Expected Values, Covariance, Correlation

If independent, corr = 0

If corr = 0, not necessarily independent

Expected Values

Discrete RVs

- $E[x] = \sum x \, P_x(x)$
- $E[g(x)] = \sum g(x) p_x(x)$
- $E[h(x,y)] = \sum h(x,y) P_{xy}(x,y)$
  ↳ joint pmf

Continuous RVs

- $E[x] = \int_{-\infty}^{\infty} x f_x(x)\,dx$
- $E[g(x)] = \int_{-\infty}^{\infty} g(x) f_x(x)\,dx$
- $E[h(x,y)] = \iint h(x,y) f_{xy}(x,y)\,dx\,dy$
  ↳ joint pdf

Covariance → how strongly two RVs change

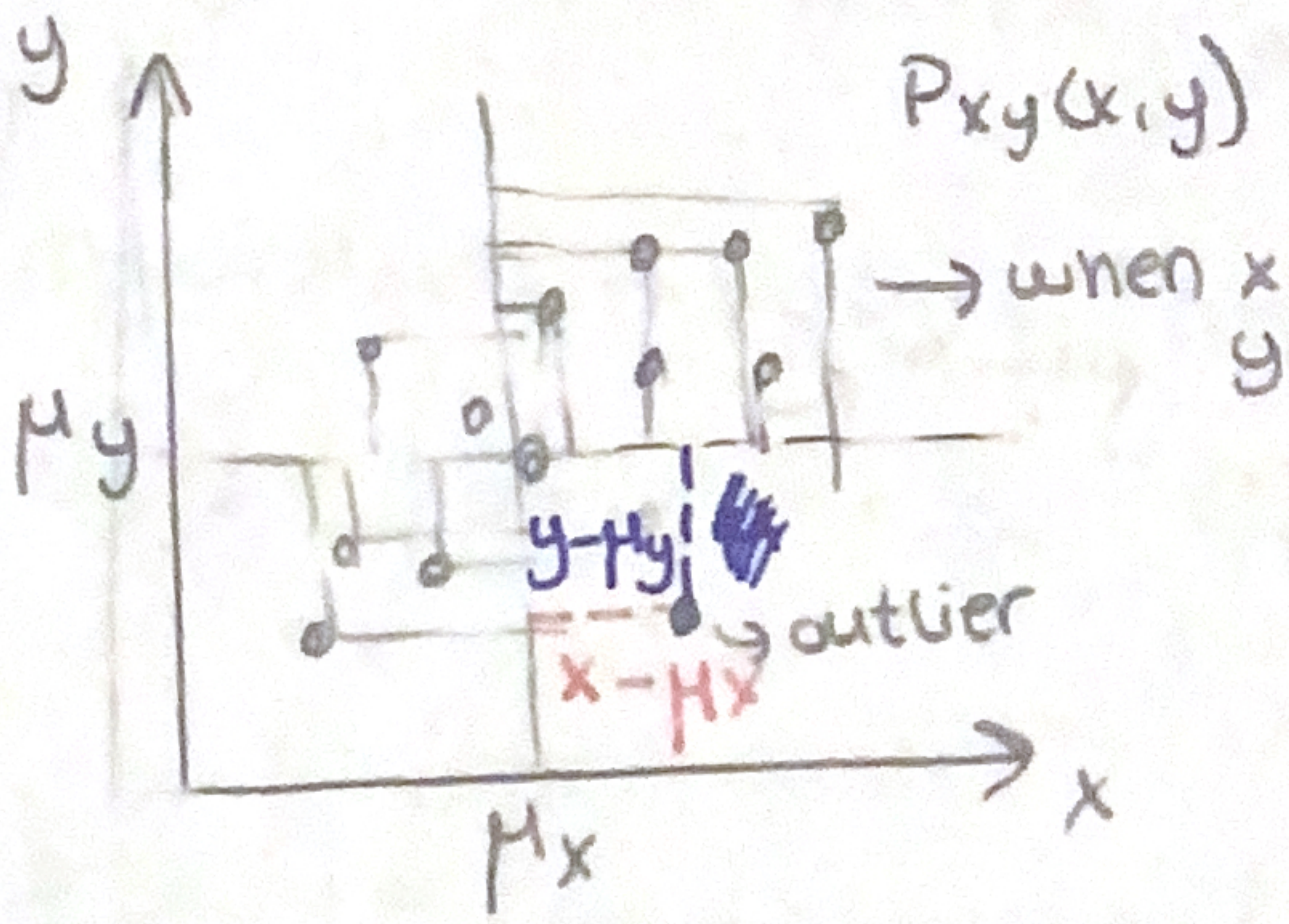
$E[x] = \mu_x$ x & y are joint random variables.

$E[y] = \mu_y$

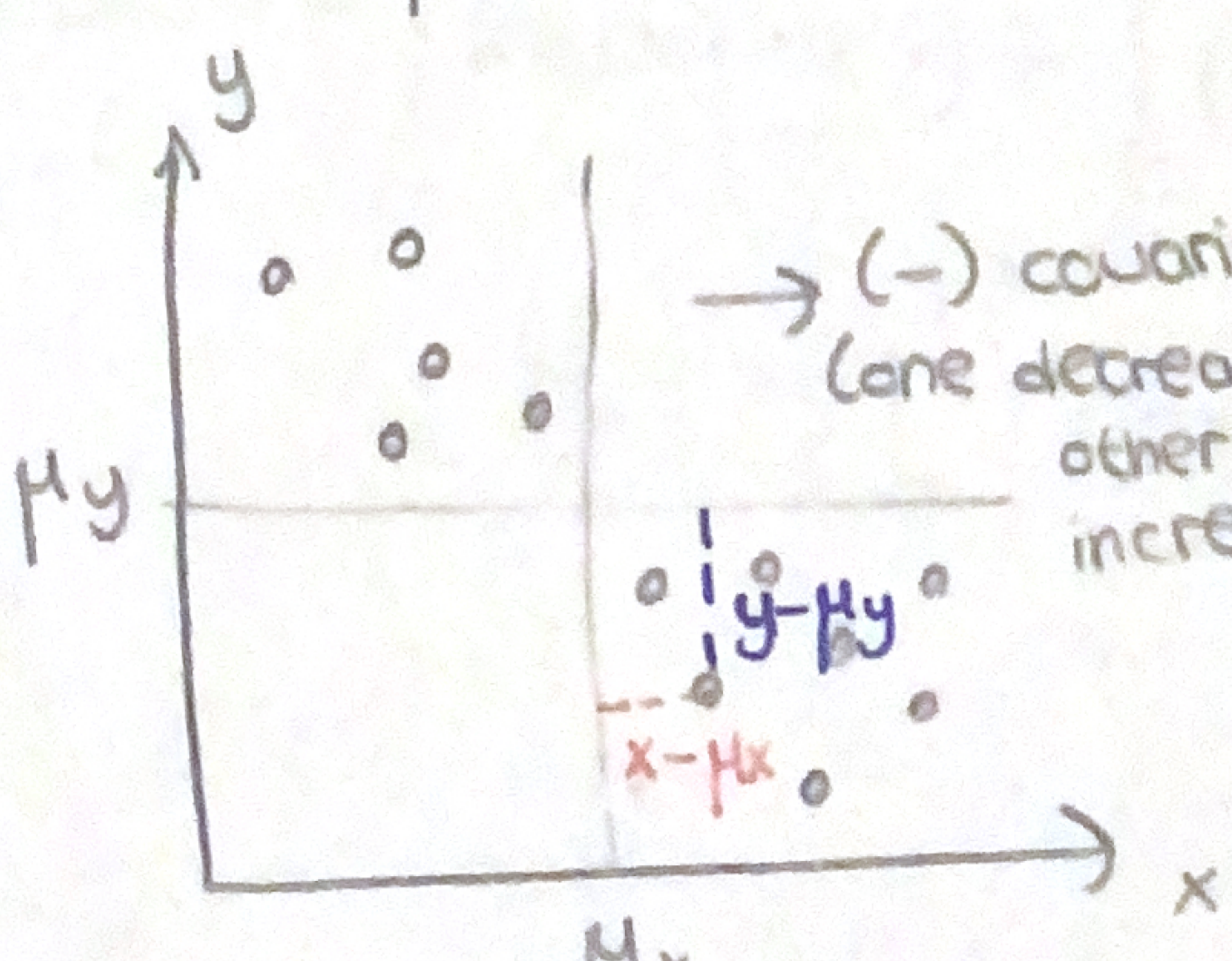
$$\text{Cov} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{N-1}$$

$$\text{Cov}(x,y) = E[(x-\mu_x)\cdot(y-\mu_y)]$$

- For Discrete x & y → $\sum_x \sum_y (x-\mu_x)(y-\mu_y) P_{xy}(x,y)$
- For Continuous x & y → $\iint (x-\mu_x)(y-\mu_y) f_{xy}(x,y)\,dx\,dy$



→ when x is (+) and (−)
y is (+) (−)
→ multiplication gives same sign, everything is positive



→ (−) covariance (one decreases, other one increases)

Covariance formula has units (e.g. kg × m) we need a metric for magnitude.

- $\rho_{x,y} = \dfrac{\text{Cov}(x,y)}{\sigma_x \sigma_y}$ } Correlation.

$$= \frac{E[(x-\mu_x)(y-\mu_y)]}{\sqrt{E[(x-\mu_x)^2]\,E[(y-\mu_y)^2]}}$$

$\left.\begin{array}{l}\rho_{x,x} = 1 \\ \rho_{x,-x} = -1\end{array}\right\}$ perfectly correlated
↳ $y = -x$

What if 2 vars are independent?

$\rho_{x,y} = 0$ (But not the other way around)