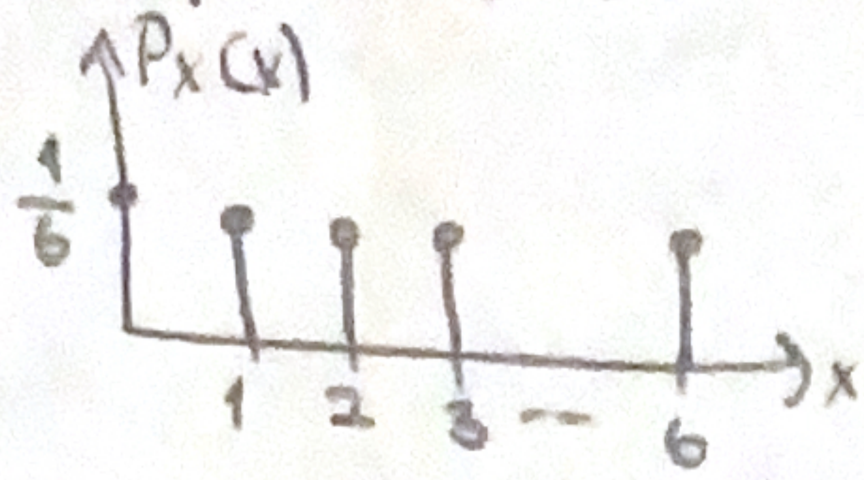
Discrete RVs

$P(x) = P(X = x)$ (pmf)

x	0	1	2 ...
$P(x)$	0.2	0.3	0.2



Cumulative Distribution Function

$F(x) = P(X \le x) = \sum_y p(y)$ (at most x)

$F(x \le 2) = 0.7 \leftarrow$ at most 2

$P(a \le X \le b) = F(b) - F(a-)$ ← $a-1$

exa $P(2 \le x \le 5) = F(5) - F(1)$

Expected Value

$E[X] = \sum x \, p_X(x) = \mu_X$

$E[h(x)] = \sum h(x) p(x)$

Variance

$V(X) = E[X^2] - E[X]^2$

Joint Distribution

$P(x, y) = P(X = x \ \& \ Y = y)$

$\sum_x \sum_y p(x, y) = 1$

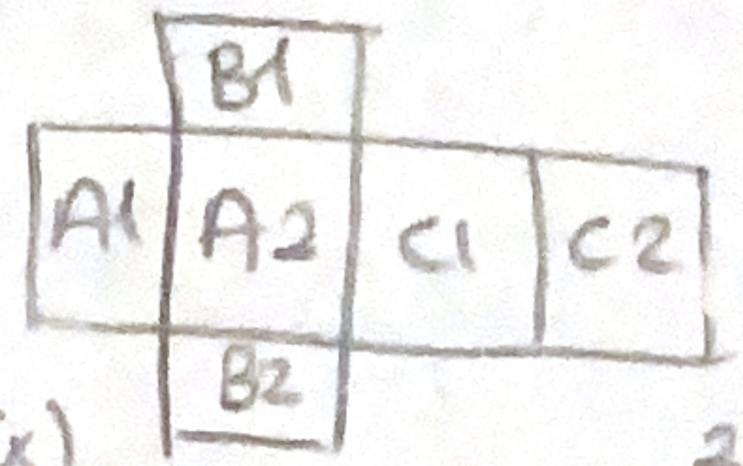
$P(x,y)$		y: 0 · 1 ...
x	5	
	10	$\rightarrow P(10,1) \rightarrow$

sum all values on row to get $X = x$

$P_X(x) = \sum_y p(x, y)$

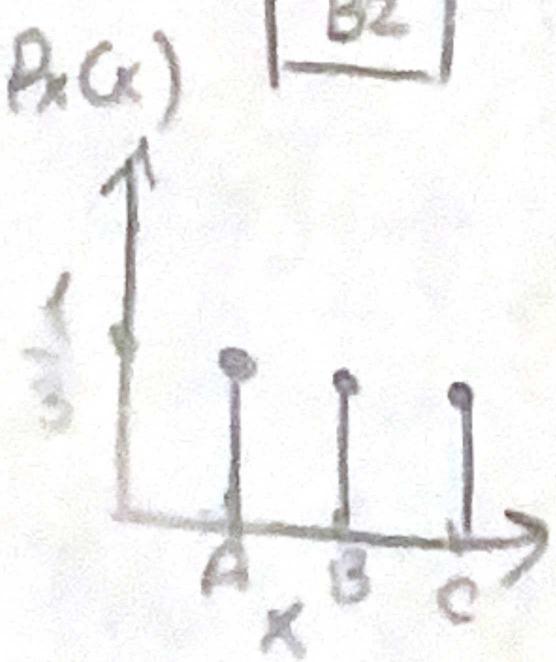
condition for independence

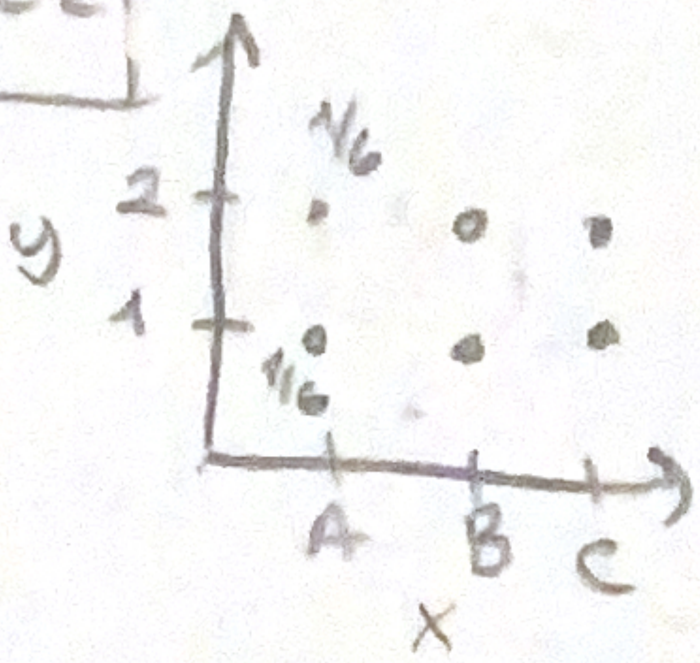
$P_{X,Y}(x, y) = P_X(x) \cdot P_Y(y)$



$X = A \ B \ C$

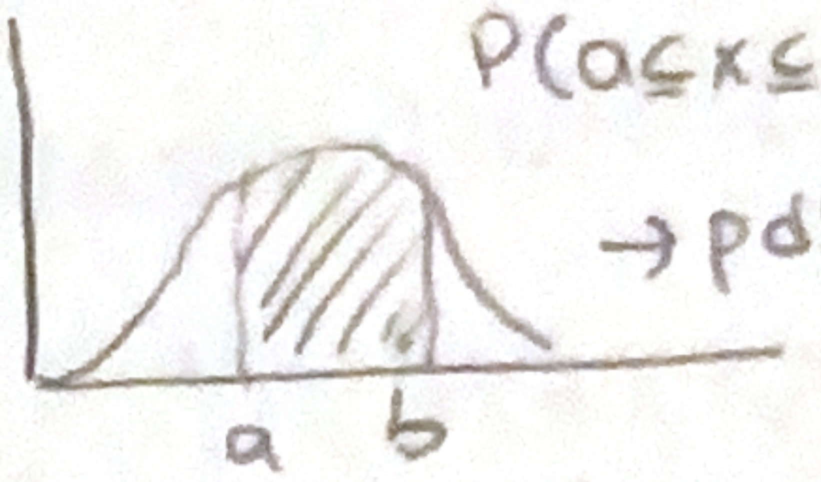
$Y = 1, 2$





$E(XY) = \sum_x \sum_y xy \, P(X = x, Y = y)$

Continuous RVs



$P(a \le x \le b) = \int_a^b f(x)\,dx$

→ pdf

$P(x = 0) \rightarrow P(a \le x \le b) = P(a < x < b)$
$= P(a \le x < b) = P(a < x \le b)$

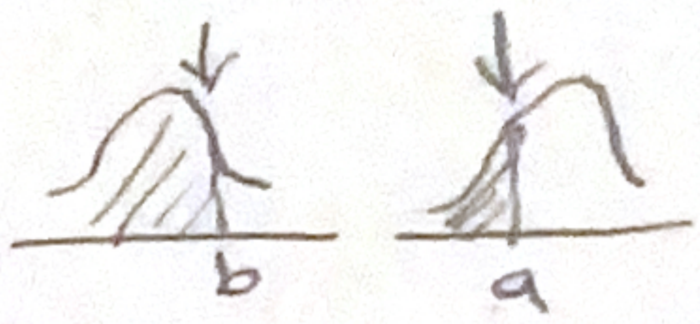
Cumulative Distribution Function

$F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)\,dy$

$P(X > a) = 1 - F(a)$

$P(a \le x \le b) = F(b) - F(a)$

PDF $\underset{\text{integrate}}{\overset{\text{derive}}{\leftrightarrows}}$ CDF



Expected Value

$\mu_X = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

$E[h(x)] = \mu_{h(x)} = \int_{-\infty}^{\infty} h(x) f(x)\,dx$

Variance

$V(X) = E[X^2] - E[X]^2$

Joint Distribution

$P(X, Y) = \iint f(x, y)\,dx\,dy$

$P(a \le x \le b, c \le y \le d) = \int_a^b \int_c^d f(x, y)\,dy\,dx$

$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\,dy$ ↳ integrate over y

↓ marginal of x

$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)\,dx$

Condition for Independence

$f_{X,Y}(x, y) = f_X(x) f_Y(y)$

$E[g(X, Y)] = \iint \underbrace{g(x, y)}_{x \cdot y} f(x, y)\,dx\,dy$

$E[XY] \longrightarrow x \cdot y.$