A new method for solving of vector problems for kinetic equations with Maxwell boundary conditions

A. V. Latyshev

Faculty of Physics and Mathematics, Moscow State Regional University, 105005, Moscow, Radio str., 10–A

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Abstract

1avlatyshev@mail.ru
In the present work the classical problem of the kinetic theory of gases (the Smoluchowsky’ problem about temperature jump in rarefied gas) is considered. The rarefied gas fills half-space over a flat firm surface. logarithmic gradient of temperature is set far from surface. The kinetic equation with modelling integral of collisions in the form of BGK-model (Bhatnagar, Gross and Krook) is used.

The general mirror-diffuse boundary conditions of molecules reflexions of gas from a wall on border of half-space (Maxwell conditions) are considered. Expanding distribution function on two orthogonal directions in space of velocities, the Smoluchowsky’ problem to the solution of the homogeneous vector one-dimensional and one-velocity kinetic equation with a matrix kernel is reduced.

Then generalization of source-method is used and boundary conditions include in non-homogeneous vector kinetic equation. The solution in the form of Fourier integral is searched. The problem is reduced to the solution of vector Fredholm integral equation of the second sort with matrix kernel.

The solution of Fredholm equation in the form of Neumann’s polynoms with vector coefficients is searched. The system vector algebraic interengaged equations turns out. The solution of this system is under construction in the form of Neumann’s polynoms. Comparison with well-known Barichello—Siewert’ high-exact results is made. Zero and the first approach of jumps of temperature and numerical density are received. It is shown, that transition from the zero to the first approach raises 10 times accuracy in calculation coefficients of temperature and concentration jump.

Key words: the Smoluchowski’ problem, collisional gas, temperature and concentration jump, vector Fredholm equation of second sort.

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1 Introduction

The problem about temperature jump is known from the end of XIX century [1]. M.Smoluhowsky has constructed in [1] the theory of temperature jump in the rarefied gas. Since then this problem the invariable attention already draws for a long time to itself (history of this question see in [2] and [3]).

This problem has been solved with use of approximate and numerical methods as for the modelling equations, and for full (non-linear) Boltzmann
In 1972 in work [19] the problem about temperature jump with use of the modelling Boltzmann equation with collisional integral BGK (Bhatnagar, Gross, Krook) and with frequency of collisions of the molecules, proportional to the module of velocities of molecules has analytically been solved.

Attempts of the exact solution of this problem about temperature jump with diffusion boundary conditions and with use of the modelling Boltzmann equation with collisional integral BGK with constant frequency of collisions of molecules [20] - [23] begin with this moment.

The solution of this problem encounters considerable difficulties. This problem is formulated in the form of a vector boundary problem. The solution of last problem meets the solution of a vector boundary value Riemann—Hilbert problem with the matrix coefficient, having points of branchings. These difficulties have been overcome only in 1990 in our work [24] where the analytical solution of the Smoluchowsky problem has been received.

By the method developed in [24], further problems about temperature jump in molecular gases [25], and also in metal [26] have been analytically solved.

Along with the Smoluchowsky’ problem the big interest represents studying of behaviour of gas at weak evaporation (condensation) from a surface. These problems are called as the generalized Smoluchowsky’ problem in view of that boundary conditions in these problems differ slightly.

Let us notice, that in works [17, 18] the various kinetic models were used, in particular, the model of Shakhov (or, S-model, see [27]).

For the solution of boundary half-space problems with accommodation some methods [28] - [30] have been developed, allowing to receive the solution of this problem with any degree of accuracy.

In the present work the generalized source-method from [30] extends on a vector case to which the problem about temperature jump is reduced. Thus the effective method of the solution of boundary problems with mirror - diffusion boundary conditions (Maxwell conditions) develops. We will notice, that the method from [30] has already been applied in problems of electro-
dynamics of plasma \[31\] and in condensate problems of Bose—Einstein.

At the heart of an offered method the idea lays to include the boundary condition in the form of a source in the kinetic equation.

The method basis consists in the following. At first in half-space \( x > 0 \) are formulated a vector problem about the temperature jump with boundary Maxwell conditions. Then unknown function continuations in conjugated half-space \( x < 0 \) in the even method on spatial and on velocity variables. In half-space \( x < 0 \) also are formulated the problem about temperature jump.

Now let us expand unknown function (which we will name also distribution function) on two composed: Chapman—Enskog’ distribution function \( h_{as}(x, \mu) \) and the second part of function distributions \( h_{c}(x, \mu) \), corresponding to continuous spectrum (see \[30\])

\[
h(x, \mu) = h_{as}(x, \mu) + h_{c}(x, \mu)
\]

\((as \equiv asymptotic, c \equiv continuous)\).

Owing to that Chapman—Enskog’ distribution function there is a linear combination of discrete solutions of the initial equation, function \( h_{c}(x, \mu) \) also is the solution of the kinetic equations. Function \( h_{c}(x, \mu) \) vanishes in zero far from wall. On a wall this function satisfies to boundary Maxwell condition.

Further we will transform the equation for function \( h_{c}(x, \mu) \). We include in this equation boundary condition on wall for function \( h_{c}(x, \mu) \) in the form of a member of source-type laying in a plane \( x = 0 \).

We will underline, that function \( h_{c}(x, \mu) \) satisfies to the received equation in both conjugated half-spaces \( x < 0 \) and \( x > 0 \).

We solve this equation in the second and the fourth quarters of a phase plane \( (x, \mu) \) as the linear differential equation of the first order, considering known the right part of the equation \( U_{c}(x) \). From the received solutions we found the boundary values of unknown function \( h^{\pm}(x, \mu) \) at \( x = \pm 0 \), entering into the equation.

Now we expand by Fourier integrals unknown function \( h_{c}(x, \mu) \), an unknown right part \( U_{c}(x) \) and Dirac delta-function. Boundary values of the unknown
functions $h_c^\pm(0, \mu)$ are thus expressed by the same integral on the spectral density $E(k)$ functions $U_c(x)$.

Substitution of Fourier integrals in the kinetic equation and expression for the right part $U_c(x)$ leads to the vector characteristic system of equations. If to exclude from this system the spectral density $\Phi(k, \mu)$ of function $h_c(x, \mu)$, we will receive vector Fredholm integral equation of the second sort.

Believing the gradient of the logarithm of temperature is setting, we will expand the unknown quantities of temperature and concentration jumps and also spectral density by polynomials on degrees of coefficient of diffusion $q$ (these are Neumann’s polynomials). On this way we receive system of the hooked equations on coefficients of polynomials for spectral density. Thus all equations on coefficients of spectral density have singularity (a pole of the second order in zero). Excepting these singularities consistently, we will construct all members of the polynomials for quantities of temperature and concentration jumps and for spectral density $E(k)$.

## 2 Statement problem

Let the rarefied one-nuclear gas occupies half-space $x > 0$ over the flat firm surface laying in a plane $x = 0$. Far from a wall the logarithmic gradient of temperature is set

$$g_T = \left( \frac{d \ln T}{dx} \right)_{x=+\infty}.$$

We take the stationary kinetic equation of relaxation type with collisional integral BGK (Bhatnagar, Gross and Krook)

$$v_x \frac{\partial f(x, v)}{\partial x} = \frac{f_{eq}(x, v) - f(x, v)}{\tau},$$

where $\tau$ is the time between two consecutive collisions of molecules, $\nu = 1/\tau$ is the collisional frequency of gaseous molecules, $f_{eq}$ is the equilibrium distribution function,

$$f_{eq}(x, v) = n(x) \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left( - \frac{m}{2kT(x)} v^2 \right),$$
where $m$ is the mass of molecule, $k$ is the Boltzmann constant, $T(x)$ is the
gas temperature,

$$T(x) = \frac{2E(x)}{3kn(x)}, \quad E(x) = \int \frac{m}{2} v^2 f(x, v) d^3v,$$

$n(x)$ is the gas number density (concentration),

$$n(x) = \int f(x, v) d^3v.$$

Further we will be linearize the kinetic equation and search distribution
function in the form

$$f(x, v) = f_0(v)(1 + \varphi(x, v), \quad (2.1)$$

where $f_0(v)$ is the absolute Maxwellian,

$$f_0(v) = n_0 \left( \frac{m}{2\pi kT_0} \right)^{3/2} \exp \left( -\frac{mv^2}{2kT_0} \right),$$

where $n_0, T_0$ are number density and gas temperature in some point, for
example, in origin of coordinates.

Let us be linierize distribution of numerical density and temperature concerning
parametres $n_0$ and $T_0$

$$n(x) = n_0 + \delta n(x), \quad T(x) = T_0 + \delta T(x).$$

According to (2.1) for distribution of numerical density we have

$$n(x) = \int f_0(v)[1 + \varphi(x, v)]d^3v = n_0 + \delta n(x),$$

where

$$n_0 = \int f_0(v)d^3v, \quad \delta n(x) = \int f_0(v)\varphi(x, v)d^3v.$$

For distribution of temperature we receive

$$T(x) = \frac{2}{3kn(x)} \int \frac{mv^2}{2} f_0(v)[1 + \varphi(x, v)]d^3v.$$

We notice that

$$\frac{n_0}{n(x)} = 1 - \frac{\delta n}{n_0} + o(h), \quad h \to 0.$$
Now for temperature we receive

\[ T(x) = T_0 \frac{2}{3n_0} \left( 1 - \frac{\delta n}{n_0} \right) \int \frac{mv^2}{2kT_0} f_0(v)[1 + \varphi(x, v)]d^3v = \]

\[ = \frac{2T_0}{3n_0} \left( 1 - \frac{\delta n}{n_0} \right) \int \frac{mv^2}{2kT_0} f_0(v)d^3v + \frac{2T_0}{3n_0} \left( 1 - \frac{\delta n}{n_0} \right) \int \frac{mv^2}{2kT_0} f_0(v)\varphi(x, v)d^3v. \]

We notice that

\[ \frac{2}{3n_0} \int \frac{mv^2}{2kT_0} f_0(v)d^3v = 1. \]

Hence, for relative change of temperature it is had

\[ \frac{\delta T}{T_0} = -\frac{\delta n}{n_0} + \frac{2}{3n_0} \int \frac{mv^2}{2kT_0} f_0(v)\varphi(x, v)d^3v. \]

We will be linearize equilibrium function of distribution

\[ f_{eq}(v) = f_0(v) \left[ 1 + \frac{\delta n(x)}{n_0} + \left( \frac{mv^2}{2kT_0} - \frac{3}{2} \right) \frac{\delta T(x)}{T_0} \right]. \]

We receive the following equation after linearizing the kinetic BGK–equation according to (2.1)

\[ v_x \frac{\partial \varphi(x, v)}{\partial x} = \nu \left[ \frac{\delta n(x)}{n_0} + \left( \frac{mv^2}{2kT_0} - \frac{3}{2} \right) \frac{\delta T(x)}{T_0} - \varphi(x, v) \right]. \]

Let us enter dimensionless velocities and parametres — dimensionless velocity of molecules \( C = \sqrt{\beta}v = \frac{v}{v_T} \), where \( \beta = \frac{m}{2kT_0} \), dimensionless time \( t_1 = \nu t \), dimensionless coordinate

\[ x_1 = \nu \sqrt{\frac{2kT_0}{m}} x = \frac{x}{v_T \tau} = \frac{x}{l}, \]

where \( l = v_T \tau \) is the mean free path of gaseous molecules, \( v_T = \frac{1}{\sqrt{\beta}} = \sqrt{\frac{2kT_0}{m}} \) is the thermal velocity of the molecules movements, having an order of velocity of a sound.

Now the kinetic equation will be transformed to the form

\[ C_x \frac{\partial \varphi}{\partial x_1} + \varphi(x_1, C) = \frac{\delta n(x_1)}{n_0} + \left( C^2 - \frac{3}{2} \right) \frac{\delta T(x_1)}{T_0}. \]
Here
\[ \frac{\delta n(x_1)}{n_0} = \frac{1}{\pi^{3/2}} \int e^{-C^2} \varphi(x_1, C) d^3C, \]
\[ \frac{\delta T(x_1)}{T_0} = \frac{2}{3\pi^{3/2}} \int e^{-C^2} \left( C^2 - \frac{3}{2} \right) \varphi(x_1, C) d^3C. \]

Further a variable \( x_1 \) we will designate again through \( x \).

Let us transform the linear kinetic equation to the form
\[ C_x \frac{\partial \varphi}{\partial x} + \varphi(x, v) = \frac{1}{\pi^{3/2}} \int K(C, C') \varphi(x, C) e^{-C'^2} d^3C' \quad (2.2) \]
with kernel
\[ K(C, C') = 1 + \frac{2}{3} \left( C^2 - \frac{3}{2} \right) \left( C'^2 - \frac{3}{2} \right). \]

It is easy to check up, that the equation (2.2) has the following partial solutions
\[ \varphi_1(x, \mu) = 1, \]
\[ \varphi_2(x, \mu) = C^2 - \frac{3}{2}, \]
\[ \varphi_3(x, \mu) = (x - C_x) \left( C^2 - \frac{5}{2} \right). \]

Let us construct asymptotic Chapman—Enskog distribution in the form of the linear combination of partial solutions of the equation (2.2) with arbitrary constants
\[ \varphi_{as}(x, \mu) = A_0 + A_1 \left( C^2 - \frac{3}{2} \right) + A_2 (x - \mu) \left( C^2 - \frac{5}{2} \right), \quad (2.3) \]
where \( A_0, A_1, A_2 \) are arbitrary constants.

For finding of these constants we will take advantage of definition of the macroscopical parameters. From definition of numerical density (concentration)
\[ n(x) = \int f(x, v) d^3v \]
follows, that the extrapolated concentration of gas on the wall is equal
\[ n_e = n_{as}(0) = \int f_{as}(0, v) d^3v = \int f_0(v) \left( 1 + \varphi_{as}(0, v) \right) d^3v = n_0 \left( \beta/\pi \right)^{3/2} \int \exp(-C^2)(1 + \varphi_{as}(0, C)) d^3v. \]
From here we have

\[
\frac{n_e}{n_0} = \pi^{-3/2} \int \exp(-C^2)(1 + \varphi_{as}(0, C)) \, d^3C,
\]
or

\[
\frac{n_e}{n_0} = 1 + \pi^{-3/2} \int \exp(-C^2)\varphi_{as}(0, C) \, d^3C.
\]

Hence, the quantity of jump of concentration is searched under the formula

\[
\frac{n_e - n_0}{n_0} = \varepsilon_n = \pi^{-3/2} \int \exp(-C^2)\varphi_{as}(0, C) \, d^3C.
\]

Substituting expression (2.3) in this equality, we have

\[
\varepsilon_n = A_0.
\]

Setting gradient of temperature far from a wall means, that temperature distribution in half-space looks like

\[
T(x) = T_e + \left(\frac{dT}{dx}\right)_{x=+\infty} \quad x = T_e + G_T x, \quad \text{где} \quad G_T = \left(\frac{dT}{dx}\right)_{x=+\infty}.
\]

This distribution we will present in the form

\[
T(x) = T_0\left(\frac{T_e}{T_0} + g_T x\right) = T_0 \left(1 + \frac{T_e - T_0}{T_0} + g_T x\right), \quad x \to +\infty,
\]
or

\[
T(x) = T_0(1 + \varepsilon_T + g_T x), \quad x \to +\infty, \quad (2.4)
\]

where

\[
\varepsilon_T = \frac{T_e - T_0}{T_0}
\]

is the required quantity of temperature jump.

From expression (2.4) it is visible, that relative change of temperature far from walls it is described by linear function

\[
\frac{\delta T_{as}(x)}{T_0} = \frac{T(x) - T_0}{T_0} = \varepsilon_T + g_T x, \quad x \to +\infty. \quad (2.5)
\]

Relative change of temperature we will present in the form

\[
\frac{\delta T(x)}{T_0} = \frac{2}{3} \pi^{-3/2} \int \exp(-C^2)(C^2 - \frac{3}{2})\varphi(x, C) \, d^3C.
\]
Far from a wall relative change of temperature transforms as follows

\[ \frac{\delta T_{as}(x)}{T_0} = \frac{2}{3\pi} \cdot 3^{3/2} \int \exp(-C^2)(C^2 - \frac{3}{2})\varphi_{as}(x, C) \, d^3C. \]

Substituting in this equality expression (2.3) for \( \varphi_{as}(x, C) \), we find, that

\[ \frac{\delta T(x)}{T_0} = A_2 + A_3x \quad (x \to +\infty). \quad (2.6) \]

Comparing expressions (2.5) and (2.6), we find, that \( A_2 = \varepsilon_T \) and \( A_3 = g_T \).

Thus, asymptotic part of function of distribution (at \( x \to +\infty \)) it is constructed and on the basis stated above transforms in the form

\[ \varphi_{as}(x, C) = \varepsilon_n + \varepsilon_T(C^2 - \frac{3}{2}) + g_T(x - C_x)(C^2 - \frac{5}{2}). \]

Let us formulate down boundary conditions to the equation (2.2). At first let us formulate mirror–diffusion boundary condition on a wall for full function of distribution

\[ f(+0, v) = qf_0(v) + (1 - q)f(+0, -v_x, v_y, v_z), \quad v_x > 0. \]

Here \( q \) is the accommodation coefficient, i.e. a part of the molecules flying after reflexion from a wall with Maxwell distribution on velocities, \( 1 - q \) is the part of the molecules reflected from a wall purely mirror.

Using (2.1), from here we receive a boundary condition of a problem onto wall

\[ \varphi(0, C) = (1 - q)\varphi(x, -C_x, C_y, C_z), \quad C_x > 0. \quad (2.7) \]

Let us demand, that far from a wall distribution function passed into Chapman—Enskog distribution with coordinate growth

\[ f(x, v) = f_0(v) \left[ 1 + \varepsilon_n + \varepsilon_T \left( \frac{mv^2}{2kT_0} - \frac{3}{2} \right) + g_T \left( x - \sqrt{\frac{m}{2kT_0}}v_x \right) \left( \frac{mv^2}{2kT_0} - \frac{5}{2} \right) \right] \quad x \to +\infty. \]

From here according to (2.1) for function \( \varphi \) we receive the following boundary conditions

\[ \varphi(x, \mu) = \varphi_{as}(x, \mu) + o(1), \quad x \to +\infty. \quad (2.8) \]
Here $\varphi_{as}(x, \mu)$ is the asymptotic Chapman–Enskog distribution, entered above.

So, the boundary problem about finding of jumps of temperature and concentration of gas (vapor) over a flat surface consists in finding of the such solution of the equation (2.2), which satisfies to boundary conditions (2.7) and (2.8).

3 Reduction to vector boundary problem

If to use substitution

$$\varphi(x, C) = h_1(x, \mu) + \left(C^2 - \frac{3}{2}\right) h_2(x, \mu), \quad \mu = C_x,$$

that equation (2.2) is reduced breaks up to two equations

$$\mu \frac{\partial h_1}{\partial x} + h_1(x, \mu) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu'^2} \left[h_1(x, \mu') + \left(\mu'^2 - \frac{1}{2}\right) h_2(x, \mu')\right] d\mu'$$

and

$$\mu \frac{\partial h_2}{\partial x} + h_2(x, \mu) = \frac{2}{3\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu'^2} \left[(\mu'^2 - \frac{1}{2}) h_1(x, \mu') + \left(\mu'^4 - \mu'^2 + \frac{5}{4}\right) h_2(x, \mu')\right] d\mu'.$$

This equation we will present in the vector form

$$\mu \frac{\partial h}{\partial x} + h(x, \mu) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu'^2} K(\mu') h(x, \mu') d\mu'.$$

Here $h(x, \mu)$ is the vector-column

$$h(x, \mu) = \begin{pmatrix} h_1(x, \mu) \\ h_2(x, \mu) \end{pmatrix},$$
and matrix kernel have the following form

\[ K(\mu) = \begin{pmatrix}
1 & \left(\mu^2 - \frac{1}{2}\right) \\
\frac{2}{3}\left(\mu^2 - \frac{1}{2}\right) & \frac{2}{3}\left[\left(\mu^2 - \frac{1}{2}\right)^2 + 1\right]
\end{pmatrix}. \]

The right part of the equation (3.2)

\[ U(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu'^2} K(\mu') h(x, \mu') d\mu' \]

has clear physical sense. The vector-column \( U(x) \) looks like

\[ U(x) = \begin{pmatrix}
\frac{\delta n(x)}{n_0} \\
\frac{\delta T(x)}{T_0}
\end{pmatrix}, \]

i.e., components of this vector consist of the relative changes of numerical density of gas and relative change temperatures (concerning equilibrium values \((n_0, T_0)\)). It is possible to present this vector in the form

\[ U(x) = U_{as}(x) + U_c(x), \]

where

\[ U_{as}(x) = \begin{pmatrix}
\frac{\delta n_{as}(x)}{n_0} \\
\frac{\delta T_{as}(x)}{T_0}
\end{pmatrix}, \quad U_c(x) = \begin{pmatrix}
\frac{\delta n_c(x)}{n_0} \\
\frac{\delta T_c(x)}{T_0}
\end{pmatrix}. \]

According to (3.1) from (2.7) and (2.8) for the vector-functions \( h(x, \mu) \) we receive the following vector boundary conditions

\[ h(+0, \mu) = (1 - q)h(+0, -\mu), \quad \mu > 0, \quad (3.3) \]

and

\[ h(x, \mu) = h_{as}(x, \mu) + o(1), \quad x \to +\infty, \quad (3.4) \]
where
\[ h_{as}(x, \mu) = \begin{pmatrix} \varepsilon_n \\ \varepsilon_T \end{pmatrix} + g_T(x - \mu) \begin{pmatrix} -1 \\ 1 \end{pmatrix}. \]

Function \( h_{as}(x, \mu) \) is the solution of the equation (3.2). Hence, if to search for the solution of the equation (3.2) in the form
\[ h(x, \mu) = h_{as}(x, \mu) + h_c(x, \mu), \quad (3.5) \]
then function \( h_c(x, \mu) \) is still searched from the equation (3.2)
\[ \mu \frac{\partial h_c}{\partial x} + h_c(x, \mu) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu'^2} K(\mu') h_c(x, \mu') d\mu'. \quad (3.6) \]
and boundary conditions (3.3) and (3.4) will be transformed thanking (3.5) to the following form
\[ h_c(+0, \mu) = h_0^+(\mu) + (1 - q) h_c(+0, -\mu), \quad \mu > 0, \quad (3.7) \]
\[ h_c(+\infty, \mu) = 0, \quad 0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (3.8) \]
Here
\[ h_0^+(\mu) = -q \begin{pmatrix} \varepsilon_n \\ \varepsilon_T \end{pmatrix} + (2 - q) g_T^+ \mu \begin{pmatrix} -1 \\ 1 \end{pmatrix}. \quad (3.9) \]

Let us solve further the problem consisting of the solution of the equation (3.6) with boundary conditions (3.7) – (3.10).

4 Kinetic equation with source

For solution of this problem the auxiliary problem is required to us in "negative" half-space \( x < 0 \). That it to formulate, we will continue function \( h(x, \mu) \) as follows
\[ h(x, \mu) = h(-x, -\mu). \quad (4.1) \]
Let us notice, that at continuation (4.1) logarithmic temperature gradient \( g_T \), which for "positive" half-spaces we will designate through \( g_T^+ \), changes the sign
\[ g_T^- = \left( \frac{d \ln T}{dx} \right)_{x=-\infty} = -\left( \frac{d \ln T}{dx} \right)_{x=+\infty} = -g_T^+. \]
Besides, we will notice, that function \( h_{as}(+0, \mu) \) automatically satisfies to equality (4.1): \( h_{as}(+0, \mu) = h_{as}(-0, -\mu) \). This equality means, that the equality (3.1) is carried out for function \( h_{as}(x, \mu): h_{as}(x, \mu) = h_{as}(-x, -\mu) \).

Hence, boundary conditions in "negative" space are formulated as follows

\[
h_c(-0, \mu) = h_0^-(\mu) + (1 - q)h_c(-0, -\mu), \quad \mu < 0,
\]

\[
h_c(-\infty, \mu) = 0.
\]

Here

\[
h_0^-(\mu) = -q \begin{pmatrix} \varepsilon_n \\ \varepsilon_T \end{pmatrix} + (2 - q)g_T^-\mu \begin{pmatrix} -1 \\ 1 \end{pmatrix}.
\]

Let us unite both problems — in "positive" and "negative" half-spaces — in one, having included boundary conditions in the kinetic equation by means of member of type of the source

\[
\mu \frac{\partial h_c}{\partial x} + h_c(x, \mu) = U_c(x) + |\mu|\delta(x) \left[ h_0^+(\mu) - q h_c(\mp 0, \mu) \right].
\]

(4.2)

Here

\[
U_c(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu^2 K(\mu)}h_c(x, \mu) d\mu,
\]

(4.3)

\[
h_0^+(\mu) = -q \begin{pmatrix} \varepsilon_n \\ \varepsilon_T \end{pmatrix} + (2 - q)g_T^+|\mu| \begin{pmatrix} -1 \\ 1 \end{pmatrix},
\]

\[
h_c(\mp 0, \mu) = \lim_{x \to \mp 0, \pm x < 0} h_c(x, \mu), \quad \pm \mu > 0.
\]

These function \( h_c(\mp 0, \mu) \) are finding from equalities

\[
h_c^+(x, \mu) = -\frac{1}{\mu} \int_{x}^{+\infty} e^{t/\mu}U_c(t)dt, \quad h_c^-(x, \mu) = \frac{1}{\mu} \int_{-\infty}^{x} e^{t/\mu}U_c(t)dt.
\]

(4.4)
5 Vector Fredholm equation of second sort

The solution of the equations (4.2) and (4.3) we search in the form of Fourier integrals

\[ U_c(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} E(k) dk, \quad \delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dk, \quad (5.1) \]

\[ h_c(x, \mu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} \Phi(k, \mu) dk. \quad (5.2) \]

From equalities (4.3) and (5.1) follows, that

\[ E(k) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu^2 K(\mu)} \Phi(k, \mu) dk. \quad (5.3) \]

Two following equalities follow from equalities (4.4)

\[ h_c^\pm(0, \mu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{E(k_1) dk_1}{1 + ik_1 \mu} = \frac{1}{\pi} \int_{0}^{\infty} \frac{E(k_1) dk_1}{1 + k_1^2 \mu^2}. \quad (5.4) \]

From the kinetic equation (4.2) by means of (5.4) it is found

\[ \Phi(k, \mu) = \frac{E(k)}{1 + ik \mu} - q \left( \begin{array}{c} \varepsilon_n \\ \varepsilon_T \end{array} \right) \frac{|\mu|}{1 + ik \mu} + \\
+ (2 - q) g \left( \begin{array}{c} -1 \\ 1 \end{array} \right) \frac{\mu^2}{1 + ik \mu} - \frac{|\mu|}{1 + ik \mu} q \int_{0}^{\infty} \frac{E(k_1) dk_1}{1 + k_1^2 \mu^2}. \quad (5.5) \]

Substituting (5.5) in (5.3), we come to the vector integral Fredholm equation of the second sort

\[ L(k) E(k) = -q \hat{T}_1(k) \left( \begin{array}{c} \varepsilon_n \\ \varepsilon_T \end{array} \right) + (2-q) g \hat{T}_2(k) \left( \begin{array}{c} -1 \\ 1 \end{array} \right) - q \int_{0}^{\infty} \hat{J}(k, k_1) E(k_1) dk_1. \quad (5.6) \]

Here \( L(k) \) is the dispersion matrix-function

\[ L(k) = E_2 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\mu^2 K(\mu)} d\mu}{1 + ik \mu} = \]
= E_2 - \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{e^{-\mu^2} K(\mu)d\mu}{1 + k^2\mu^2} = E_2 - \hat{T}_0(k),

where \( E_2 \) is the unit matrix of the second order,

\[
\hat{T}_n(k) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{e^{-\mu^2} K(\mu)\mu^n d\mu}{1 + k^2\mu^2}, \quad n = 1, 2, \cdots,
\]

the matrix kernel of integral Fredholm equation is defined by integral expression

\[
\hat{J}(k, k_1) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{e^{-\mu^2} K(\mu)\mu d\mu}{(1 + k^2\mu^2)(1 + k_1^2\mu^2)}.
\]

It is obvious, that

\[
\hat{J}(k, 0) = \hat{T}_1(k), \quad \hat{J}(0, k_1) = \hat{T}_1(k_1).
\]

6 Solution of vector Fredholm equation

For the solution of the equation (5.6) we will search in the form of Neumann’s polynomials

\[
E(k) = (2 - q)g_T \left[ E_0(k) + E_1(k)q + E_2(k)q^2 + \cdots + E_m(k)q^m \right]
\]

(6.1)

\[
\begin{pmatrix} \varepsilon_n \\ \varepsilon_T \end{pmatrix} = \frac{2 - q}{q} g_T \begin{pmatrix} \varepsilon_n^o + \varepsilon_n^1q + \varepsilon_n^2q^2 + \cdots + \varepsilon_n^mq^m \\ \varepsilon_T^o + \varepsilon_T^1q + \varepsilon_T^2q^2 + \cdots + \varepsilon_T^mq^m \end{pmatrix}.
\]

(6.2)

Let us substitute (6.1) and (6.2) in the equation (5.6). We receive system of the hooked equations

\[
L(k)E_0(k) = -\hat{T}_1(k) \begin{pmatrix} \varepsilon_1^o \\ \varepsilon_1^T \end{pmatrix} + \hat{T}_2(k) \begin{pmatrix} -1 \\ 1 \end{pmatrix},
\]

(6.3)

\[
L(k)E_1(k) = -\hat{T}_1(k) \begin{pmatrix} \varepsilon_1^o \\ \varepsilon_1^T \end{pmatrix} - \frac{1}{\pi} \int_0^\infty \hat{J}(k, k_1)E_0(k_1)dk_1,
\]

(6.4)

\[
L(k)E_m(k) = -\hat{T}_1(k) \begin{pmatrix} \varepsilon_m^o \\ \varepsilon_m^T \end{pmatrix} - \frac{1}{\pi} \int_0^\infty \hat{J}(k, k_1)E_{m-1}(k_1)dk_1, \quad m = 1, 2, \cdots
\]

(6.5)
Let us calculate in an explicit form the matrixes entering into the equation (5.6)

\[
L(k) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\mu^2} \begin{pmatrix} 1 & \frac{\mu^2 - 1}{2} \\ \frac{2}{3}(\mu^2 - \frac{1}{2}) & \frac{2}{3}(\mu^4 - \mu^2 + \frac{5}{4}) \end{pmatrix} \frac{d\mu}{1 + k^2\mu^2} = \\
= \begin{pmatrix} 1 - T_0(k) & -T_2(k) + \frac{1}{2}T_0(k) \\ -\frac{2}{3}(T_2(k) - \frac{1}{2}T_0(k)) & 1 - \frac{2}{3}(T_4(k) - T_2(k) + \frac{5}{4}T_0(k)) \end{pmatrix}.
\]

Here integrals are entered

\[
T_m(k) = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} \frac{e^{-t^2} \mu^m d\mu}{1 + k^2\mu^2}, \quad m = 0, 1, 2, \ldots.
\]

Let us notice, that the dispersion matrix \(L(k)\) is proportional to \(k^2\). Let us notice, that

\[
\frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2} K(t)dt = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} K(t)dt = E_2.
\]

Therefore the dispersion matrix–function is equal

\[
L(k) = E_2 - \hat{T}_0(k) = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2} K(t)dt - \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} \frac{e^{-t^2} K(t)}{1 + k^2t^2}dt = \\
= k^2 \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} \frac{e^{-t^2} K(t)t^2 dt}{1 + k^2t^2} = k^2 \hat{T}_2(k).
\]

(6.6)

Let us write out matrix elements \(\hat{T}_2(k)\)

\[
T_{11}^2(k) = T_2(k), \quad T_{12}^2(k) = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} \frac{e^{-t^2}(t^2 - 1/2)t^2 dt}{1 + k^2t^2} = T_4(k) - \frac{1}{2}T_2(k),
\]

\[
T_{21}^2(k) = \frac{2}{3} T_{12}^2(k) = \frac{2}{3} (T_4(k) - \frac{1}{2}T_2(k)), \quad T_{22}^2(k) = \frac{2}{3} (T_6(k) - T_4(k) + \frac{5}{4}T_2(k)).
\]
Therefore
\[ \hat{T}_2(k) = \begin{pmatrix} T_2(k) & T_4(k) - \frac{1}{2} T_2(k) \\ \frac{2}{3} \left( T_4(k) - \frac{1}{2} T_2(k) \right) & \frac{2}{3} \left( T_6(k) - T_4(k) + \frac{5}{4} T_2(k) \right) \end{pmatrix}. \]

Further we find a matrix \( \hat{T}_1(k) \)
\[ \hat{T}_1(k) = \begin{pmatrix} T_1(k) & T_3(k) - \frac{1}{2} T_1(k) \\ \frac{2}{3} \left( T_3(k) - \frac{1}{2} T_1(k) \right) & \frac{2}{3} \left( T_5(k) - T_3(k) + \frac{5}{4} T_1(k) \right) \end{pmatrix}. \]

Let us start the solution of zero approximation. From the equation (6.3) with by the help (6.6) it is received, that
\[ E_0(k) = \frac{1}{k^2} \hat{T}_2^{-1}(k) \left[ - \hat{T}_1(k) \begin{pmatrix} \varepsilon_n^o \\ \varepsilon_T^o \end{pmatrix} + \hat{T}_2(k) \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right] = \]
\[ = -\frac{1}{k^2} \left[ \hat{T}_2^{-1}(k) \hat{T}_1(k) \begin{pmatrix} \varepsilon_n^o \\ \varepsilon_T^o \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right]. \quad (6.7) \]

For existence of the zero solution (6.7) we will eliminate at it solution a pole of the second order.

Let us notice, that \( T_m(k) \) it is possible to present integrals in the form
\[ T_m(k) = T_m(0) - k^2 T_{m+2}(k), \]
where
\[ T_m(0) = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-\mu^2} \mu^m d\mu, \quad m = 0, 1, 2, \ldots. \]

By means of these equalities we will transform matrixes \( \hat{T}_1(k) \) and \( \hat{T}_1(k) \)
\[ \hat{T}_1(k) = \frac{1}{\sqrt{\pi}} \begin{pmatrix} 1 & 1 \\ 2 & 3 \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix}, \]
\[ -k^2 \begin{pmatrix} T_3(k) & T_5(k) - \frac{1}{2} T_3(k) \\ \frac{2}{3} \left( T_5(k) - \frac{1}{2} T_3(k) \right) & \frac{2}{3} \left( T_7(k) - T_5(k) + \frac{5}{4} T_3(k) \right) \end{pmatrix}. \]
and

\[ \hat{T}_2(k) = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 2 & 7 \\ 3 & 3 \end{pmatrix} \]

\[ -k^2 \begin{pmatrix} T_4(k) & T_6(k) - \frac{1}{2} T_4(k) \\ \frac{2}{3}(T_6(k) - \frac{1}{2} T_4(k)) & \frac{2}{3}(T_8(k) - T_6(k) + \frac{5}{4} T_4(k)) \end{pmatrix} \].

Let us rewrite these equalities in the matrix form

\[ \hat{T}_1(k) = \hat{T}_1(0) - k^2 \hat{T}_3(k), \quad \hat{T}_2(k) = \hat{T}_2(0) - k^2 \hat{T}_4(k). \]

Here

\[ \hat{T}_1(0) = \frac{1}{\sqrt{\pi}} \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 3 & 3 \end{pmatrix}, \quad \hat{T}_2(0) = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 2 & 7 \\ 3 & 3 \end{pmatrix}. \]

\[ \hat{T}_3(k) = \begin{pmatrix} T_3(k) & T_5(k) - \frac{1}{2} T_3(k) \\ \frac{2}{3}(T_5(k) - \frac{1}{2} T_3(k)) & \frac{2}{3}(T_7(k) - T_5(k) + \frac{5}{4} T_3(k)) \end{pmatrix}, \]

\[ \hat{T}_4(k) = \begin{pmatrix} T_4(k) & T_6(k) - \frac{1}{2} T_4(k) \\ \frac{2}{3}(T_6(k) - \frac{1}{2} T_4(k)) & \frac{2}{3}(T_8(k) - T_6(k) + \frac{5}{4} T_4(k)) \end{pmatrix}. \]

Let us return to equality (4.7) and by means of the previous equalities let us write down it in the form

\[ E_0(k) = \frac{1}{k^2} \hat{T}_2^{-1}(k) \left[ - \hat{T}_1(0) \begin{pmatrix} \varepsilon_n^o \\ \varepsilon_T^o \end{pmatrix} + \hat{T}_2(0) \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right] + \]

\[ + \hat{T}_2^{-1}(k) \left[ \hat{T}_3(k) \begin{pmatrix} \varepsilon_n^o \\ \varepsilon_T^o \end{pmatrix} - \hat{T}_4(k) \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right]. \]

For existence of zero approach we will demand, that the following equality was carried out

\[ \hat{T}_1(0) \begin{pmatrix} \varepsilon_n^o \\ \varepsilon_T^o \end{pmatrix} = \hat{T}_2(0) \begin{pmatrix} -1 \\ 1 \end{pmatrix}. \]
So, in zero approach taking into account (6.8) we receive, that
\[
\begin{pmatrix}
\varepsilon_n \\
\varepsilon_T
\end{pmatrix} = \frac{2 - q}{q} g_T \begin{pmatrix}
\varepsilon_n^o \\
\varepsilon_T^o
\end{pmatrix},
\]
and
\[
E(k) = (2 - q) g_T E_0(k),
\]
where
\[
E_0(k) = \hat{T}_2^{-1}(k) \begin{bmatrix}
\hat{T}_3(k) & \begin{pmatrix}
\varepsilon_n^o \\
\varepsilon_T^o
\end{pmatrix} - \hat{T}_4(k) \begin{pmatrix}
-1 \\
1
\end{pmatrix}
\end{bmatrix} = \\
= \hat{T}_2^{-1}(k) \begin{bmatrix}
\hat{T}_3(k) & \hat{T}_1^{-1}(0) \hat{T}_2(0) - \hat{T}_4(k)
\end{bmatrix} \begin{pmatrix}
-1 \\
1
\end{pmatrix}.
\]

7 The first and the higher approximations of solution

From the equation (6.4) by means of (6.6) it is received the following equation
\[
k^2 \hat{T}_2(k) E_1(k) = -\hat{T}_1(k) \begin{pmatrix}
\varepsilon_n^1 \\
\varepsilon_T^1
\end{pmatrix} - \frac{1}{\pi} \int_0^\infty \hat{J}(k, k_1) E_0(k_1) dk_1. \tag{7.1}
\]

Here the vector–column \(E_0(k_1)\) is defined by expression (6.9).
Let us notice, that
\[
\hat{J}(k, k_1) = \hat{T}_1(k_1) - k^2 \hat{J}_3(k, k_1).
\]
The equation (7.1) by means of this equality we will rewrite in the form
\[
k^2 \hat{T}_2(k) E_1(k) = -\hat{T}_1(0) \begin{pmatrix}
\varepsilon_n^1 \\
\varepsilon_T^1
\end{pmatrix} - \frac{1}{\pi} \int_0^\infty \hat{T}_1(k_1) E_0(k_1) dk_1 + \\
+ k^2 \begin{bmatrix}
\hat{T}_3(k) & \begin{pmatrix}
\varepsilon_n^1 \\
\varepsilon_T^1
\end{pmatrix} + \frac{1}{\pi} \int_0^\infty \hat{J}_3(k, k_1) E_0(k_1) dk_1
\end{bmatrix}. \tag{7.2}
\]

From the equation (7.2) it is visible, that for existence of the first approximation we should impose on free parametres of the solution the following vector
condition
\[
\left( \begin{array}{c}
\varepsilon_n^m \\
\varepsilon_T^m
\end{array} \right) = -\hat{T}_1^{-1}(0) \frac{1}{\pi} \int_0^\infty \hat{T}_1(k_1)E_0(k_1)dk_1. 
\] (7.3)

Then from the equation (7.2) it is found spectral density of our problem in the first approximation
\[
E_1(k) = \hat{T}_2^{-1}(k) \left[ \hat{T}_3(k) \left( \begin{array}{c}
\varepsilon_n^1 \\
\varepsilon_T^1
\end{array} \right) + \frac{1}{\pi} \int_0^\infty \hat{J}_3(k, k_1)E_0(k_1)dk_1 \right]. 
\] (7.4)

So, as first approximation the problem solution looks as the following the form
\[
E(k) = (2 - q)g_T \left[ E_0(k) + E_1(k)q \right],
\]
\[
\left( \begin{array}{c}
\varepsilon_n \\
\varepsilon_T
\end{array} \right) = \frac{2 - q}{q} g_T \left( \begin{array}{c}
\varepsilon_n^0 + \varepsilon_n^1q \\
\varepsilon_T^0 + \varepsilon_T^1q
\end{array} \right). 
\]

Let us consider approximation of an arbitrary \( m \)th order. From the equations (7.5) by means of (6.6) we will write down
\[
k^2\hat{T}_2(k)E_m(k) = -\hat{T}_1(0) \left( \begin{array}{c}
\varepsilon_n^m \\
\varepsilon_T^m
\end{array} \right) - \frac{1}{\pi} \int_0^\infty \hat{T}_1(k_1)E_{m-1}(k_1)dk_1 +
\]
\[
+k^2 \left[ \hat{T}_3(k) \left( \begin{array}{c}
\varepsilon_n^m \\
\varepsilon_T^m
\end{array} \right) + \frac{1}{\pi} \int_0^\infty \hat{J}_3(k, k_1)E_{m-1}(k_1)dk_1 \right], \quad m = 1, 2, \cdots. 
\] (7.5)

For existence \( m \)-th approximation we will impose the following condition on free parameters of solutions
\[
\left( \begin{array}{c}
\varepsilon_n^m \\
\varepsilon_T^m
\end{array} \right) = -\hat{T}_1^{-1}(0) \frac{1}{\pi} \int_0^\infty \hat{T}_1(k_1)E_{m-1}(k_1)dk_1, \quad m = 1, 2, \cdots. 
\] (7.6)

Now from the equation (7.5) it is found spectral density in \( m \)-th approximation
\[
E_m(k) = \hat{T}_2^{-1}(k) \left[ \hat{T}_3(k) \left( \begin{array}{c}
\varepsilon_n^m \\
\varepsilon_T^m
\end{array} \right) + \frac{1}{\pi} \int_0^\infty \hat{J}_3(k, k_1)E_{m-1}(k_1)dk_1 \right], \quad m = 1, 2, \cdots. 
\] (7.5)
8 Numerical calculations and comparison with exact solution

From the equation (6.8) it is found in zero approximation free parameters of the solution

\[
\begin{pmatrix}
\varepsilon_n^o \\
\varepsilon_T^o
\end{pmatrix} = \hat{T}_1^{-1}(0)\hat{T}_2(0) \begin{pmatrix}
-1 \\
1
\end{pmatrix} = \begin{pmatrix}
-0.55389 \\
1.10778
\end{pmatrix},
\]

or

\[
\varepsilon_n^o = \frac{-\varepsilon_T^o}{2} = -\frac{5\sqrt{\pi}}{16} \approx -0.55389, \quad \varepsilon_T^o = \frac{5\sqrt{\pi}}{8} \approx 1.10778.
\]

For comparison we will give exact values of the dimensionless coefficients of temperature jump and jump of numerical density [3] at diffusion reflection of molecules from the wall \( \varepsilon_T = 1.30272 \) и \( \varepsilon_n = -0.74428 \).

Error in zero approximation for temperature jump makes 15 %, and for jump of numerical density makes 25.5 %.

Let us consider the first approximation. From the equation (7.3) it is found

\[
\varepsilon_n^1 = -\frac{3\sqrt{\pi}}{8}(3D_1 - D_2), \quad \varepsilon_T^1 = \frac{\sqrt{\pi}}{4}(D_1 - 3D_2),
\]

where \( D_1 \) and \( D_2 \) are defined by the following expression

\[
\begin{pmatrix}
D_1 \\
D_2
\end{pmatrix} = \frac{1}{\pi} \int_0^\infty \hat{T}_1(k)E_0(k)dk.
\]

According to (6.9) for vector–column \( E_0(k) \) we receive the expression

\[
E_0(k) = \hat{T}_2^{-1}(k)C(k),
\]

where

\[
C(k) = \hat{T}_3(k) \begin{pmatrix}
\varepsilon_n^o \\
\varepsilon_T^o
\end{pmatrix} - \hat{T}_4(k) \begin{pmatrix}
-1 \\
1
\end{pmatrix}.
\]

From here we find elements of vector–column \( C(k) \)

\[
C_1(k) = \varepsilon_T^o[T_5(k) - T_3(k)] + \frac{3}{2}T_4(k) - T_6(k),
\]
and
\[ C_2(k) = \frac{2}{3} \left[ \varepsilon_T^* \left( T_7(k) - \frac{3}{2} T_5(k) + \frac{3}{2} T_3(k) \right) + 2T_6(k) - T_8(k) - \frac{7}{4} T_4(k) \right]. \]

Now according to (8.4) we find elements of the vector–column \( E_0(k) \)
\[ E_0^1(k) = \frac{1}{\det \hat{T}_2(k)} \left[ \frac{2}{3} C_1(k)(T_6(k) - T_4(k)) + \frac{5}{4} T_2(k) - C_2(k) \left( T_4(k) - \frac{1}{2} T_2(k) \right) \right], \]
and
\[ E_0^2(k) = \frac{1}{\det \hat{T}_2(k)} \left[ -\frac{2}{3} C_1(k)(T_4(k) - \frac{1}{2} T_2(k)) + C_2(k) T_2(k) \right], \]
where
\[ \det \hat{T}_2(k) = \frac{2}{3} \left[ T_2(k) T_6(k) - T_4^2(k) + T_2^2(k) \right]. \]

According to (8.3) it is found
\[ D_1 = \frac{1}{\pi} \int_0^\infty \left[ T_1(k) E_0^1(k) + \left( T_3(k) - \frac{1}{2} T_1(k) \right) E_0^2(k) \right] dk \]
and
\[ D_2 = \frac{1}{\pi} \int_0^\infty \left[ \frac{2}{3} \left( T_3(k) - \frac{1}{2} T_1(k) \right) E_0^1(k) + \frac{2}{3} \left( T_5(k) - T_3(k) + \frac{5}{4} T_1(k) \right) E_0^2(k) \right] dk. \]

Under formulas (8.1) and (8.2) it is found as the first approximation, that
\[ \varepsilon_n^1 = -0.21018 \quad \text{and} \quad \varepsilon_T^1 = 0.21378. \]

Thus, as a first approximation we find, that
\[ \begin{pmatrix} \varepsilon_n \\ \varepsilon_T \end{pmatrix} = \frac{2-q}{q} g_T \begin{pmatrix} -0.55389 - 0.21018q \\ 1.10778 + 0.21378q \end{pmatrix}. \]

Comparison with exact result at \( q = 1 \) shows, that as the first approximation an error in finding of temperature jump coefficient makes 1.4 %, and in finding of numerical density jump coefficient makes 2.7 %.

Let us spend comparison of the received results in the present work with the results received in [4] by high-precision method of discrete ordinates. Let us notice, that value of temperature jump coefficient at \( q = 1 \) from [4] in accuracy to equally exact value (see [3], p. 228).
Fig. Dependences $\varepsilon_T = \varepsilon_T(q)$ and $\varepsilon_n = \varepsilon_n(q)$. The first approximation of solution.

From given below table it is visible, that with decreasing of accommodation coefficient accuracy of the first approximation grows and at $q = 0.1$ the error makes hundredth fraction of percent.

| Table |
|-------|
| $q$ | 1   | 0.9 | 0.7 | 0.6 | 0.5 | 0.3 | 0.1 |
| [4]  | 1.30272 | 1.57026 | 2.31753 | 2.86762 | 3.62922 | 6.63051 | 21.45012 |
| Present article | 1.32156 | 1.58911 | 2.33522 | 2.88411 | 3.64401 | 6.64085 | 21.45400 |
| Error,% | -1.4% | -1.2% | -0.75% | -0.58% | -0.44% | -0.16% | -0.018% |

**Conclusion**

In the present work the Smoluchowsky’ problem about temperature jump with mirror–diffusion boundary conditions is solved. The kinetic equation is
used, received as a linearization result of the modelling kinetic Boltzmann equation in relaxation approximation (BGK–equation). Then the problem is reduced to the solution of half-space boundary problem for the vector kinetic equation with matrix kernel. The generalized method of a source develops. This method has been offered in [30]. Comparison with well-known Barichello–Siewert’ high-exact results is made. Zero and the first approach of jumps of temperature and numerical density are received. It is shown, that already the first approach leads to the results close to the exact. Further in this direction it is offered to use the models leading to correct Prandtl number, for example, S-model of Shakhov [27].

REFERENCES

[1] *Smoluchowski M.* Über Wärmeleitung in verdünnten Gasen// Ann. Phys. Chem. 1896. B. 64. S. 101-130.

[2] *Kolenchits O. A.* Thermal accommodation of systems gas–firm body// Minsk. Publishing house "Science and Technics". 1977. 126 pp.[russian]

[3] *Latyshev A.V., Yushkanov A.A.* Analytical methods in linetic theory. Monograph. Moscow.: Publishing "Moscow state regional university". 280 pp. 2008.[russian]

[4] *Barichello L.B., Siewert C.E.* The temperature–jump problems in rarefied gas–dynamics//Euro. J. Appl. Math. 2000. V. 11. P. 353-364.

[5] *Barichello L.B., Bartz A.C.R., Camargo M., Siewert C.E.* The temperature jump problems for a variable collision frequency model//Physics of Fluids. 2002. V. 14. №1. P. 383-391.

[6] *Loyalka S.K.* Slip and jump coefficients for rarefied gas flows: variational results for Lennard-Jones and n(r)–6 potentials//Physica A. 1990. V. 163. P. 813-821.
[7] Loyalka S.K., Siewert C.E., Thomas J.R., jr. Temperature jump problem with arbitrary accommodation//Phys. Fluids. 1978. V. 21. №5. P. 854-855.

[8] Siewert C.E., Thomas J.R., jr. Half-space problems in the kinetic theory of gases//Phys. Fluids. 1973. V. 16. №9. P. 1557-1559.

[9] Sharipov F., Seleznev V. Data on internal rarefied gas flows// J. Phys. Chem. Ref. Data, 1998, 27, p. 657-706.

[10] Thomas J.R., jr. Temperature jump problem with arbitrary accommodation//Phys. Fluids. 1973. V. 16. №1. P. 1162-1164.

[11] Thomas J.R., jr., Valougeorgis D. The $F_N$-Method in kinetic theory. 1. Half-Space Problems// Transport Theory and Stat. Physics. 1985. V. 14. №4. P. 485-496.

[12] Welander P. On the temperature jump in rarefied gas// Arkiv for Fysik. 1954. Bd. 7. № 44. P. 507–564.

[13] Sone Y., Ohwada T., Aoki K. Temperature jump and Knudsen layer in a rarefied gas over a plane wall: Numerical analysis of the linearized Boltzmann equation for hard–sphere molecules//Phys. Fluids A. 1989. V.1. P. 363-370.

[14] Onishi Y. Kinetic theory analysis for temperature and density fields of a slightly binary gas mixture over a solid wall//Phys. Fluids. 1997. V. 9. №1, P. 226-238.

[15] Loyalka S.K. Momentum and temperature-slip coefficients with arbitrary accommodation at the surface//J. Chem. Phys. 1968. V. 48. P. 5432-5436.

[16] Siewert C.E. The linearized Boltzmann equation: a concise and accurate solution of the temperature-jump problem//J. Quant. Spec. Rad. Trans. 2003. V. 77. P. 417-432.
[17] Kalempa D., Sharipov F. Temperature jump in rarefied gaseous systems//Proc. 10-th Brazilian Congress of Thermal Sciences and Engineering. 2004. Nov. 29 - Dec. 03. P. 1-7.

[18] Knackfuss R.F., Barichello L.B. On the temperature-jump problem in rarefied gas dynamics: the effect of the Cercignani—Lampis boundary conditions//SIAM J. Appl. Math. 2006. V. 66. №6, P. 2149-2186.

[19] Cassell J.S., Williams M.M.R. An exact solution of the temperature slip problem in rarefied gases//Transport Theory and Statistical Physics. 1972. V. 2(1). P. 81-90.

[20] Kriese J.T., Chang T.S., Siewert C.E. Elementary solutions of coupled model equations in the kinetic theory of gases//Intern. J. Engen. Sci. 1974. V. 12. №6. 441-470.

[21] Cercignani C. Analytic solution of the temperature jump problem for the BGK model//Transport Theory and Stat. Physics. 1977. V. 6. №1. P. 29-56.

[22] Siewert C.E., Kelley C.T. An analytic solution to a matrix Riemann–Hilbert problem//Transport Theory and Stat. Physics. 1980. V. 6. P. 344-351.

[23] Cercignani C., Siewert C.E. On partial indices for a matrix Riemann—Hilbert problem// J. Appl. Math. Phys. 1983. V. 33. P. 297-299.

[24] Latyshev A.V. Application of Case’ method to the solution of linear kinetic BGK equations in a problem about temperature jump// Appl. math. and mechanics. 1990. Т.54. Вып. 4. С. 581-586.[russian]

[25] Latyshev A.V., Yushkanov A.A. The temperature jump and slow evaporation in molecular gases// J. of experimental and theoretical physics. 1998, September. V. 87. №3. P. 518–526.

[26] Latyshev A.V., Yushkanov A.A. Smolukhovski problem for electrons in a metal// Theor. and Mathem. Phys. 2005. V. 142 (1). P. 79–95.
[27] Shakhov E.M. Method of research of movements of the rarefied gas. – Moscow. Science. 1974. 209 pp. [russian]

[28] Latyshev A.V., Yushkanov A.A. A Method for Solving Boundary Value Problems for Kinetic Equations// Comput. Maths and Math. Physics. Vol. 44. No. 6. 2004, pp. 1051–1061.

[29] Latyshev A.V., Yushkanov A.A. The Method of Singular Equations in Boundary Value Problems in Kinetic Theory// Theor. and Mathem. Physics. 2005. 143(3). P. 855–870.

[30] Latyshev A.V., Yushkanov A.A. A new method for solving the boundary value problem in kinetic theory // Zh. Vychisl. Mat. Mat. Fiz., 2012, 52:3, 539–552.

[31] Latyshev A.V., Yushkanov A.A. Electric Field in the Smoluchowski Problem in a Metal with an Arbitrary Coefficient of Specular Reflection// Comp. Maths and Math. Physics, 2010, Vol. 50, pp. 481–494.

[32] Latyshev A.V., Yushkanov A.A. Temperature jump in degenerate quantum gases with the Bogoliubov excitation energy and in the presence of the Bose–Einstein condensate// Theoretical and Mathematical Physics, 165(1): 1359–1371 (2010).