Slowly rotating pulsars

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In the present work we investigate one possible variation on the usual static pulsars: the inclusion of rotation. We use a formalism proposed by Hartle and Thorne to calculate the properties of rotating pulsars. All calculations were performed for zero temperature and fixed entropy equations of state.

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I. INTRODUCTION

Pulsars are believed to be the remnants of supernova explosions. They have masses 1–2\(M_\odot\), radii \(\sim 10\text{km}\), and a temperature of the order of \(10^{11}\text{K}\) at birth, cooling within a few days to about \(10^{10}\text{K}\) by emitting neutrinos. Pulsars are normally known as neutron stars. Qualitatively, a neutron star is analogous to a white dwarf star, with the pressure due to degenerate neutrons rather than degenerate electrons. The assumption that the neutrons in a neutron star can be treated as an ideal gas is not well justified: the effect of the strong force needs to be taken into account by replacing the equation of state (EoS) for an ideal gas by a more realistic EoS. The composition of pulsars remains a source of speculation, with some of the possibilities being the presence of hyperons \([1,2,3]\), a mixed phase of hyperons and quarks \([4,5,6,7,8]\), a phase of deconfined quarks or pion and kaon condensates \([9]\). Another possibility would be that pulsars are, in fact, quark stars \([11]\). In conventional models, hadrons are assumed to be the true ground state of the strong interaction. However, it has been argued \([12,13,14,16]\) that strange matter composed of deconfined \(u, d\) and \(s\) quarks is the true ground state of all matter. In the present work we have opted to use the term quark stars instead of strange stars because in more refined nuclear structure models, as the Nambu-Jona-Lasinio model for instance, the \(s\) quark appears in a much smaller quantity than in the very naive MIT model, where it is considered in equal footing with \(u\) and \(d\) quarks \([11]\). In the stellar modeling, the structure of the star depends on the assumed EoS, which is different in each of the above mentioned cases. An important distinction between quark stars and conventional neutron stars is that the quark stars are self-bound by the strong interaction, whereas neutron stars are bound by gravity. This allows a quark star to rotate faster than would be possible for a neutron star \([11,16]\).

Once an adequate EoS is chosen, it is used as input to the Tolman-Oppenheimer-Volkoff (TOV) equations \([17]\), which are derived from Einstein’s equations in the Schwarzschild metric for a static, spherical star. Some of the stellar properties, as the radius, gravitational and baryonic masses, central energy densities, etc. are obtained. These results are then tested against some of the constraints provided by astronomers and astrophysicists \([18,19]\) and some of the EoS are shown to be inappropriate for describing pulsars \([5,6,9]\).

On the other hand, it is well known from the Doppler broadening of the pulsars spectral lines that they rotate. Some pulsars are observed to rotate with periods as small as 1.56 ms \([10]\). Newly born hot pulsars can rotate fast and hence undergo instabilities. The effect of a rotating star on spacetime is commonly known as the Lense-Thirring effect, which refers to the dragging of local inertial frames. Rotation breaks spherical symmetry, but a rotating star is still axially symmetric. In this case, the TOV equations are no longer valid. Hartle and Thorne proposed a perturbative approximation to treat rotating stars \([20]\). The method was further developed and it is valid not only for slowly rotating stars, but also for stars rotating up to the Kepler frequency \([21]\). In a more recent work \([22]\) the Hartle-Thorne approximation was tested against a full general relativistic numerical model available as part of the numerical relativity library LORENE and it was shown that it is reliable for most astrophysical applications. In \([22]\) some EoS were used to compute rotating star properties, all of them obtained at zero temperature.

In the present work we use the Hartle-Thorne formalism to calculate the maximum mass, moment of inertia and eccentricity of all different classes of possible pulsars described above. In previous works \([22,23,24]\), many EoS have been investigated, but they were restricted to hadronic matter at \(T = 0\). In the present paper we investigate all possible classes of pulsars (hadronic, hybrid and quark stars) at zero and finite temperature, to account also for protoneutron stars. It is important to distinguish between the EoS during the short time period when neutrinos are still trapped in the star, and the EoS after the neutrinos escape. Next we restrict ourselves to the second case, when pulsars are believed to be already stable stars. Notice also that the temperature in the interior of the star is not constant \([8,25]\), but the entropy per baryon is. This is the reason for choosing fixed entropies to take the temperature effects into account. The maximum entropy per baryon (\(S\)) reached in the core of a new born star is about 2 (in units of Boltzmann’s constant) \([20]\). We then use EoS obtained with \(S = 0\) (\(T = 0\)), 1 and 2.
Other important motivations for revisiting rotating stars is the fact that as they slow down, the decreasing centrifugal force leads to increasing core pressure and density. A softening of the EoS takes place and it can be the result of a phase transition to quark matter [27]. If a first order phase transition takes place at central densities, it was shown that the moments of inertia diverge in a characteristic way and the braking index diverges [28]. Gamma ray bursts are also linked with stellar phase transitions [29] and the relation between gamma ray bursts and phase transitions in stellar matter under rotation should also be extensively considered. The present work is the seed for this investigation since we consider only a few possibilities for the EoS at $T = 0$.

This paper is organized as follows: in Sec. II the formalisms of the Hartle-Thorne approximation are revisited and the results are presented. In Sec. III the results are discussed and the main conclusions are drawn.

II. FORMALISM AND RESULTS

As the first step we need to know the EoS of the system,
\[ \epsilon = \epsilon(p), \quad n = n(p), \]
where $p$ is the pressure, $\epsilon$ is the energy density, and $n$ is the number density of baryons. Once an adequate EoS is obtained, it can be used to provide the stellar properties. If we opt for a non-rotating configuration, the Tolman-Oppenheimer-Volkoff (TOV) equations [17], which are derived from Einstein’s equations in the Schwarzschild metric for a static, spherical star are used to compute the stellar properties, as the radius, gravitational and baryonic masses and central energy densities. If we opt for its rotating counterpart, the same EoS is used, but the Hartle-Thorne formalism is used instead of the TOV approximation.

A. Hartle-Thorne formalism

The simplest nontrivial form that Einstein’s equations take is the form for spherically symmetric stars. In this case, the line element has the form,
\[ ds^2 = e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \] (1)

However for rotating stars the spherical symmetry is broken, but the star maintains axial symmetry. The expression for the line element for axially symmetric spacetime is given by
\[ ds^2 = e^{\nu(r, \theta)} dt^2 - e^{\lambda(r, \theta)} dr^2 - e^{\mu(r, \theta)} [r^2 d\theta^2 + r^2 \sin^2 \theta (d\phi - \omega(r, \theta) dt)^2]. \] (2)

The perfect fluid has a stress-energy tensor
\[ T^{ab} = (\epsilon + p) u^a u^b + pg^{ab}, \] (3)
where $u^a$ is the 4-velocity of the fluid. By using (2), (3) and the Ricci tensor we obtain a system of partial differential equations in which the solution is numerically complicated. In the Hartle-Thorne method, rotation is treated as a perturbation on the non-rotating configuration of the star and it gives spherical and quadrupole deformations. Within this method the expansion of the metric has the form
\[ e^{\lambda(r, \theta)} = e^\lambda \left[ 1 + 2\left(m_0 + m_2 P_2(\cos(\theta))\right)\right], \]
\[ e^{\nu(r, \theta)} = e^\nu \left[ 1 + 2(h_0 + h_2 P_2(\cos(\theta))\right), \] (4)
\[ e^{\mu(r, \theta)} = e^\mu \left[ 1 + 2(v_2 - h_2) P_2(\cos(\theta))\right], \]
where $P_2$ is the Legendre polynomial of second order; and $h_0, h_2, m_0, m_2$ and $v_2$ are all functions of $r$.

A non-rotating configuration is computed by integrating the Tolman-Oppenheimer-Volkoff equations [17] of hydrostatic equilibrium for the pressure, $p(r)$, and the mass interior to a given radius, $m(r)$:
\[ \frac{dp}{dr} = -(\epsilon + p) \frac{m + 4\pi r^3 p}{r(r - 2m)}, \]
\[ \frac{dm}{dr} = 4\pi r^2 \epsilon, \]
\[ \frac{dr}{d\nu} = -2(\epsilon + p)^{-1} \frac{dp}{dr}, \] (5)
where the boundary conditions are $m = 0$ at $r = 0$, the radius surface $R$ is defined so that $\nu = 0$ when $r \to \infty$. Here, $\nu$ is the metric function. In this work we use natural units where $G = c = 1$.

The quantity $\varpi = \Omega - \omega$ is the angular velocity of the fluid relative to the local inertial frame. In the classical mechanics the magnitude of the centrifugal force is determined by the angular velocity $\Omega$. However, in the general relativity the magnitude of the centrifugal force is determined by $\varpi$:
\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 j \frac{d\varpi}{dr} \right) + \frac{2}{r} \frac{dj}{dr} \varpi = 0, \] (6)
where
\[ j(r) = e^\nu/2 [1 - 2m/r]^{1/2}. \] (7)
The boundary conditions $\varpi = \varpi_c$ and $d\varpi_c/dr = 0$ are imposed. When $r = R$ we can determine the angular velocity, $\Omega$, and angular momentum, $J$, corresponding to $\varpi_c$:
\[ J = \frac{1}{6} R^4 \left( \frac{d\varpi}{dr} \right)_{r=R}, \quad \Omega = \varpi(R) + \frac{2J}{R^3}. \] (8)
To obtain a different value of $\Omega$ we need to rescale the function $\varpi_c(r)$:
\[ \varpi_{new}(r) = \varpi_{old}(\Omega_{new}/\Omega_{old}). \] (9)
The metric function $h_0$ obeys the following algebraic relations

\[
\begin{align*}
    h_0 &= -\frac{m_0 + J^2/r^3}{r - 2m} + \frac{J^2}{r^3(r - 2m)}, \quad \text{outside the star,} \\
    h_0 &= -p_0 + \frac{1}{3} r^2 e^{-\psi} + h_0c, \quad \text{inside the star,}
\end{align*}
\tag{10}
\]

where $h_0c$ is a constant determined by imposing $h_0$ to be continuous at $r = R$. The mass perturbation factor $m_0$ and the pressure perturbation factor $p_0$ are calculated by integrating:

\[
\begin{align*}
    \frac{dm_0}{dr} &= 4\pi r^2 \frac{d\epsilon}{dp} (\epsilon + p)p_0 + \frac{1}{12} J^2 r^4 \left( \frac{d\psi}{dr} \right)^2 - \frac{1}{3} r^3 \frac{dj^2}{dr} \left( \frac{d\omega}{dr} \right)^2, \\
    \frac{dp_0}{dr} &= -m_0(1 + 8\pi r^2 p) \frac{(r - 2m)}{(r - 2m)^2} - \frac{4\pi (\epsilon + p) r^2}{(r - 2m)} p_0 + \frac{1}{12} \frac{r^4 j^2}{(r - 2m)} \left( \frac{d\psi}{dr} \right)^2 + \frac{1}{3} \frac{dj^2}{dr} r^3 \left( \frac{d\omega}{dr} \right)^2 - \frac{1}{3} r^3 \frac{d\psi}{dr} - \frac{1}{4\pi r^2} \left[ \frac{1}{12} j^2 r^4 \left( \frac{d\psi}{dr} \right)^2 - \frac{1}{3} \frac{dj^2}{dr} r^3 \omega^2 \right],
\end{align*}
\tag{11}
\]

where the boundary conditions are that $m_0(0) = p_0(0) = 0$. The mass correction at first order is given by

$$\delta m = m_0 + J^2/R^3.$$ 

Thus the total mass of a rotating neutron star with corrections up to the first order is given by

$$m(R) + \delta m = m(R) + m_0(R) + J^2/R^3,$$

where $R$ is the radius of the star surface.

According to \[20\], the binding energy of a relativistic star in a non-rotating configuration is the difference between its baryon mass and its total mass-energy

$$E_B = A - M,$$

where $A$ is the total baryon mass and $m_0$ is the nucleon rest mass. To calculate the binding energy for the non-rotating configuration we need to solve the Tolman-Oppenheimer-Volkoff equation. The first order correction in the binding energy for rotating configuration is given by

$$\delta E_B = \frac{J^2}{R^3} + \int_0^R 4\pi r^2 B(r) dr,$$

where $\epsilon_i$ is the internal energy density,

$$\epsilon_i = \epsilon - m_n n.$$

Thus the baryon mass of rotating stars is given by

$$M_B = A + \delta E_B + \delta m.$$ 

The metric functions $h_2$ and $v_2$ are given by
\[
\frac{dv_2}{dr} = \left(1 + \frac{1}{2} \frac{dv}{dr}\right) \left[-\frac{1}{3} \frac{d^2 j}{dr^2} + \frac{1}{6} \frac{1}{r^2} \frac{d}{dr} \left(\frac{d\varpi}{dr}\right)^2 \right] - \frac{dv}{dr} h_2,
\]
\[
dh_2 = \left[\frac{dv}{dr} + \frac{r}{2m} \left(\frac{d}{dr}\right)^{-1} \left(8\pi (e + p) - 4m/r^3\right)\right] h_2 - \frac{4v_2}{r(r-2m)} \left(\frac{d}{dr}\right)^{-1} - \frac{1}{3} \left[\frac{1}{2} \frac{dv}{dr} + \frac{1}{r-2M} \left(\frac{dv}{dr}\right)^{-1}\right] r^2 \frac{dr}{d\varpi} \left(\frac{d\varpi}{dr}\right)^2,
\]

where the boundary conditions are \(h_2(0) = v_2(0) = 0\). The non-radial mass and pressure perturbation factors, \(m_2\) and \(p_2\), are determined from the algebraic relations

\[
m_2 = (r - 2m) \left[-h_2 - \frac{1}{3} \frac{d^2 j}{dr^2} \varpi^2 + \frac{1}{6} r^2 j^2 \left(\frac{d\varpi}{dr}\right)^2\right],
\]
\[
p_2 = -h_2 - \frac{1}{3} r^2 e^{-\varpi^2}.
\]

To calculate the eccentricity \(e\) we use

\[
e = \left(1 - \frac{R_p^2}{R_c^2}\right)^{1/2},
\]

where \(R_p\) and \(R_c\) are the polar and equatorial radii. To obtain \(R_p\) and \(R_c\) we use the relations

\[
S(\theta) = r + \xi_0(r) + \xi_2(r) P_2(\cos(\theta)),
\]
\[
\xi_0 = -\rho_0(e + p)/(dp/dr),
\]
\[
\xi_2 = -\rho_2(e + p)/(dp/dr).
\]

where \(S(\theta)\) is a surface of constant density.

In table I results for slowly rotating stars are presented. The \(M_{\text{max}}\) and \(R\) are respectively the maximum mass and the radius of an analogous non-rotating configuration. The \(M_{\text{max}}^1\) is the maximum mass corrected up to first order for a star with angular velocity \(\Omega\). \(R\) is the radius of the non-rotating star. \(R_p\) is the equatorial radius, \(R_c\) is the polar radius, \(\epsilon\) is the assumed central energy density, \(I\) is the calculated moment of inertia and \(e\) is the eccentricity. The EoS for hadronic and hybrid stars were taken from [8] and the EoS for quark stars were taken from [11]. As these EoS have already been extensively discussed in the literature, we refrain from further explanations here. The only point worth mentioning refers to the parametrizations used. For hadronic and hybrid stars we have chosen to work with a parametrization that describes the properties of saturating nuclear matter proposed in [30] since other common parameter sets for the non-linear Walecka model namely, TM1 [31] and NL3 [32] proved to be inadequate because, due to the inclusion of hyperons, the nucleon mass becomes negative at relatively low densities [3, 4]. The chosen parameters are \(g_{\pi}/m_\pi = 11.79\) fm\(^2\), \(g_{\omega}/m_\omega = 7.148\) fm\(^2\), \(g_{\rho}/m_\rho = 4.41\) fm\(^2\), \(\kappa/M = 0.005896\) and \(\lambda = -0.0006426\), for which the binding energy is -16.3 MeV at the saturation density \(\rho_0 = 0.153\) fm\(^{-3}\), the symmetry coefficient is 32.5 MeV, the compression modulus is 300 MeV and the effective mass is 0.7\(M\). For the meson-hyperon coupling constants we have chosen them constrained by the binding of the \(\Lambda\) hyperon in nuclear matter, hypernuclear levels and neutron star masses \((x_\pi = 0.7\) and \(x_\rho = x_\rho = 0.783)\) and have assumed that the couplings to the \(\Sigma\) and \(\Xi\) are equal to those of the \(\Lambda\) hyperon [30, 32]. For the construction of the EoS for hybrid stars, the hadronic phase was obtained with the non-linear Walecka model and the parameters above and the quark phase with the MIT bag model with \(Bag = (180\ MeV)^3\). For the quark stars within the MIT model, we have used \(m_u = m_d = 5.5\ MeV\), \(m_s = 150.0\ MeV\) and \(Bag = (180\ MeV)^3\). For the purpose of the present work, the value of the bag parameter does not play an important role. Concerning quark stars within the NJL model, the set of parameters were chosen in order to fit the values in vacuum for the pion mass, the pion decay constant, the kaon mass and the quark condensates [34, 35]: \(\Lambda = 631.4\ MeV\), \(g_S\Lambda^2 = 1.824\), \(g_D\Lambda^5 = -9.4\), \(m_u = m_d = 5.6\ MeV\) and \(m_s = 135.6\ MeV\) which were fitted to the following properties: \(m_\pi = 139\ MeV\), \(f_\pi = 93.0\ MeV\), \(m_K = 495.7\ MeV\), \(f_K = 98.9\ MeV\), \(\langle uu\rangle = (dd) = -246.7\ MeV^3\) and \(\langle ss\rangle = -(266.9\ MeV)^3\).

One can easily see from table I that for all models considered, the rotating stars bears a slightly larger mass than its non-rotating counterpart. Eccentricities are rather uniform for all models (0.43-0.49). Quark stars are bound by the strong force and hence they can rotate faster than hadronic or hybrid stars that are bound by the gravitational force. According to (19), the value of the Kepler frequency can be obtained from the values of the mass and the radius of the corresponding non-rotating star and its empirical relation reads

\[
\Omega = (0.63 - 0.65)(M/R^3)^{1/2}.
\]

In table I the Kepler frequency for each EoS is specified. While hadronic and hybrid stars rotate with similar frequencies, the values for quark stars are indeed much higher.
III. RESULTS AND CONCLUSIONS

Let’s now go back to our results in order to compare them with what is found in the literature and draw the conclusions.

We first analyzed the results of the rotating pulsars. The values shown in table I for zero entropy and hadronic stars show the same behavior observed in [24], i.e., there is a small increase in the maximum mass of the rotating star as compared with the non-rotating configuration. Notice, however, that in [24], the usual NL3 and TM1 parametrizations could be used because only protons and neutrons were considered and the central density is somewhat lower than in our case.

In [23], the models named O,P,Q and R given in tables I can again be compared with our result for the hadronic star at $T = 0$ and they are indeed very similar. The central density and the moment of inertia are of the same order and the maximum mass is very similar. As far as we know there is no result for rotating stars with entropy different from zero (finite temperature) in the literature, but we can examine its effect on the stellar properties from table I. As entropy increases the maximum masses decrease for all kinds of pulsars, except for the very simple and unrealistic MIT model, the eccentricity of a rotating star remains practically unchanged. As expected, the moments of inertia of quark stars are much lower than of hadronic and practically unchanged. As expected, the moments of inertia of quark stars are much lower than of hadronic and hybrid stars at a fixed baryonic mass. For $\Delta E$ to be positive, the gravitational mass of the metastable star, at a fixed baryonic mass, has to be larger than the gravitational mass of the stable star. We have calculated the released energies in the conversion mechanism for the hadronic to the hybrid, hadronic to the quark and hybrid to the quark stars with $S = 0$ in the rotating configuration at fixed baryon mass. The released energy is always negative, except in the conversion mechanism of the hadronic (MS) to the hybrid star (SS) at a fixed baryonic mass of $1.56M_\odot$, which yields $\Delta E = 1.14 \times 10^{53}\text{erg}$, allowing for the required energy measured in a short gamma ray burst.

In the present work we have investigated one possible variation on the usual static neutral pulsars: the inclusion of rotation. We have observed that the behaviors shown in previous works with much simpler EoS were also observed here. The influence of the temperature was also investigated in both cases. We are now in a position to calculate the energy released from the conversion of a metastable star (hadronic or hybrid) to a stable star (hybrid or quark) under the influence of slow rotation. For the simple calculations done so far with stars in a rotating configuration at $S = 0$, only a hadronic star could convert into a hybrid star releasing an energy compatible with a short gamma ray burst. A more detailed and complete investigation, with a smaller bag parameter in the MIT bag model is under way.

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TABLE I: Rotating compact stars properties with Kepler angular velocity $\Omega = 0.65(M/R^3)^{1/2}$.

| Type            | Entropy | $M_{\text{max}}$ ($M_\odot$) | $M_{\text{max}}'$ ($M_\odot$) | $R$ (km) | $R_e$ (km) | $R_p$ (km) | $\epsilon_c$ (g/cm$^3$) | $I$ (gcm$^2$) | $\epsilon$ | $\Omega$ (Hz) |
|-----------------|---------|-------------------------------|--------------------------------|----------|------------|------------|------------------|---------------|----------|----------------|
| Hadronic 0      | 2.04    | 2.07                          | 11.73                          | 12.06    | 10.86      | 1.98 x 10$^{15}$      | 2.39 x 10$^{45}$ | 0.43        | 1661.37       |
| Hadronic 1      | 1.96    | 2.00                          | 11.02                          | 11.33    | 10.21      | 2.23 x 10$^{15}$      | 2.11 x 10$^{45}$ | 0.43        | 1791.66       |
| Hadronic 2      | 1.93    | 1.96                          | 10.94                          | 11.22    | 10.11      | 2.24 x 10$^{15}$      | 2.03 x 10$^{45}$ | 0.43        | 1801.74       |
| Hybrid 0        | 1.63    | 1.65                          | 12.33                          | 12.78    | 11.19      | 1.57 x 10$^{15}$      | 1.90 x 10$^{45}$ | 0.48        | 1381.09       |
| Hybrid 1        | 1.50    | 1.53                          | 11.32                          | 11.73    | 10.28      | 1.75 x 10$^{15}$      | 1.56 x 10$^{45}$ | 0.48        | 1503.07       |
| Hybrid 2        | 1.50    | 1.52                          | 11.76                          | 12.20    | 10.65      | 1.58 x 10$^{15}$      | 1.65 x 10$^{45}$ | 0.49        | 1418.80       |
| Quarkonic(MIT) 0| 1.22    | 1.25                          | 6.76                           | 6.97     | 6.23       | 5.14 x 10$^{15}$      | 0.53 x 10$^{45}$ | 0.45        | 2938.89       |
| Quarkonic(MIT) 1| 1.22    | 1.25                          | 6.76                           | 6.97     | 6.23       | 5.17 x 10$^{15}$      | 0.53 x 10$^{45}$ | 0.45        | 2941.47       |
| Quarkonic(MIT) 2| 1.23    | 1.26                          | 6.79                           | 6.99     | 6.25       | 5.08 x 10$^{15}$      | 0.54 x 10$^{45}$ | 0.45        | 2929.40       |
| Quarkonic(NJL) 0| 1.20    | 1.23                          | 7.87                           | 8.12     | 7.24       | 3.45 x 10$^{15}$      | 0.69 x 10$^{45}$ | 0.45        | 2316.71       |
| Quarkonic(NJL) 1| 1.17    | 1.18                          | 7.71                           | 7.81     | 7.45       | 3.68 x 10$^{15}$      | 0.64 x 10$^{45}$ | 0.45        | 2363.51       |
| Quarkonic(NJL) 2| 1.10    | 1.13                          | 7.18                           | 7.39     | 6.62       | 4.58 x 10$^{15}$      | 0.51 x 10$^{45}$ | 0.44        | 2552.36       |