromagnetic' theory of gravitation

It was Robert Dicke [9]⁶ who first thought of combining the dependence (5) with a variable speed of light, apparently having been unaware of Sciama's previous efforts. Though Dicke obviously left this path in favor of the much more prominent scalar-tensor-theories, we shall investigate this very different first approach here only. Dicke's proposal belongs to the 'conservative' VSL theories that do not postulate exotic dependencies of c but widely agree with general relativity (GR) in the sense that a variable c in a flat background metric generates a curved space. While recent VSL theories had to suffer a couple of objections, these do not apply to the present 'bimetric' type, since the notion of VSL is implicitly present in GR (see [15], ref. 70, with numerous excerpts of GR textbooks). Even before developing the definite version of GR, Einstein [16] considered that case. Dicke realized that the failure of Einstein's attempts (see

⁵ Eqs. (1) and (5a) with a change of notation.

⁶ Unfortunately, this paper was published with the misleading title 'Gravitation without a principle of equivalence' which tells very little about the inspiring content.

also [17]) were due to neglecting the variation of length scales λ (Einstein considered varying time scales only)⁷, and noted that the classical tests could be described by

$$\frac{\delta c}{c} = \frac{\delta \lambda}{\lambda} + \frac{\delta f}{f}, \quad (7)$$

assuming further $\frac{\delta \lambda}{\lambda} = \frac{\delta f}{f}$. Dicke started from Einstein's idea of light deflection caused by a lower c in the vicinity of masses [16]:

'... that the velocity of light in the gravitational field is a function of the place, we may easily infer, by means of Huyghens's principle, that light-rays propagating across a gravitational field undergo deflexion'.

Dicke introduced therefore a variable index of refraction ([9], Eq. (5))

$$\epsilon = 1 + \frac{2GM}{rc^2}. \quad (8)$$

While the second term on the r.h.s. is related to the gravitational potential of the sun, Dicke was the first to raise the speculation on the first term having 'its origin in the remainder of the matter in the universe'. In Sect. A.1, the reader will find a brief description how Dicke's tentative theory may provide a formulation of spacetime geometry equivalent to GR and compatible with the classical tests.⁸ In Sect. A.2, it will be outlined how Newton's law of gravitation arises from Sciama's hypothesis (5) and can be embedded in Dicke's model.

The Cosmological redshift in Dicke's proposal is a cornerstone that distinguishes drastically from standard cosmology. He described the idea as follows:

'The cosmological principle was taken to be a fundamental assumption of the theory. Namely, from any fixed position of a Newtonian frame the universe is assumed to be on the average uniform. This implies that matter is on the average fixed in position relative to the Newtonian coordinate frame, for motion would introduce a lack of uniformity as seen by an observer located where the matter would be moving. In like manner the scalar field variable ϵ [polarizability of the vacuum] and matter density must be position independent.' ([9], p. 374 left).

Though not stated explicitly, the increase of the horizon $R(t)$ must be governed by $\dot{R}(t) = c(t)$, since there is no other possibility for a horizon increase:

'Although all matter is at rest in this model there is a galactic red shift. With increasing ϵ [and decreasing c], the photon emitted in the past has more energy than its present counterpart. This might be thought to cause a "blue shift". However, a photon loses energy at twice the rate of loss characteristic of an atom, hence there is a net shift toward the red ([9], p. 374 right).

The decrease in c is due to new masses dropping into the horizon. The corresponding decrease of length scales appears as a net expansion which becomes visible as cosmological redshift. Astonishingly, Dicke did not clearly follow that path and derived a total number of particles proportional to $\epsilon^{\frac{3}{2}}$, in contrast to Dirac II.⁹ It seems that this was due to the quite arbitrary assumption in Eq. (94) that led to the complicated form of (95) which turns out to be in conflict with the differential equation that will be derived from (5). Contrarily, we shall see below that the density that arises from Sciama's assumption (and Dicke's 'rest' of the theory) matches indeed Dirac's second hypothesis on the number of particles.

⁷ It should be noted that though c being a scalar field here, this theory is not a 'scalar' theory *coupled to matter* to which Einstein later expressed general caveats; these reservations were however put into question by [18].

⁸ Yilmaz [19] reports a private communication that Dicke believed the perihelion of mercury to come out with a (wrong) factor, a problem which seems to be settled by the calculations of [20].

⁹ There is a brief correspondence on this topic [21]. To fix that difference, Dicke introduced another quantity $\frac{\epsilon}{\epsilon_0}$ which later played an important role in the so-called Brans-Dicke theory.

3 Dirac-Sciama-Dicke (DSD) cosmology

3.1 Units and measurement

In the following, we assume an absolute, Euclidean space¹⁰ and an absolute, undistorted time. The time t and the distances r expressed in this absolute units however are mathematical parameters not directly observable. All time and distance *measurements* instead are performed in relative, dynamical units defined by the frequencies $f(t)$ and wavelengths $\lambda(t)$ of atomic or nuclear transitions. These perceived or relative quantities measured by means of $f(t)$ and $\lambda(t)$ shall be called t' and r' . In that absolute space, all matter is assumed to be at rest having a uniform density ρ (particles per absolute volume). In the next subsection we shall consider an evolution of the horizon $R(t)$ (absolute distance) with the assumption¹¹ $\dot{R}(t) = c(t)$ starting at $R(t = 0) = 0$ *everywhere* in Euclidean space. To obtain (arbitrarily chosen) time and length scales for the absolute units, we define $\lambda_0 > 0 = \lambda(t_0 > 0)$ by the condition $\frac{4}{3}\pi\rho\lambda_0^3 = 1$. Equivalently, we may say the horizon $R(t_0) = \lambda_0$ contains just one particle. $\dot{R}(t_0) = c(t_0) =: c_0$ is then the speed of light at $t = t_0$ in absolute units and we may define the frequency $f(t_0) = f_0$ by the identity¹² $\lambda_0 f_0 = c_0$.

3.2 Temporal evolution

Expressing Dicke's index of refraction in (8) as $\epsilon = \frac{c+\delta c}{c}$ and taking into account the smallness of δc , with $\delta c^2 = 2c\delta c$ we may write

$$\frac{c^2 + \delta c^2}{c^2} = 1 + \frac{4GM}{rc^2}. \quad (9)$$

Slightly modifying Sciama's proposal (5) we use $\frac{c^2}{4G} = \sum_i \frac{m_i}{r_i}$, leading to¹³

$$1 + \frac{\delta c^2}{c^2} = 1 + \frac{\frac{M}{r}}{\sum_i \frac{m_i}{r_i}}. \quad (10)$$

Since δ indicates the difference of values far from and nearby the sun, we compare the l.h.s. and r.h.s in (10) with $\frac{M}{r} = \delta \sum \frac{m_i}{r_i}$ and assume all elementary particles to have the same mass ($m_i = 1$). After integration and cancelling of the arising logarithms, this leads to a spatiotemporal dependency of the speed of light

$$c(\vec{r}, t)^2 = \frac{c_0^2}{\sum_i \frac{1}{|\vec{r}_i - \vec{r}|}}, \quad (11)$$

whereby the sum is taken over all particles i and $|\vec{r}_i - \vec{r}|$ denoting the (time-dependent) distance to particle i , measured in absolute units. I shall abbreviate (11) as $c_0^2/c^2 = \Sigma$ for simplicity¹⁴. The expansion rate $c(t)$ depends therefore on the number of visible particles and will decrease while the horizon increases. Without loss of generality, $r = 0$ is assumed, thus we also shall use the approximation

¹⁰ called 'Newtonian' by [9].

¹¹ The time derivative refers to the absolute time.

¹² To be precise, t_0 coincides with $\frac{1}{f(t_0)}$ only by a factor, since we did not introduce further assumptions on $R(t)$ for the period $0 < t < t_0$. For the choice of units, this factor does not do any harm. Physically, the (only reasonable) definition of measurable time by atomic and nuclear transitions is not possible as long as the transition is not completed at $t = t_0$.

¹³ Sciama explicitly ([14], p. 38 below) allowed such a factor.

¹⁴ To be in precise agreement with Sect. 3.1, the sum Σ should be replaced by $\Sigma + 1$, since one particle is visible at t_0 . Since we shall consider only large values of Σ in the following, this will be omitted.

$$\Sigma \approx \int_0^R \frac{4\pi\rho r^2 dr}{r} = 2\pi\rho R^2. \quad (12)$$

Keeping in mind that $\dot{\rho} = 0$, after inserting $\dot{R}(t) = c(t)$, (11) transforms to

$$\dot{R}(t)^2 = \frac{c_0^2}{2\pi\rho R(t)^2}, \quad (13)$$

which after taking the square root, reduces to the simple form

$$\frac{d}{dt}R(t)^2 = \text{const.} \quad (14)$$

with the solution¹⁵

$$R(t) \sim t^{\frac{1}{2}}; \quad c(t) \sim t^{-\frac{1}{2}}. \quad (15)$$

This evolution is the central difference to FL cosmology with $\frac{d}{dt}R(t) = c = \text{const.}$ ¹⁶

3.3 Change of measuring rods

Since the locally observed speed of light $c' = \lambda' f'$ is a constant¹⁷, the agreement with the classical tests of GR (see Sect. A.1) requires $\lambda \sim t^{-\frac{1}{4}}$ and $f \sim t^{-\frac{1}{4}}$, that means both wavelengths and frequencies of atomic transitions become smaller during the evolution of the universe. The intervals τ we actually use to measure time change according to $\tau = t^{\frac{1}{4}}$. Since for the relative, measured time t' the condition $t'\tau = t\tau_0$ holds ($\tau_0 = 1$ by definition), the relative time $t' = \frac{t}{\tau}$ shows a dependence $t' \sim t^{\frac{3}{4}}$ (mind that constancy, $\tau \sim t^0$ would lead to the usual $t' \sim t^1$). The measuring value of the perceived epoch is $t' = 10^{39}$ now¹⁸, therefore the ‘true’ epoch, in absolute units, must be $t = 10^{52}$ at present.

Dimensionful units and change of further quantities The change of time and length scales has further consequences. Firstly, all measurements of velocities and accelerations will be affected. This is already clear for those arising in atoms, otherwise the scale-defining decline of wavelengths and frequencies could not happen. Thinking in absolute units, the same particles, undergoing smaller accelerations, have an apparent *inertial mass* which accordingly increases. Developing further this principle of measurement with dynamical scales, almost all dimensionful physical units turn out to have a time evolution, thus we may imagine the dependency directly ‘attached’ to a unit like m or s . This eases to find the consistent trend but also elucidates why the change of physical quantities may be hidden at a first glance in conventional physics. A list of the respective change of physical quantities for the static case has already been given by [9], p. 366. A corresponding overview is given below in Table 1. All quantities at t_0 are normalized to 1.

3.4 Observational consequences

Cosmological redshift As Dicke ([9], p. 374) points out, in the context of a VSL light propagation the following properties hold: ∇c with approximately $\dot{c} = 0$ affects λ , while f remains unchanged. Vice versa, when ∇c vanishes, \dot{c} will change f and leave λ constant. Therefore, assuming an isotropic DSD universe while analyzing the large-scale evolution, a propagating photon will change its frequency only, while λ is kept fixed.

¹⁵ This describes approximately the evolution, since detailed assumptions for $0 < t < t_0$ cannot be given.

¹⁶ In the matter-dominated epoch.

¹⁷ $c' = 299792458$ m/s is used for the SI definition.

¹⁸ To avoid fractional exponents, we shall approximate 10^{40} by 10^{39} .

Table 1 Temporal evolution of quantities.

Quantity	Symbol	evolution t^γ	present epoch
abstract time	t	t^1	10^{52}
horizon	R	$t^{\frac{1}{2}}$	10^{26}
speed of light	c	$t^{-\frac{1}{2}}$	10^{-26}
wavelengths	λ	$t^{-\frac{1}{4}}$	10^{-13}
frequencies	f	$t^{-\frac{1}{4}}$	10^{-13}
actual time interval	τ	$t^{\frac{1}{4}}$	10^{13}
velocities	v	$t^{-\frac{1}{2}}$	10^{-26}
accelerations	a	$t^{-\frac{3}{4}}$	10^{-39}
perceived horizon R'	$\frac{R}{\lambda}$	$t^{\frac{3}{4}}$	10^{39}
perceived epoch t'	$\frac{t}{\tau}$	$t^{\frac{3}{4}}$	10^{39}
particles	N	$t^{\frac{3}{2}}$	10^{78}
perceived particle density	ρ'	$t^{-\frac{3}{4}}$	10^{-39}
masses	m	$t^{\frac{3}{4}}$	10^{39}

Consider now a photon emitted at t_1 with $c(t_1) = \lambda(t_1)f(t_1)$, in brief $c_1 = \lambda_1 f_1$. It is detected later at t_2 when other photons (*) of the same atomic transition obey $\lambda_2^* f_2^* = c_2^*$ with

$$c_2^* = c_1 \left(\frac{t_2}{t_1} \right)^{-\frac{1}{2}} ; \quad \lambda_2^* = \lambda_1 \left(\frac{t_2}{t_1} \right)^{-\frac{1}{4}} . \quad (16)$$

Since the arriving photon still has $\lambda_2 = \lambda_1$ (and $c_2 = c_2^*$), it will appear redshifted by the factor

$$(z + 1) := \frac{\lambda_1}{\lambda_2^*} = \left(\frac{t_2}{t_1} \right)^{\frac{1}{4}} . \quad (17)$$

Its frequency decreased by $\frac{f_2}{f_1} = (z + 1)^{-2}$ with respect to emission, but is lower only by $\frac{f_2}{f_2^*} = (z + 1)^{-1}$ with respect to other photons (*) generated at t_2 .

Dirac's second hypothesis on the total number of particles Since we have assumed an Euclidean space with constant density ρ in which the horizon increases according to $R(t) \sim t^{\frac{1}{2}}$ (15), for the total number of visible particles

$$N(t) = \rho V(t) = \frac{4}{3} \pi \rho R(t)^3 \sim t^{\frac{3}{2}} \quad (18)$$

holds. Taking into account that the perceived time shows the dependency $t' \sim t^{\frac{3}{4}}$, Dirac's second hypothesis

$$N(t) \sim t'^2 \quad (19)$$

follows. Of course, the same result is obtained considering the shortening of length scales $\lambda \sim t^{-\frac{1}{4}}$ causing the perceived horizon to be at the relative distance of $R' = \frac{R}{\lambda} \sim t^{\frac{3}{4}}$. Then for the number of particles

$$N = \frac{4}{3} \pi \rho' R'(t)^3 \sim \rho' t'^{\frac{3}{4}} \sim \rho' t'^3 \quad (20)$$

holds, which coincides with (18) because $\rho' \sim t^{\frac{3}{4}} \sim t'^{-1}$, an equivalent form of Dirac's second observation.

A possible apparent constancy of the gravitational constant G There is some observational evidence [12] against a temporal variation of G . In the DSD evolution developed above however, Dirac's postulate of a variation of G turns out to be premature. First we have to ask what observational evidence supports $\dot{G} \approx 0$. Exemplarily, we consider the absence of increasing radii in the Earth-moon¹⁹ and the Sun-Mars orbit (e.g., [23, 24]), since these are the most simple ones to discuss. In the DSD picture, frequencies and wavelengths of atomic transitions contract according to $f \sim \lambda \sim t^{-\frac{1}{4}}$. Hence, in the classical limit of orbiting electrons, Bohr's radius²⁰ has to decline like $r_b \sim \lambda \sim t^{-\frac{1}{4}}$ and the respective centripetal acceleration according to $a_z \sim t^{-\frac{3}{4}}$. On the other hand, the gravitational acceleration is proportional to ∇c^2 . Since all gradients are taken with respect to the dynamic units $\lambda \sim t^{-\frac{1}{4}}$, they appear bigger by the factor $t^{\frac{1}{4}}$, while $c \sim t^{-\frac{1}{2}}$. Therefore, the gravitational acceleration (see Sect. A.2) $a_g \sim t^{-\frac{3}{4}}$ has at least the dependence required for a decrease of the radius $r \sim t^{-\frac{1}{4}}$ in a sun-planet orbit. This contraction synchronous with length scales would result in an apparent absence of any change in distance; two-body systems, the 'planetary clocks', would run slower and contract their orbits in the same manner as the atomic clocks do. This result is still in agreement with Kepler's 2nd law, since the angular momentum $\vec{l} = m\vec{v} \times \vec{r}$, with $m \sim t^{\frac{3}{4}}$, $v \sim t^{-\frac{1}{2}}$ and $r \sim t^{-\frac{1}{4}}$ yields a time-invariant quantity even in absolute Euclidean units. From other considerations (see Sect. A.1) there are good reasons to assume Planck's constant h , whose units correspond to \vec{l} , to be unchanged in time.

Contrarily to the speed of light, the factor $\frac{F_c}{F_g}$ will yield different measuring values dependent on the epoch, and therefore the measuring value of G , too. The experimental bounds of absolute G determinations by far do not exclude such a possibility. The commonly expected constancy of G and the underlying assumptions of the respective observations must be reconsidered in DSD cosmology.

4 Discussion

Dirac's hypotheses and agreement with GR phenomenology The most convincing property of DSD cosmology seems the agreement with Dirac's large number hypotheses. In particular, also the second one is obtained while providing a mechanism for an apparent constancy of G , which has been used as an argument against Dirac's first hypothesis so far. Mach's principle is fully encompassed while the cosmological redshift becomes an intrinsic necessity in DSD cosmology. A critical point to be evaluated further will be the agreement of the underlying tentative gravity model with GR from a theoretical and experimental point of view. For the latter, as far as the classical tests are concerned, DSD cosmology does not seem to predict any differences to GR. However, general covariance can hardly be achieved since a minute variation of the gravitational constant is suggested (see A.2). If ever, a consistent formulation must be obtained along the flat-space formulations of GR, the bimetric theories. Though there is a long history (e.g. [25–27]), the representations in terms of a spatially varying speed of light (e.g. [15, 20, 28–30]) have to gain yet broad acceptance.²¹

In general, there is a wide-ranging observational agreement with conventional cosmology due to the dynamics of physical units, whose relations to each other change so slowly that observational differences, if any, remain minute. This conjecture has still to be verified for the impact of DSD gravity on electrodynamics, since nobody would expect c_{GR} to be different from c_{EM} (see [32] for a systematic review of the different meanings of c). Though Dicke [9], p. 372 already proposed in an explicit way how to modify Maxwell's equations, we cannot go into details here.

¹⁹ An anomaly related to this issue was reported by [22].

²⁰ which is equal to the de-Broglie wavelength of the orbiting electron divided by 2π .

²¹ For possible experimental tests, see [31].

Energy conservation is no longer a valuable condition for the evolution of the universe. Taking a general perspective, this is not heavily surprising, because energy is a concept introduced to describe the time-independence of physical laws.²² While this is true for the snapshot of the universe we are observing, the clumping of matter suggests that the universe is anything but stationary. Though the differential equation $\frac{d}{dt}R^2 = \text{const.}$ seems to be a simple principle, a general formulation, possibly by means of a Lagrangian, has still to be given.

The flatness and horizon problem These cosmological puzzles triggered the revival of modern VSL theories ([34, 35], for an overview [36]) which provided solutions alternative to inflation. Here we restrict to the fact that flatness is closely related to the observation of the approximate coincidence (4). As it is evident from (5), the apparent G must have an according value in the same order of magnitude. It does not make sense at the moment to relate DSD predictions to the WMAP data which set tight bounds on flatness like $\Omega = 1 \pm 0.02$. Given that deviations from $\Omega = 1$ at primordial times should cause huge deviations at present, approximate coincidence following from Sciama's ansatz (5) is a step towards an explanation of 'flatness'.

In Friedman-Lemaitre cosmology, gravity acts as a contracting force which slows down the Hubble expansion. It is precisely that slowdown that causes new masses to drop into the horizon and raises the question how masses, without having causal contact, could show a highly uniform behavior like the CMB emission. Contrarily, in DSD cosmology, since all matter is initially at rest, masses attract due to gravitational interaction, but this does not affect the apparent redshift. Consequently, the problem of slowing down the 'expansion' does not even arise.

Cosmic microwave background It is interesting to investigate the impact of the present proposal for the WMAP data of the cosmic microwave background. According to common cosmology, the CMB is a signal from the recombination period at $z \approx 1100$, commonly assumed to be 380000 years after the big bang. Assuming an nonuniform evolving time like in DSD cosmology, $\frac{\lambda'}{\lambda} - 1 = z \approx 1100$ corresponds, since $\lambda \sim t^{-\frac{1}{4}}$, to an epoch of $t = 3 \cdot 10^{42}$, while at present $t = 10^{53}$ holds. Measured in units of the 'local' time, that epoch corresponds to $t' = 10^{31}$, i.e. about one year. This is a dramatic difference and must carefully be compared to the observations. A calculation of the power spectrum that has to take into account different temperature and density assumptions, would be premature at this stage. As far as the amplitude of CMB fluctuations is concerned, one expects much tinier fluctuations in DSD cosmology since there is much more time left for the fluctuations to evolve to galaxies. One should keep in mind that before the COBE data had been analyzed, much greater fluctuations were expected, a riddle which was resolved in the following by assuming corresponding dark matter fluctuations.

Big Bang Though we were not able to discuss the details shortly after $t = 0$, some substantial differences to FL cosmology should be noted. The absolute scale λ_0 was defined above by the condition of a single particle being contained in the horizon. If one assumes this particle to be a baryon, its rest energy corresponds to the zero energy E_0 of a particle closed in a quantum well of the size of the horizon: $E_0 = \frac{h}{t_0} = \frac{hc}{\lambda_0} \approx m_p$. In this case, the evolutionary equation in absolute units writes as²³ $\frac{d}{dt}R^2 = \frac{h}{m_p}$. In general, a density equal to the density of nuclear matter seems to require much less extrapolation of physical laws than the densities that arise in FL cosmology shortly after the big bang.

²² Conceptual problems of this kind are addressed in [33].

²³ Given that m_p increased by a factor 10^{40} in the meantime (see Table 1), the perceived h/m_p accordingly decreased to the actual value $5 \cdot 10^{-7} \text{ m}^2/\text{s}$.

5 Outlook

The present proposal based on the ideas of Dirac, Sciama and Dicke is a first framework for a cosmology based on a tentative alternative gravity model. Regarding the quantity of observations in agreement with a theoretical framework, the DSD proposal is unable to compete with standard FL cosmology with its currently accepted Λ CDM model. DSD cosmology may only gain importance if one is disposed to raise doubts to (1) the validity of the standard model with its considerable extrapolation of the laws of nature and its increasing number of free parameters (2) the suggestion of the standard model that Mach's principle and Dirac's enigmatic hypotheses being just numerical coincidences (3) the conviction of the constants of nature being fixed but arbitrary numbers; this last condition seems the most entrenched one. However the idea that we are observers living inside a prison of dynamic measuring instruments, which in first approximation cause a blindness for the perception of change, is certainly not unfamiliar.

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A Appendix

A.1 Tentative VSL formulation of GR

It is a known feature of GR that in a gravitational field clocks run slower and a shortening of measuring rods occurs with respect to clocks and rods outside the field. Defining c with respect to the latter scales, one can equivalently say that c is lowered²⁴ in the gravitational field (cfr. [37], p. 111, and [15], ref. 70). Based on that point of view, an equivalent description of GR by means of VSL theories can be tried, e.g. [20] (and references above) which is known as 'polarisable vacuum' (PV) representation of GR. The only necessary postulate is that since $c = f\lambda$ and $\delta c = f\delta\lambda + \lambda\delta f$, the relative change

$$\frac{\delta c}{c} = \frac{\delta f}{f} + \frac{\delta\lambda}{\lambda}, \quad (21)$$

is equally²⁵ distributed to f and λ , that means $\frac{\delta f}{f} = \frac{\delta\lambda}{\lambda}$. We shall denote the quantities outside the gravitational field as c, λ, f and the lower quantities in the field as c^*, λ^*, f^* . Hence, in a gravitational field, clocks run slower by the relative amount

$$f^* = \alpha^{-1}f; \quad \alpha := \left(1 + \frac{GM}{rc^2}\right) \quad (22)$$

and wavelengths λ shorten by the same factor α : $\lambda^* = \alpha^{-1}\lambda$, which is a well-known result of GR. According to (21), $c^* = \alpha^{-2}c$ has to be lowered by $\alpha^2 \approx \left(1 + \frac{2GM}{rc^2}\right)$ in a weak-field approximation. While that idea has first been developed by [9], the results of [20], Sect. III, suggest that all classical tests of GR can be described in this manner, see also [38], Sect. 3–5²⁶. To give an example, we briefly describe the gravitational redshift of the sun [39].

One can imagine the process as follows: consider a photon travelling from the gravitational field of the sun to the earth (with approximately zero gravity). Starting at f^*, λ^*, c^* , while travelling it keeps its (lowered) frequency f^* . At earth, i.e. outside the gravitational field where $c = \alpha^2 c^*$ is higher by the double amount (21), the photon has to adjust its λ , and raise it with respect to the value λ^* at departure. Since the adjustment α^2 to c overcompensates the originally lower $\lambda^* = \alpha^{-1}\lambda$, we detect the photon as gravitationally redshifted with $\alpha^2 \lambda^* = \alpha\lambda$.

²⁴ This is sometimes called 'non-proper' speed of light, see [17].

²⁵ In 1911, Einstein considered fixed λ 's only, thus $\frac{\delta c}{c} = \frac{\delta f}{f}$, being in accordance with the 'Newtonian' value that failed to match the famous data of Eddington's eclipse observation in 1919.

²⁶ I do not uphold any longer the considerations in Sect. 6.

Change of measuring rods Time and length measurements naturally affect accelerations ($a \sim \alpha^{-3}$), and surprisingly, masses, too. Photon and rest masses, hf and mc^2 have to behave in the same manner, and since $f \sim \alpha^{-1}$, $c^2 \sim \alpha^{-4}$, $m \sim \alpha^3$ must hold. This is in agreement with Newton's second law according to which masses have to be proportional to inverse accelerations. An overview on the relative change of various quantities inside the gravitational field (cfr. [9], p. 366 and [27]) is given in Table 2 below. α denotes a factor of $(1 + \frac{GM}{rc^2})$:

Table 2 Relative change of quantities inside the gravitational field.

Quantity	symbol	unit	rel. change
speed of light	c	$\frac{\text{m}}{\text{s}}$	α^{-2}
frequency	f	$\frac{1}{\text{s}}$	α^{-1}
time	t	s	α
length	λ	m	α^{-1}
velocity	v	$\frac{\text{m}}{\text{s}}$	α^{-2}
acceleration	a	$\frac{\text{m}}{\text{s}^2}$	α^{-3}
mass	m	kg	α^3
force	F	N	α^0
pot. energy	E_p	Nm	α^{-1}
ang. mom.	l	$\frac{\text{kg m}^2}{\text{s}}$	α^0

A.2 Newton's law from a variable c

Once time and length measurement effects of GR are described by a spatial variation of c , all gravitational phenomena should be encompassed by the same framework. However, $c \sim \alpha^{-2}$ requires (eqns. 8, 9) in first approximation

$$\delta(c^2) = 2c\delta c = -\frac{4GM}{r}. \quad (23)$$

This leads to a Newtonian gravitational potential of the form

$$\phi_{\text{Newton}} = \frac{1}{4}c^2, \quad (24)$$

which differs²⁷ by a factor 4 from Sciama's potential. Sciama's proposal was however always considered as approximate by the author ([14], p. 38 below). Since (11)

$$c(\vec{r})^2 = \frac{c_0^2}{\sum_i \frac{1}{|\vec{r}_i - \vec{r}|}}, \quad (25)$$

for the acceleration of a test mass

$$\vec{a}(\vec{r}) = -\frac{1}{4}\nabla c(\vec{r})^2 = \frac{c_0^2}{4\Sigma^2} \sum_i \frac{\vec{r}_i - \vec{r}}{|\vec{r}_i - \vec{r}|^3} \quad (26)$$

holds. Assuming without loss of generality $|\vec{r}| = r = 0$ and substituting c_0^2 ,

$$a = \frac{c^2}{4\Sigma} \sum_i \frac{\vec{e}_i}{r_i^2} \quad (27)$$

²⁷ An early investigation on the Mach-Sciama approach ([40], p. 93) deduces $\frac{1}{3}c^2$, which seems to be incompatible with the Newtonian limit.

follows, yielding the inverse-square law. Thus c_0 does not appear any more and the Newtonian force is perceived in the local, dynamic units. As one easily verifies, (27) does not depend on the units in which masses and distances are measured. Thus (25), while setting $r = 0$, may be rewritten in SI quantities (with an new reference c_n):

$$c^2 = \frac{c_n^2}{\sum_i \frac{m_i}{r_i}}, \quad (28)$$

The ‘gravitational constant’ is then given by the quantity

$$G = \frac{c^2}{4 \sum_i \frac{m_i}{r_i}}, \quad (29)$$

in accordance with [14]. From (29) and the assumption of an homogeneous universe, elementary integration over a spherical volume yields $\sum \frac{m_i}{r_i} \approx \frac{3m_u}{2r_u}$, and therefore

$$m_u \approx \frac{c^2 r_u}{6G} \quad (30)$$

holds, which is in approximate agreement with the amount of baryonic matter.

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