Towards Collinear Evolution Equations in Electroweak Theory

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Abstract

We consider electroweak radiative corrections to hard inclusive processes at the TeV scale, and we investigate how collinear logarithms factorize in a spontaneously broken gauge theory, similarly to the DGLAP analysis in QCD. Due to the uncancelled double logs noticed previously, we find a factorization pattern which is qualitatively different from the analogous one in QCD. New types of splitting functions emerge which are needed to describe the initial beam charges and are infrared-sensitive, that is dependent on an infrared cutoff provided, ultimately, by the symmetry breaking scale. We derive such splitting functions at one-loop level in the example of $SU(2)$ gauge theory, and we also discuss the structure functions' evolution equations, under the assumption that isospin breaking terms present in the Ward identities of the theory are sufficiently subleading at higher orders.
1 Introduction

The recent analysis of enhanced electroweak corrections at the TeV scale \[1\] has shown that, for \(s \gg M^2\), even inclusive observables take large contributions from both collinear and infrared singularities, arising in the limit of vanishing symmetry breaking scale \[2\]. The magnitude of such enhancements, involving the effective coupling \(\alpha_s \log^2 \frac{M^2}{M_W^2}\), raises the question of taking into account single logarithmic collinear contributions, a problem which corresponds to the usual DGLAP \[3\] analysis in QCD.

In a spontaneously broken gauge theory, the analysis of mass singularities is complicated by two peculiar features: firstly, the fact that initial states, like electrons and protons in the accelerator beams, carry nonabelian (isospin) charges and may be mixed charge states \[4\]. This feature, due to symmetry breaking at low energies, causes the very existence of double logs, i.e. the lack of cancellation of virtual corrections with real emission in inclusive observables. Secondly, the gauge theory Ward Identities are broken by Goldstone boson contributions \[5\] which are, however, proportional to the symmetry breaking scale and are therefore expected to be suppressed, to a certain extent, in the high energy limit. In a series of papers \[6\] we have analyzed the problem above at the leading double-log level by using, for \(\sqrt{s} \gg M_W\), Ward Identity restoration, and therefore isospin and charge conservation in the squared matrix elements. At this level, double-log effects are classified according to the total isospin \(T\) and/or total charge in the \(t\)-channel of the inclusive overlap matrix being considered.

In this paper we analyze, under the same assumptions, collinear singularities at leading single-logarithmic level and we argue that Ward identities are restored at least at one-loop level: this is sufficient in order to provide the analogue of DGLAP splitting functions for a broken gauge theory. A key feature of the result is the existence of a new kind of splittings, which correspond to a non-vanishing \(t\)-channel isospin \(T \neq 0\) and are infrared singular, in the sense that they are explicitly dependent on the infrared cutoff provided by the symmetry breaking scale, and are thus responsible for the double logs. We derive here the set of splitting functions in the example of a spontaneously broken SU(2) gauge theory. We limit ourselves to the case of initial light fermions and transverse bosons; longitudinal bosons and the Higgs sector will be considered elsewhere \[7\]. By further assuming, in this example, collinear factorization to all orders we also discuss the solution of the ensuing evolution equations, including the novel ones with \(T \neq 0\). Ultraviolet logarithms leading to running coupling effects are neglected throughout the paper.

2 Collinear Ward Identities

Consider for definiteness a lepton initiated Drell Yan process of type \(e^+(p_1) e(p_2) \rightarrow q(k_1)\bar{q}(k_2) + X\) where \(s = 2p_1 \cdot p_2\) is the total invariant mass and \(Q^2 = 2k_1 \cdot k_2\) is the hard scale. The differential cross section of such process is related to the overlap function \(\mathcal{O}_{\alpha_1\beta_1,\alpha_2\beta_2}(p_1, p_2, k_1, k_2)\) for \(\alpha_1 = \beta_1 = e\) and \(\alpha_2 = \beta_2 = e^+\), and we are interested in the collinear singular radiative corrections to the Born level overlap function \(\mathcal{O}^H\).

In order to compute such corrections, we first concentrate on the behavior of the contribution of an emitted boson closely parallel to an initial fermion leg \(p_1\). In the Feynman 't Hooft gauge three kinds of unitarity diagrams contribute to collinear singularities (Fig. \[8\]). The contributions in Fig. \[8\] contain a double pole factor due to the denominators \(2p_1 \cdot k\) and can be explicitly evaluated by performing the usual Dirac algebra. The one in Fig. \[8\], with a single denominator, includes the sum of vector boson insertion diagrams on all interfering legs but \(p_1\). It can be evaluated in the collinear limit \(k_\mu \propto p_1\mu\) thanks to the gauge theory Ward Identities.

The method of the collinear Ward Identities (CWI), introduced by Amati, Petronzio and Veneziano in QCD \[9\] consists in factoring the contributions in Fig. \[1\] by relating them to the remaining insertion on the outgoing \(p_1\) leg, plus Goldstone boson contributions. At one-loop level, the latter are of the type in Fig. \[2\], carrying a coupling of order \(M^2/s\) and are suppressed by a power \(M^2/s\) for initial light fermions, which do not couple explicitly to Goldstones (Fig. \[2\]). On the other hand, for initial transverse bosons, the suppression factor is rather \(M^2/k_T^2\) (Fig. \[2\]); therefore, upon \(k_T\) integration, they do not provide any collinear logs at all. Goldstone contributions are thus negligible in either case, at this level.

By then using unbroken CWI, the diagrams in Fig. 1(a,b), coupling the fermionic overlap matrix with itself, are evaluated as follows

\[
\delta \mathcal{O}_{\alpha_1,\beta_1} = \alpha W \int_0^s \frac{d^2 k}{k^2} \frac{d^2 z}{z} \int_0^1 \frac{k}{z} \left(1 - z + \frac{2z}{1 - z}\right) \left[t^\alpha \mathcal{O}^H(zp_1)t^\beta\right]_{\alpha_1,\beta_1}
\]

where \(t^\alpha\) denote the isospin matrices in the fundamental representation, and the first (second) term in square brackets correspond to Fig. 1(a) (1(b)) respectively. Let us note that the longitudinal fraction \(1 - z\) is here cutoff by \(\epsilon = \frac{k}{k_T}\).

The precise value of such cutoff is important for disentangling double and single logarithms and can be derived as follows. The emitted boson admits the Sudakov parameterization \(k^\mu = (1 - z)p_1^\mu + \bar{z}p_2^\mu + k_\perp\); \(\bar{z} = \frac{k^2 + M^2}{(1 - z)s}\). It is clear then that the angular region with the boson emitted in the forward hemisphere defined by \(p_1\) corresponds to \(1 - z > \bar{z} = \frac{k^2 + M^2}{1 - z s}\) so that we can set \(1 - z > \frac{k}{\sqrt{s}} > \frac{M}{\sqrt{s}}\), which are the integration boundaries appearing in 1(a).

*Here and in the following we adopt definitions and conventions used in 1(b).
Figure 1: Unitarity diagrams for one-loop real emission corrections to the inclusive hard overlap matrix $\mathcal{O}^{Hf}$. The symmetrical counterpart of diagram b) is not shown.

Figure 2: Examples of one-loop Goldstone boson contributions to CWI for (a) fermion-initiated and (b) boson-initiated processes. The wavy (dashed) lines label vector boson (Goldstone boson) exchanges, the small circles denote the $M$ couplings arising from the use of CWI's.
important to note that eq. (1) was derived in the collinear region $1 - z \gg \frac{k}{\sqrt{s}}$, but is still valid in the infrared region $1 - z = O\left(\frac{k}{\sqrt{s}}\right)$ because of the eikonal approximation derived in our previous work. Therefore the precise value of the cutoff is well determined by the above argument.

By including the contributions of fig. 1(c) and virtual contributions 3, we obtain

$$
\delta \mathcal{O}_{\alpha_1 \beta_1}^f = \frac{\alpha_W}{2\pi} \int_{M_2}^\infty d\ell^2 k_\perp^2 \int_0^1 \frac{dz}{z} \left\{ P_{ff}^R(z) \theta(1 - z - \frac{k}{\sqrt{s}}) \left[ t^a \mathcal{O}^{Hf}(zp_1) t^a \right]_{\alpha_1 \beta_1} + P_{ff}^R(z) \left[ t^A \mathcal{O}^{Hf}(zp_1) t^B \right]_{\alpha_1 \beta_1} + C_f P_{ff}^V(z, \frac{k}{\sqrt{s}}) \left[ \mathcal{O}^{Hf}(zp_1) \right]_{\alpha_1 \beta_1} \right\}
$$

(2)

The real-emission splitting function $P_{Rf}(z)$ occurring in eq. (2) is the same occurring in the unbroken theory; this is not surprising since we have assumed restored collinear Ward Identities. On the other hand the broken theory differs as follows:

- The structure function $F(z)$ is kept fixed, we will omit it from now on, with the understanding that, for instance, the structure functions will satisfy evolution equations with respect to a collinear cutoff.
- If nevertheless the factorization formula (4) is assumed, and CWI are supposed to be restored at higher orders as well, we have defined the convolution $P_{ff}^V(z, \frac{k}{\sqrt{s}})$ given in eq.(3) and
- $P_{bb}^R(z)$, $P_{bb}^R(z)$, $P_{bb}^V(z, \frac{k}{\sqrt{s}})$ where

$$
P_{bb}^R(z) = 2 \left( z(1 - z) + \frac{1 - z}{z} + \frac{z}{1 - z} \right) ; \quad P_{bb}^R(z) = z^2 + (1 - z)^2 ; \quad P_{bb}^V(z, \epsilon) = -\delta(1 - z) \left( \log \frac{1}{\epsilon} + \frac{11}{6} + \frac{n_f}{6} \right)
$$

(6)

Here $n_f$ is the number of fermion doublets, and the real emission cutoff $1 - z > \frac{k}{\sqrt{s}}$ is understood.

Because of the assumed recovered isospin invariance, eq. (4) can be decomposed in t-channel isospin components as follows:

$$
\mathcal{O}_{ij}(p_1, p_2; k_1, k_2) = \int \frac{dz_1 dz_2}{z_1 z_2} \sum_{s, M^2} \left\{ f_{k_1}^{(T)}(z_1; s, M^2) \mathcal{O}^{H(T)}_{k_1}(z_1 p_1, z_2 p_2; k_1, k_2) f_{k_2}^{(T)}(z_2; s, M^2) \right\}
$$

(7)

Then, by using the following projections, with a definite value of the t-channel isospin $T = 0, 1, 2$

$$
f_f^0 = \frac{1}{2} \text{Tr} \{ F_f \} = \frac{f_+ + f_-}{2} \quad f_f^1 = \text{Tr} \{ F_f t_3 \} = \frac{f_+ f_+ - f_-}{2} \quad f_f^2 = \frac{f_+ + f_- - 2 f_3}{2}
$$

(8)

$$
f_b^0 = \frac{f_+ + f_3 + f_-}{3} \quad f_b^1 = \frac{f_+ - f_-}{2} \quad f_b^2 = \frac{f_+ + f_- - 2 f_3}{6}
$$

(9)

3 Evolution Equations

If nevertheless the factorization formula (4) is assumed, and CWI are supposed to be restored at higher orders as well, the structure functions will satisfy evolution equations with respect to a collinear cutoff $\mu$ parameterizing the lowest value of $k_\perp$, as follows (see fig. 3 $t = \log \mu^2$):

$$
- \frac{\partial \mathcal{F}^{\alpha \beta}_{if}}{\partial t} = \frac{\alpha_W}{2\pi} \left\{ C_{if} \mathcal{F}^{\alpha \beta}_{if} \otimes P_{V}^f + (t^C \mathcal{F}^{\alpha \beta}_{if} t^C) \otimes P_{Rf}^f + (t^A \mathcal{F}^{AB}_{if} t^B) \otimes P_{Ab}^f \right\}
$$

(5a)

$$
- \frac{\partial \mathcal{F}^{\alpha \beta}_{if}}{\partial t} = \frac{\alpha_W}{2\pi} \left\{ C_{if} \mathcal{F}^{AB}_{if} \otimes P_{V}^f + (t^C \mathcal{F}^{AB}_{if} T^C) \otimes P_{Ab}^f + \text{Tr}(t^A \mathcal{F}^{AB}_{if} t^B) \otimes P_{Ab}^f \right\}
$$

(5b)

In these equations $T^a$ ($t^a$) denote the isospin matrices in the adjoint (fundamental) representation and $\mathcal{F}^{\alpha \beta}_{ij}$ denotes the distribution of a particle $i$ (whose isospin indices are omitted) inside particle $j$ (with isospin leg indices $\alpha, \beta$); the trace is taken, and here in the following, with respect to the indices of the soft scale leg $j$. Since the index $i$ is always kept fixed, we will omit it from now on, with the understanding that, for instance, $\mathcal{F}_{Tf}$ collectively denotes all $\mathcal{F}_{Tf}$ with any value of $i$. Furthermore, we have defined the convolution $[f \otimes P](x) = \int x P(z) f(\frac{z}{x}) \frac{dz}{z}$.

The relevant distributions are $P_{Rf}^f(z)$, $P_{Rf}^f(z)$, $P_{Rf}^f(z)$ given in eq.(3) and

$$
P_{bb}^R(z) = \frac{1}{2} \left( (1 - z)^2 + \frac{1 - z}{z} + \frac{z}{1 - z} \right) ; \quad P_{ab}^V(z, \epsilon) = -\delta(1 - z) \left( \log \frac{1}{\epsilon} + \frac{11}{6} + \frac{n_f}{6} \right)
$$

(6)
Therefore, notice however that, for $T$ region isospin quantum numbers is done [2], is apparent in the first (virtual) term of eqs. (10), for behavior due to the singularity at $z=1$ is exposed. This lack of compensation, due to the fact that no averaging over isospin quantum numbers is done [2], is apparent in the first (virtual) term of eqs. (10), for $T=1$. For instance we have:

$$-\frac{d}{dt} f^0_{T} = \frac{\alpha W}{2\pi} \left\{ \frac{3}{4} f^0_\alpha \otimes (P_{ff}^R + P_{ff}^V) + \frac{3}{4} f^0_\beta \otimes P_{ff}^R \right\}$$

(10a)

$$-\frac{d}{dt} f^0_{T} = \frac{\alpha W}{2\pi} \left\{ 2 f^0_\alpha \otimes (P_{bb}^R + P_{bb}^V) + \frac{1}{2} (f^0_\alpha + f^0_\beta) \otimes P_{bb}^R \right\}$$

(10b)

$$-\frac{d}{dt} f^1_{T} = \frac{\alpha W}{2\pi} \left\{ f^1_\alpha \otimes P_{ff}^V - \frac{1}{4} f^1_\alpha \otimes (P_{ff}^R + P_{ff}^V) + \frac{1}{2} f^1_\alpha \otimes P_{bb}^R \right\}$$

(10c)

$$-\frac{d}{dt} f^1_{T} = \frac{\alpha W}{2\pi} \left\{ f^1_\alpha \otimes P_{bb}^R + f^1_\beta \otimes (P_{bb}^R + P_{bb}^V) + \frac{1}{2} (f^1_\alpha + f^1_\beta) \otimes P_{bb}^R \right\}$$

(10d)

$$-\frac{d}{dt} f^2_{T} = \frac{\alpha W}{2\pi} \left\{ 3 f^2_\alpha \otimes P_{bb}^V - f^2_\beta \otimes (P_{bb}^R + P_{bb}^V) \right\}$$

(10e)

with similar equations holding for $f^0_{T}$. Notice that for $T=0$, real and virtual contributions carry the same charge factor so that the distribution $P_{ii}^R(z) + P_{ii}^V(z)$ is regularized and the cutoff $k^2_T$ is irrelevant in the strong-ordering region $M \ll k_T \ll \sqrt{s}$. On the other hand for $T \neq 0$ real and virtual emission do not cancel and the double-log behavior due to the singularity at $z=1$ is exposed. This lack of compensation, due to the fact that no averaging over isospin quantum numbers is done [2], is apparent in the first (virtual) term of eqs. (10), for $T=1$.2.

Eqs. (10) can be partly diagonalized by introducing the valence-like quantities $V^T = \frac{f^T_f - f^T_e}{2}$, in terms of which we have:

$$-\frac{dV^0}{dt} = \frac{\alpha W}{2\pi} \frac{3}{4} V^0 \otimes (P_{ff}^R + P_{ff}^V) \quad -\frac{dV^1}{dt} = \frac{\alpha W}{2\pi} V^1 \otimes \left\{ P_{ff}^V - \frac{1}{4} (P_{ff}^R + P_{ff}^V) \right\}$$

(11)

Therefore, $f^0_{T}, V^0$ and $V^1$ satisfy single-channel evolution equations, while the remaining ones couple sea-like and boson distributions. Notice however that, for $T=1$ these are not the usual “valence” or “sea” combinations appearing in QCD, because of the $t_3\beta$ weight in eq. (11). For instance we have:

$$V^0 = \frac{f_\nu - f_\bar{\nu}}{2} + \frac{f_\bar{e} - f_\bar{\bar{e}}}{2} \quad V^1 = \frac{f_\nu + f_\bar{\nu}}{2} - \frac{f_\bar{e} + f_\bar{\bar{e}}}{2}$$

(12)

The explicit solution of eqs. (11) is obtained as in QCD for $T=0$ where only single logarithms occur. For $T \neq 0$, on the other hand, it is always possible to extract the double-log contribution by defining new distributions $\tilde{f}$ whose evolution equations are regular and can be solved as in QCD. For instance, for fermions we set:

$$f^T_{f}(x; s, M^2) = \exp \left[ \frac{\alpha W}{4\pi} \frac{T(T+1)}{2} \int_{0}^{1} dz \int_{M^2}^{s} \frac{dk_{\perp}^2}{k_{\perp}^2} P_{ff}^V(z, \frac{k_{\perp}}{\sqrt{s}}) \right] f^T_{f}(x; s, M^2)$$

$$f^T_{e}(x; s, M^2) = \exp \left[ -\frac{\alpha W}{4\pi} \frac{T(T+1)}{2} (\log \frac{s}{M^2} - 3 \log \frac{s}{M^2}) \right] f^T_{f}(x; s, M^2)$$

(13)
where we have taken $k_\perp = M$ ($k_\perp = \sqrt{s}$) as lower (upper) bound of the $k_\perp$ integration. It is then straightforward to check that indeed the $f_f^T$'s satisfy regularized evolution equations, whose splitting functions are read off in the second term of eqs. (10) for $T = 1$.

Finally, the overlap matrix $O_{ij}$ itself is determined from eq. (11) on the basis of the hard overlap matrix $O_{kl}^{H(T)}$, to be calculated perturbatively for the various $T$ values, given the initial isospin states of the process.

To summarize, we have derived the splitting functions and written down the evolution equations for radiative corrections of electroweak type in a spontaneously broken theory. Our analysis in the various t-channel isospin $T$ components shows that for $T = 0$ the equations are quite similar to the DGLAP ones in QCD. However, for $T \neq 0$, the infrared singularity is exposed, originating double logarithms that are absent in the unbroken theory. Further developments require showing that indeed Goldstone boson insertions in the Ward Identities can be neglected at higher orders, as assumed in this paper. They also require considering longitudinal degrees of freedom, which mix with Higgs states and have a more complicated group structure, as noticed in [9] at double-log level. But, on the whole, the route for writing the collinear evolution equations in the full Standard Model is open.

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1 One could have taken $k_\perp = Q$ instead of $k_\perp = \sqrt{s}$ as upper bound. This difference does not contribute any logarithms to eq. (11), because $Q/\sqrt{s} = O(1)$, and is therefore a subleading effect.