A Sybil-Resilient Mechanism for an Egalitarian and Just Digital Currency

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Abstract. We envision a self-sovereign community that defines itself from the grass roots, bottom up, in a decentralized manner. In a different paper we show how a digital community can grow by using a trust graph and maintaining a bound on the number of fake identities. In another paper we show how this trust graph can integrate into a consensus protocol, to maintain a distributed ledger of the history of the community, while maintaining governance over the list of members. Next, we show in this paper how the community can maintain self-sovereignty over the economy, by transferring the role of minting money to the hands of every registered member of the community. Such minting serves both as the origin point of sovereignty over the economy, as well as a form of universal basic income.

Yet, a distributed construction of an online community which is bigger than the reach of personal acquaintances is strongly susceptible to sybils. This paper assumes that the community has some means for occasionally exposing sybils. It then shows, based on this assumption, that there are means to balance back the economy such that all money minted by sybils is eventually retrieved and liquidated, so that – asymptotically – the amount of money minted equals the number of genuine identities, multiplied by the amount of time they were minting. Furthermore, we argue that this approach can serve as an incentive mechanism that deters the creation of sybils and incentivizes sybil hunting.

Keywords: eDemocracy; sybil resilience; incentive mechanism; Universal Basic Income; cryptoeconomy

1 Introduction

The rise of the internet in the late 20th century revolutionized the way people communicate. Global mass communication between individuals and broadcasting of one individual to the rest of the world, instantaneously, is now a trivial part of our lives. The appearance of bitcoin, about a decade ago, made another revolutionary leap in social interactions, by showing how to create value distributively, without a central authority. It also modified the meaning of objective truth, or objective reality – with blockchain technology, whatever is written in the ledger is the indisputable state of affairs. What is still missing is distributed governance – a method for a group of people to reach agreement together, which naturally spawns from the opinions of the individuals, without authoritative coercion.
The first step in community governance is setting the boundaries of the community and deciding who is a member and who is not. In a paper from 1997, “On the question ‘who is a J’” [15], Ariel Rubinstein and Asa Kasher propose to settle this question by aggregating the views of the members of the community. In the first paper in the line of work presented here [13], we present a method for community growth, where each candidate or member of the community points on other individuals which she trusts to be part of the community. The method then approves new members based on the connectivity of the underlying trust graph. Our main theorem, there, shows that, if the community manages to maintain some bound $\gamma$ of corrupt identities (that may point on misfits) and the underlying trust graph has a good enough connectivity (vertex expansion higher than some value $\Phi$), then the community can grow while maintaining a bound of $\sigma$ of misfits that penetrate in, where $\sigma < \frac{\gamma}{\Phi} - \gamma$. In the contexts of this work, the ‘misfits’ are sybils – fake or duplicate digital identities (in contrast to genuine identities that are one to one identifiers of genuine individuals).

Bounding the number of sybils in the community is a good start, but it is not enough for taking democratic decisions together, where the idiom ‘one person one vote’ is highly desirable. Our work presented here shows how money can be used to further eradicate sybils that have penetrated into the group. The starting point of the mechanism is that money is minted by the individuals periodically. This by itself is a big step towards distributed, coercion free, economy, as it puts the governance of the economy in the hands of the individuals, rather than in the hands of a central bank. Once money is minted by individuals, the first concern is that some of them may try to operate sybils, in order to mint more money than their right share. Note that an automatic mechanism is limited in its ability to enforce actions on individuals. It might seem desired that once a sybil is caught, all the coins it minted will be automatically collected and destroyed by the mechanism, but this is problematic, as the coins were probably assimilated into the economy by the time the sybil is caught, and at least some of them reached the accounts of honest individuals. Our approach to overcome this issue is to collect the amount minted by sybils, post factum, by preventing the corrupt identities that have edges with the sybils in the trust graph from further minting coins. We regard these corrupt identities as those responsible for introducing the sybils into the community. Either they represent the individuals that operate the sybils, or individuals that collude with them. There are two assumptions in the mechanism. The first is that the number of sybils to begin with is bounded, and the second is that sybils get caught from time to time. Based on these two assumptions the mechanism guarantees the following:

1. The amount of money minted by sybils is retrieved post factum by preventing corrupt identities from minting. In the long run, the total amount minted equals the number of genuine identities, multiplied by the elapsed time, plus a constant that is time dependent.
2. Corrupt individuals that supported the sybils not only retrieve the fault money, but are also fined an additional sum that acts as a deterring punish-
ment against sybil creation. This fine is gathered collectively and can budget the sybil exposure mechanism, for example by incentivizing sybil hunters.

Remark 1. A side effect of the proposed minting process is that it gives the people control over redistribution of wealth. This can be achieved in at least two different ways. One is by tax collecting. If every minting account is taxed on every round, say, proportional to the amount of money in the account ($x$ percentage of all the coins in circulation is collected), and then burned, following with minting of the exact same amount, in equal share, by all the identities in the community, then the total amount of coins in circulation remains fixed, and wealth is redistributed. The second option is not to collect such a tax, but simply mint new money in every round. This will entail a steady rate of inflation, that will cause a similar effect (existing money looses value by some factor and newly minted money is distributed equally). The community then has the power to regulate this steady flow of redistribution of wealth by setting the rate of money minting. Set high enough, if economically desired, and this minted money can serve as a mean of a universal basic income.

This paper focuses on a single community and a single currency. In particular, we start with a degenerate model of a static community, with the maximal number of sybils (according to the bound derived from the community growth paper [13]), where the only change over time is that sybils get exposed occasionally. The model shows that all money minted by sybils is collected and nullified, deterministically. We then elaborate the model in steps, towards a probabilistic model in which not only sybils are exposed (with some probability), but also genuine identities cease to exist (with some probability). The result is a dynamic community that keeps changing. Yet we show that under such conditions the minting process also guarantees the above requirements (asymptotically).

2 Related Work

We first mention the work of Shahaf et al. [18] in which they study economic properties of a currency network, aiming at utilizing digital economy to support equality, while assuming some mechanism to fight sybils. We furthermore discuss several pillars of related work.

2.1 eDemocracy

The bigger scope of our work is e-Democracy, which is the use of information and communications technology to augment democracy. There are many forms of democracy, some depend more on technology than others. Jean-Jacques Rousseau discusses direct democracy already in 1762 [14], yet he regards it as ineffectual. The obstacles he saw, distance, diversity and impossibility of determining the general will, all seem solvable in the age of technology [20]. Delegative Democracy, also known as Liquid Democracy [10], can be seen as a midpoint between direct and representative democracies. It also leans on technology, if only for keeping track
of the delegation graph. Representative democracy, which is the most common type in western countries today, can also benefit from the use of technology. Estonia is an impressive example of state issued digital identities and digital governmental services.

But e-Democracy goes further than more sophisticated voting. With the implementation of smart contracts it seems that we cannot yet fully comprehend how far we can go with programming the social contract and empowering the individual within a community.

2.2 Sybil Attacks

Digital identities are prone to sybils. The first to coin this term for duplicate identities is Douceur who showed some impossibility results for detecting sybils without a central authority. Levine et al surveyed multiple domains where sybil attacks are applicable, and different approaches for dealing with the attacks. Yu et al were probably the first to propose a protocol for bounding the number of sybils which is based on graph connectivity, with several variations to follow. Viswanath et al, and later Alvisi et al, surveyed these and other graph based protocols and showed that they don’t differentiate well between sybils and other isolated but genuine communities in social networks. Most of these works differentiate the population into two types – sybils and not sybils.

These mentioned papers mostly discuss the detection of sybils by analyzing a given graph. We are not aware of any work that deals with punishing sybils with a fine, neither works that handle sybils as an ongoing process on a dynamic graph that changes whenever a sybil is exposed.

2.3 Cryptocurrencies

Permissionless cryptocurrencies are indifferent to sybils. With Bitcoin, Ethereum and the like, it does not matter how many accounts a user holds. An account has no identifying characteristic, and is only a place holder for some amount of tokens. The consensus protocols themselves are also indifferent. Proof-of-Work depends on the processing power a user holds, and Proof-of-Stake on the total amount of tokens he has. This is not the case with e-Democracy. Not only it is required to have some (preferably anonymous) identification of the user, to maintain the ‘one person one vote’ paradigm, it is also preferable to divide the governance of the distributed ledger itself equally among the participants.

There are several cryptocurrencies that involve a universal basic income. GoodDollar is an Ethereum based token, planned to be launched in the fall of 2020 on an Ethereum side chain called Fuse. It relies on a 3rd party called Facetec, for identity verification, which uses face recognition for this purpose. Circles is a personal currency. Each participant mints her own token and mark on a trust graph which other personal tokens she trusts. She is then required to accept these tokens on a one-to-one conversion ratio. These are just two examples out of many. Several of those require unique identities for participation, still
we believe the work presented here is the first to integrate the use of a UBI as a mechanism to incentivize unique identities.

3 Preliminaries

We focus our study on a model in which there is only a single community. Furthermore, identities vouch for each other to join the community (i.e., identities can point to other identities, declaring some form of trust towards them). We use a graph notation where vertices are identities and edges are mutual sureties between identities. We differentiate between genuine identities and sybil identities. A genuine identity that gives surety to a sybil one is considered corrupt. We assume the corrupt identities are the beneficiaries of the sybil identities they vouch for and regard them as responsible for the crime of adding the sybil identities into the community. See the community growth paper for more details. We treat the case where corrupt identities and their controlled sybils become majority, as a tilting point for the democratic governance of the community. Further, we assume that it is feasible for the community growth mechanism to achieve a bound of 1\(\frac{1}{2}\) between sybils and corrupt. Hence all the following scenarios assume a ratio of 1:2:3 between sybil:corrupt:honest identities.

We assume that all identities in the community graph participate in a joint distributed ledger, with an internal clock, that mints 1 coin of some currency for each identity on each clock tick, subject to the different conditions. It is our goal to show that, under these conditions, the corrupt identities will be fined according to all the money minted by sybils, and more; this will serve a form of balance in the economic system. How much more is dependent on the stage of the model and will be defined specifically for each stage.

4 A Static Community

We start with a static community, saturated with the maximal ratio of corrupt and sybil identities as described above. During the minting process an external oracle will expose sybils from time to time. Once exposed, the minting process will withdraw the sybil from the community and impose a fine on the identities with edges to the exposed sybil (that is, the corrupt agents surrounding it). We will show that, after all sybils are exposed, all the money they minted will be collected back from the group of corrupt identities.

4.1 The Model

We start with our formal definition of a community graph, with vertices for agents in the community and trust edges among them.

Definition 1 (Community Graph). A community graph is a undirected, finite, labeled graph \(G = (V,E)\) where the label of each vertex is of the form \((\text{type,properties})\), with the following characteristics:
1. $G$ is connected.
2. $\text{type} \in \{H,C,S\}$, meaning that the identity represented by this vertex is either honest, corrupt, or sybil.
3. \[\{(v_1,v_2) \in E : \text{type}_{v_1} = H, \text{type}_{v_2} = S\} = \emptyset;\] i.e., there are no edges between honest and sybils; put differently, the corrupt identities are those that are genuine but are connected to sybils.
4. $\text{properties}$ is a tuple of the form $(x,m,f)$, representing the exposure status of $v$, the amount of money it minted and the fine imposed on it. The exact usage and meaning of these properties will be defined by the following minting mechanism.
5. There are at most $\gamma$ fraction of vertices with type C.
6. The vertex expansion of $G$ is $\phi$.

In the following we use the notation $x_v,m_v,f_v$ to refer to the $x,m,f$ values of a given node $v$ in such a graph.

As the minting mechanism that follows exposes sybils, it will need to traverse the graph to follow the path of already exposed sybils, until it reaches higher degree neighbours that are still participating in the community (either genuine identities that are corrupt, or sybils that are not yet exposed). For this purpose we define the conditional boundary of a vertex, to include all neighbours that meet some condition.

**Definition 2 (Conditional boundary).** The conditional boundary $\partial_{\text{cond}}v$ of a vertex $v$ is the set \{u : $\exists$ path from u to v, s.t. only u satisfies cond\}. If the neighbours of $v$ are changing over time (as later we will consider a dynamic setting), then $\partial_{\text{cond}}(v,t)$ is the conditional boundary of $v$ at time $t$.

Similarly, the conditional boundary of a set of vertices is defined as follows:
\[\partial_{\text{cond}}U := \bigcup_{u \in U} \partial_{\text{cond}}u.\]

The method by which sybils are exposed is external to this paper; we do mention that it might be realized with some ML-based algorithm. Of course, such algorithms are never perfect; here, for simplicity, we assume such a “perfect” machine that is never wrong.

**Definition 3 (Sybil exposure method).** A sybil exposure method $\text{Expose}(v)$ receives a node $v \in V$. It returns true if $v$ is a sybil and false otherwise.

### 4.2 The Mechanism

Mechanism II presents the minting mechanism for a static community. Though we do not go into the implementation details, assume it is implemented as a smart contract running over a distributed ledger. The loop in line 4 can be regarded as a trigger activated on every transaction, or block of transactions, added to the ledger. There is no need for an actual distributed clock to be implemented. It is enough to synchronize the amount of time the loop is called as the ledger evolves. The pseudocode does not care how much money is transferred at the end to the user. The property $m_v$ collects how much money she minted, not how much she
Mechanism 1 Static community minting mechanism

1: $G \leftarrow$ a community graph
2: $\forall v \in G$, $x_v = m_v = f_v = 0$
3: $in\text{Circulation} = tax\text{Collected} = 0$
4: loop $\triangleright$ once per time unit
5: // the minting part
6: for all $v \in G$ such that $x_v = 0$ do
7: $m_v \leftarrow m_v + 1$
8: pay $\leftarrow \min(1, f_v)$
9: $tax\text{Collected} \leftarrow tax\text{Collected} + \frac{pay}{2}$
10: $f_v \leftarrow f_v - pay$
11: $in\text{Circulation} \leftarrow in\text{Circulation} + 1 - pay$
12: end for
13: // the fine imposing part
14: $v \leftarrow$ pick at random from $G$.
15: if $\text{Expose}(v)$ is true then
16: $x_v \leftarrow 1$
17: fine $\leftarrow 2 \cdot m_v + f_v$
18: for all $u \in \partial_{x_u=0} v$ do
19: $f_u \leftarrow f_u + \frac{\text{fine}}{|\partial_{x_u=0} v|}$
20: end for
21: end if
22: end loop

owns. It does care (for the following simulations) how much money all users own collectively, which is reflected by the $in\text{Circulation}$ parameter. The $tax\text{Collected}$ parameter can be regarded as a publicly owned account that is further governed by other smart contracts for the benefit of the community. Specifically, we regard this amount as a source for budgeting sybil exposure procedures, like paying for sybil hunters.

The first part of the mechanism (lines 7-13) is the minting loop over all nodes. Each node that is not yet exposed as a sybil mints 1 coin, but if this node has an induced fine, then this coin is used to pay the fine. half of the payment is collected as tax and the other half is nullified (does not go into circulation). The second part of the mechanism (lines 16-23) is the fine calculation whenever a node is exposed as sybil. The fine is double the money this node ever minted, plus any fine it had not yet pay. The amount it minted is doubled to ensure that not only this amount is retrieved and nullified, but an additional sum is fined to punish the operators of this sybil. The mechanism then traverses the graph to find the neighbours of this node (at distance 1 or more) that are not yet exposed as sybils (eventually reaching the corrupt nodes that surround the group of sybils) and induce the fine on these neighbours in equal parts.

The main result we want to show is that the introduction of sybils does not benefit its operator (the corrupt) with more money. That is, for every coin minted
by a sybil, some neighbouring corrupt node will pay with a coin that is nullified
(minted, but does not go into circulation). In the static community model this
result is deterministic and final. At some point in time all sybils are exposed, and
in a following point in time all money minted by these sybils is retrieved back.

Claim 1 (Money minted by sybils is nullified) Let $t$ be a loop counter for
the loop in mechanism $\lbrack 7 \rbrack$. Let $g$ be the number of genuine identities in the graph
$g = |\{v : \text{type}_v \neq S\}|$. Then, there exists a loop count $t_e$ such that $\forall t \geq t_e$ the
following holds: $\text{inCirculation} + \text{taxCollected} = t \cdot g$.

Proof. As the graph is finite, and in every time unit another vertex is checked,
by some point in time $t_l$, the last sybil is exposed. The graph $G$ is fixed (does not
change over time, or from loop to loop) and connected, so there is always a set of
vertices in the boundary of the set of exposed identities that receives the fine. As
the fine is finite, given enough time (or program loops), all fine will be collected.
It rests to show that the amount of fine collected covers precisely the amount of
coins minted by sybils. More precisely, for each coin minted by a sybil, two coins
are registered as fine. When this fine is paid, one coin is registered as
$\text{taxCollected}$ and one coin is not registered, neither as $\text{taxCollected}$, nor as $\text{inCirculation}$. We
say in this case that this coin is nullified. What we show is that the money that
is registered equals the money minted by genuine identities. We use induction on
the number of sybils and analyze the code to do so.

Assume there is only one sybil $v$ and it has $d$ neighbours. When it is exposed
at time $t_l$, it managed to mint exactly $t_l$ coins, and at the point of exposure
$f_v = 0$. Therefore the fine imposed on its neighbours is exactly $2 \cdot t_l$ (line $\lbrack 19 \rbrack$).
Let $t_e$ be a point in time where all fine was payed back. Let $n$ be the number of
nodes in the graph and $g = n - 1$ the number of genuine nodes. Then by time $t_e$,
all nodes except the sybil one minted $t_e \cdot g$ coins. Note that by line $\lbrack 12 \rbrack$ for every
coin of fine there is one less coin in circulation. So the money in circulation is
$t_e \cdot g + t_l - 2 \cdot t_l$, that is the money minted by genuine identities, plus the money
minted by the sybil, minus the money not accounted due to the fine. In line $\lbrack 10 \rbrack$
half of the fine is collected in $\text{taxCollected}$, so there are $t_l$ coins in tax. Together
we get $\text{inCirculation} + \text{taxCollected} = t_e \cdot -t_l + t_l = t_e \cdot g$.

Now suppose by induction that the claim holds for $k$ sybils. Let $S$ be the
set of sybils in the graph, $|S| = k + 1$, and $C$ be the set of neighbours of
sybils that are not sybil by themselves. The number of genuine identities is now
$g = n - k - 1$. Let $v$ be the last $(k + 1)$ sybil to be exposed, and $t_l$ be the point
in time of its exposure. If $v$ was not a sybil, then by time $t_e$ we would have
$\text{inCirculation}_{t_e} + \text{taxCollected}_{t_e} = t_e \cdot (n - k)$. Let $\text{fine}^* = \sum_{u \in C} f_u + f_v$, be the
cumulative registered fine at time $t_l$, just before $v$ is exposed. By line $\lbrack 12 \rbrack$ we have
$\text{inCirculation}_{t_e} = \text{inCirculation}_{t_l} + (t_e - t_l) \cdot (n - k) - \text{fine}^*$. By line $\lbrack 10 \rbrack$ we have
$\text{taxCollected}_{t_e}^* = \text{taxCollected}_{t_l} + \frac{\text{fine}^*}{2}$. Now let $\text{fine}$ be the accumulated fine after
Fig. 1. Coins in circulation minted by sybils (on the left) and corrupt identities (on the right). The sybil diagram shows that every sybil node stops minting at some point, as it is being exposed. The corrupt diagram shows that every corrupt node holds minting from time to time (as it is being fined for neighbouring sybils) and once all sybils are exposed continuous to mint uninterrupted.

$v$ is exposed. By line 19 we have \( \text{fine} = \text{fine}^* + 2 \cdot t_l \). We can now calculate:

\[
\begin{align*}
\text{inCirculation}_{t_e} &= \text{inCirculation}_{t_l} + (t_e - t_l) \cdot (n - k - 1) - \text{fine} = \\
&= \text{inCirculation}_{t_e}^* - (t_e - t_l) + (\text{fine}^* - \text{fine}) \\
\text{taxCollected}_{t_e} &= \text{taxCollected}_{t_l} + \frac{\text{fine}}{2} = \\
&= \text{taxCollected}_{t_e}^* - \frac{\text{fine}^* - \text{fine}}{2}.
\end{align*}
\]

Summing together and applying the induction claim we get:

\[
\begin{align*}
\text{inCirculation}_{t_e} + \text{taxCollected}_{t_e} &= t_e \cdot (n - k) - t_e + t_l - \frac{(\text{fine} - \text{fine}^*)}{2} = \\
&= t_e \cdot (n - k - 1) + t_l - t_l = t_e \cdot g.
\end{align*}
\]

By now we showed the claim holds after exactly \( t_e \) time loops. Since all sybils are exposed by this point in time, and stop minting, the condition also holds for any \( t \geq t_e \).

4.3 Experimental Evaluation

We ran a simulation of mechanism 1 with a community of 120 identities, of which 20 are sybils and 40 are corrupt identities (may be neighbours of sybils in the trust graph). Figure 1 shows the amount of coins minted by each identity over time. The diagram on the left shows the 20 sybil identities. Once exposed, each sybil stops minting, hence its line becomes flat. The last sybil to be exposed lasted slightly less than 300 loops and minted almost 300 coins until exposed.
The lines with multiple climbs are sybils caught as neighbours of other sybils and paying their fine. The diagram on the right shows the 40 corrupt identities. Once all sybils are exposed and all the fine is payed, all lines climb with slope 1 (minting 1 coin per loop).

The main purpose of the simulation was to verify that the fine accounting in mechanism 1 is correct and also that claim 1 holds. Indeed, the total amount of coins (in circulation and as collected tax) after 500 loops was exactly 50,000 coins (500 loops times 100 genuine, non-sybil identities). That is, any money minted by sybils was eventually retrieved and nullified.

5 A Community with Regenerating Sybils

In reality communities are not static. They evolve. In the next step we assume that the adversary (we assume the corrupt identities can collude, so we treat them all as a single adversary entity) is fast to replace each exposed sybil with a new one. As a result, the community remains saturated with the maximal number of corrupt identities and maximal number of sybil identities (see section 3), at every step. We regard it as the worst case for the defender, under the assumption that genuine identities remain static in the community (an assumption that will be removed in the next section). Starting from this point we can no longer show that all money minted by sybils is nullified, as there will always be some sybils not yet exposed. What we will show instead is that the money minted by unexposed sybils, together with fine not yet payed, are bounded by a constant. Therefore, as time continues to advance, this constant will become negligible to the amount of money in circulation. In other words, the ratio of money minted by sybils compared to all the minted money becomes infinitesimal as time goes on.

5.1 The model

As we are now dealing with the dynamics of the community, we provide definitions that capture the changes in the graph over time.

**Definition 4 (Community History).** A community history is a series of community graphs \( G_t = (V_t, E_t) \).

Note that, as each community graph in a community history represents one time unit, so do the labels of this graph. For example, \( m_{v_t} \) represents the coin minted by \( v \) at time \( t \). In particular, if each identity mints one coin per one unit of time, then \( m_{v_t} \) can be either 0 or 1. Similarly, \( m_{v_{t+1}} \) is a different label, representing the coins (0 or 1) minted by \( v \) at time \( t + 1 \). Each of these different labels may by itself evolve over time, as for example \( f_{v_t} \) may be 0 at point \( t \), then grow to 2 at a later point in time, when a neighbor of \( v \) (at time \( t \)) is exposed as a sybil, then it can go back to 0, after \( v \) pays the fine. The following defines the transitions between consecutive steps of a community history. In the simulation, this is just a method that the minting mechanism calls, to simulate changes over time. In an implementation, this will probably be handled by some
smart contracts that implement social interactions in the trust graph, with some interface to the minting smart contracts.

**Definition 5 (Community transition method).** A community transition method is a function $G' = \text{Transition}(G)$ that receives a community graph and returns a new community graph. In this subsection it is restricted as follows:

1. $\forall v \in V', x_v = 0$; i.e., exposed sybils are removed from the graph and cannot penetrate back.
2. $V \setminus \{v \in V | x_v = 1\} \subseteq V'$; i.e., nodes in $V$ that are not exposed as sybils remain in $V'$.

**5.2 The mechanism**

From the definition of a community graph, the ratio of corrupt identities is bounded by $\gamma$, and since the vertex expansion is bounded below by $\phi$, so does the ratio between corrupt and sybils (see section 1). As before, we assume that the community is always saturated – the number of corrupt and sybils is at their maximum. Since genuine identities remain in the community history forever (for now), we refer to this model as a model with a regenerating sybil community. Mechanism 2 describes the minting mechanism. The main principle of mechanism 2 is that it does the same as mechanism 1 but separately for each time unit. Each coin that a sybil has minted is returned as a fine on the nodes that were neighbours of the sybil at the time it minted that coin. Each coin is accounted separately. In the first part of the mechanism, the minting part (lines 7-21), there is an additional internal loop (lines 13-18) that handles the fine payment correctly for each elapsed time unit. In the second part (lines 24-33) there is another internal loop to calculate fine inducement for each elapsed time unit. The mechanism ends by updating the graph before the next time step (line 34).

It may be hard to show, as sybils are immediately replaced, that the rate of sybil money minting is not higher than the rate of paying the fine. It may be the case that corrupt identities continuously pay the fine, while their neighbouring sybils continue to mint more coins faster than the payment rate. To overcome this difficulty, we will prove the above claim for a slightly modified mechanism, where all fine payers divide the fine equally between them. The simulations show that the two versions of mechanism 2 behave similarly, so the next definition is purely for the simplification of the claim that follows. We also parametrize the fine factor for a more general claim and proof.

**Definition 6 (Mechanism 2*).** Mechanism 2* is similar to mechanism 4 with the following modifications:

1. The total fine at every time unit is distributed evenly between all fine payers.
2. The fine (line 27) is $\text{fine} \leftarrow \alpha \cdot m_{v_{t_2}} + f_{v_{t_2}}$.

Like in the previous section, we again want to show that the introduction of sybils does not benefit their operators (the corrupts) with more money. Since
Mechanism 2: Spawning sybil minting mechanism

1: $G_0 \leftarrow$ initial community graph
2: $\forall v \in G, \forall t, x_{vt} = m_{vt} = f_{vt} = 0$
3: $\text{inCirculation} = \text{taxCollected} = 0$
4: loop $t$ ◁ once per time unit
5:
6: // the minting part
7: for all $v_t \in G_t$ do
8: $m_{vt} \leftarrow 1$
9: if $\sum_{v_t} f_{v_{vt}} = 0$ then
10: $\text{inCirculation} \leftarrow \text{inCirculation} + 1$
11: else
12: $\text{payment} \leftarrow 1$
13: for $t_2 := 0$ to $t - 1$ do
14: $\text{pay} \leftarrow \min(\text{payment}, f_{v_{t2}})$
15: $\text{taxCollected} \leftarrow \text{taxCollected} + \frac{\text{pay}}{2}$
16: $f_{v_{t2}} \leftarrow f_{v_{t2}} - \text{pay}$
17: $\text{payment} \leftarrow \text{payment} - \text{pay}$
18: end for
19: $\text{inCirculation} \leftarrow \text{inCirculation} + \text{payment}$
20: end if
21: end for
22:
23: // the fine imposing part
24: $v \leftarrow$ pick at random from $G$.
25: if $\text{Expose}(v)$ is true then
26: $x_v \leftarrow 1$
27: for $t_2 := 0$ to $t - 1$ do
28: $\text{fine} \leftarrow 2 \cdot m_{v_{t2}} + f_{v_{t2}}$
29: for all $u \in \partial_{x_v = 0} v_{t2}$ do
30: $f_u \leftarrow f_u + \frac{\text{fine}}{\partial_{x_v = 0} v_{t2}}$
31: end for
32: end for
33: end if
34: $G_{t+1} \leftarrow \text{Transition}(G_t)$
35: end loop
now the sybil community is regenerating, we can no longer show that all their
money is retrieved. We show instead that the money not yet retrieved is bounded
by a constant.

**Claim 2** In mechanism 2 consider the case where $\alpha = \frac{\phi}{1-\phi}$ and assume $\phi > 0.5$.
Let $\sigma$ be the ratio of sybils in the community. The amount of coins minted by
sybils and not yet recovered is bounded by $O(\sigma \cdot n^2)$, where $n = |V_0|$.

**Proof.** Since every vertex is eventually tested for being a sybil, then any sybil $v$
in the graph at time $t$ is necessarily exposed by time $t + t_I$, where $t_I = |V_0|$ (the
size of the graph does not change over time). Also, any sybil neighbour of $v$
at time $t$ is necessarily exposed by time $t + t_I$. It follows that every coin minted by
sybil at time $t$ has turned into fine registered on a corrupt identity by time $t + t_I$
or before, as the fine propagates from sybil to sybil, as they are exposed, until
reaching a non-sybil neighbour. Given that the vertex expansion of a community
graph is $\phi$, it follows that any set of sybils $S$ has a conditional boundary of
non-sybil of size:

$$\frac{|\partial_{type} = c S|}{|\partial_{type} = c S| + |S|} \geq \phi$$

or:

$$|\partial_{type} = c S|(1 - \phi) \geq \phi |S|$$

$$|\partial_{type} = c S| \geq \frac{\phi}{1 - \phi} |S| = \alpha |S|$$

Now, let $S_0$ be the set of all sybils at time $t = 0$. The fine for the coins minted by
$S_0$ is $\alpha |S_0|$, and by time $0 + t_I$, there are at least $\alpha |S_0|$ corrupt identities evenly
carrying this fine. As the fine is payed oldest first (by line 13), it follows that by
time $t_I + 1$ all coins minted at time $t = 0$ by sybils will be recovered (assuming
$\alpha \geq 1$) by time $t_I + 1$. Iteratively, this holds for any $t$. Any coin minted at time $t$
is recovered by time $t + t_I + 1$. The amount of coins in circulation, minted by
sybils and not yet recoverd, is at most $\sigma \cdot n \cdot (t_I + 1) = O(\sigma \cdot n^2)$. □

### 5.3 Experimental evaluation

We ran a simulation of mechanism 2 with a community of 120 identities, of
which 20 are sybils and 40 are corrupt identities. Note that this time the fine is
calculated separately for each loop, that is for each coin minted by a sybil, the
mechanism looks for the not yet exposed neighbours of that sybil, at the time
the coin was minted. The payment of the fine is also calculated per loop. We
ran the simulation for 10,000 loops to be convinced that the amount of coins in
circulation, minted by sybils, is bounded. Note that the simulation divides the
fine between the neighbours of the sybil, and as such demonstrates mechanism
2 rather then the variant defined in definition 6. Figure 2 shows the results of
this simulation. The graph on the left shows on its left axis how many sybils in
each loop are exposed. It shows that by the time the simulation ended, all sybils
older then the last 120 loops have been exposed, as expected. As a result, coins
Fig. 2. Excess coins in circulation. The diagram on the left shows how many sybils in each loop are exposed and how much fine per loop is not yet paid. Note that the graph zooms in on the last 1000 loops for visibility. The diagram on the right shows the overall amount of sybil coins in circulation in every loop.

minted by sybils in the last 120 loops are still partially in circulation as these sybils are not yet exposed. This is the first source of excess coins in circulation (coins minted by sybils and not yet retrieved). On its right axis the graph shows the fine per loop not yet paid. It shows that all fine that derives from coins minted, up to the last about 230 loops is already paid, while fine that derives from the last 230 loop is only partially paid. This is the second source of excess coins in circulation. The diagram on the right shows the accumulated amount of excess coins per loop. It shows that the amount of excess coins is bounded around roughly 2000, which is slightly less than the bound calculated by claim 2. Again, the simulation demonstrates that our theory holds.

6 A Probabilistic Model

The assumption that every sybil has a bounded lifespan is too strong. A more relaxed assumption is to assume that every sybil has a probability $p$ to get exposed at every cycle. We should also relax the assumption on the immortality of genuine identities, so for this section we also assume that every genuine identity ceases to exist with probability $q$ at every cycle.

6.1 The model

First we need to modify the sybil exposure method. Note that until now the mechanism picked a node at random, and exposed it with probability 1 if it was a sybil. From now on each node draws its random coin independently, so multiple sybils may be exposed in the same round (or none at all).

Definition 7 (Probabilistic sybil exposure method). A probabilistic sybil exposure method $\text{ProbExpose}(v)$ receives a node $v \in V$. If $v$ is a sybil, then it returns true with probability $p$. It returns false otherwise.
We also need a method to notify when a genuine node ceases to exist. This will simulate death of the genuine individual represented by this node, or, if permitted, leaving the community.

**Definition 8 (Genuine terminator method).** A genuine terminator method $\text{Ceased}(v)$ receives a node $v \in V$. If $v$ is genuine, it returns true with probability $q$. It returns false otherwise.

### 6.2 The mechanism

The difference between mechanism 3 and mechanism 2 lies in line 24 where now the mechanism loops on all nodes in the second part and checks the status of each node. Additional handling of the case where a genuine node ceases to exist is introduced in line 35.

Proving theoretical claims on a stochastic model is harder, so for now we settle for demonstrating the behavior of this model with simulations.

### 6.3 Experimental analysis

As before, we use a graph with 60 honest nodes, 40 corrupts and 20 sybils, as we consider this distribution of nodes to be the extreme case for our model. This ratio is maintained all the time. Whenever a node is removed from the graph, a new node of the same type (but with different neighbours) is introduced. The details of how we generate the graph for the simulations is given by method 1. It is a random graph generator that picks at random two nodes which degree is less than the target degree, and connects them as long as these are not an honest and a sybil node. It then continues in a loop until all nodes have the required degree. The generated graph has maximal degree $d$ and minimal degree $d - 1$. Such a graph is slightly simpler to construct compared to a $d$-regular graph, and it seems adequate enough for these simulations. The method also cleans the graph at every round by removing redundant edges between two nodes with degree $d$.

Our simulations use this mechanism to construct both the initial graph $G_0$, as well as the next iteration of the graph (The Transition method), after each loop (after the exposed sybils and terminated genuine identities are removed from the graph).

The first simulation repeats the result from the previous section, to make sure it still holds with the probabilistic approach. For this run $p = 0.034$ and $q = 0$, so sybils are exposed with half life of about 20 loops, and genuine identities are still immortal. The desired goal is to mint 100 coins per loop asymptotically, equal to the number of genuine identities. Figure 3 shows the difference between this expected number and the actual amount of coins in circulation. The graph shows that the amount of excess coins (money minted by sybils, that is not yet retrieved as a fine) is bounded (slightly below 1000), as time progress. Consequently, the ratio of coins minted by sybils goes to 0 over time, as the money minted by genuine identities is linear in $t$, and therefore grows to infinity.
Mechanism 3 Stochastic mechanism

1: \( G_0 \leftarrow \) initial community graph
2: \( \text{inCirculation} = \text{taxCollected} = 0 \)
3: \( \text{loop } t \)  \( \triangleright \) once per time unit
4: \( \forall v \in G_t, x_v = m_{v,t} = f_{v,t} = 0 \)
5: 
6: // the minting part
7: for all \( v \in G_t \) do
8: \( m_{v,t} \leftarrow 1 \)
9: if \( \sum_{\tau=2}^t f_{v,\tau} = 0 \) then
10: \( \text{inCirculation} \leftarrow \text{inCirculation} + 1 \)
11: else
12: \( \text{payment} \leftarrow 1 \)
13: \( \text{for } \tau := 0 \text{ to } t - 1 \text{ do} \)
14: \( \text{pay} \leftarrow \min(\text{payment}, f_{v,\tau}) \)
15: \( \text{taxCollected} \leftarrow \text{taxCollected} + \frac{\text{pay}}{2} \)
16: \( f_{v,\tau} \leftarrow f_{v,\tau} - \text{pay} \)
17: \( \text{payment} \leftarrow \text{payment} - \text{pay} \)
18: \( \text{end for} \)
19: \( \text{inCirculation} \leftarrow \text{inCirculation} + \text{payment} \)
20: \( \text{end if} \)
21: \( \text{end for} \)
22: 
23: // the fine imposing part
24: for all \( v \in G_t \) do
25: if \( \text{ProbExpose}(v) \) is true then
26: \( x_v \leftarrow 1 \)
27: \( G_t \leftarrow G_t \setminus v \)
28: \( \text{for } \tau := 0 \text{ to } t - 1 \text{ do} \)
29: \( \text{fine} \leftarrow 2 \times m_{v,\tau} + f_{v,\tau} \)
30: \( \text{for all } u \in \partial_{x_u=0}(v, t) \text{ do} \)
31: \( f_u \leftarrow f_u + \frac{\text{fine}}{|\partial_{x_u=0}(v, t)|} \)
32: \( \text{end for} \)
33: \( \text{end for} \)
34: else if \( \text{Ceased}(v) \) is true then
35: \( G_t \leftarrow G_t \setminus v \)
36: \( \text{end if} \)
37: \( \text{end for} \)
38: \( G_{t+1} \leftarrow \text{Transition}(G_t) \)
39: \( \text{end loop} \)
Method 1 Random graph generation method

1: function RandomGraphGen(G, degree, honest, corrupt, sybil)
2: \hspace{1em} G ← G + missing nodes
3: \hspace{1em} while minimal degree < degree − 1 do
4: \hspace{2em} \hspace{1em} v ← pick a random node with deg(v) < degree − 1
5: \hspace{2em} \hspace{1em} U ← all nodes u with deg(u) < degree
6: \hspace{2em} \hspace{1em} U ← U \{v\}
7: \hspace{2em} \hspace{1em} U ← U \{neighbours of v\}
8: \hspace{2em} \hspace{1em} if v is sybil then
9: \hspace{2em} \hspace{2em} \hspace{1em} U ← U \{honests\}
10: \hspace{2em} \hspace{1em} else if v is honest then
11: \hspace{2em} \hspace{2em} \hspace{1em} U ← U \{sibils\}
12: \hspace{2em} \hspace{1em} end if
13: \hspace{1em} \hspace{1em} if U is empty then
14: \hspace{2em} \hspace{2em} \hspace{1em} continue while loop
15: \hspace{1em} \hspace{1em} end if
16: \hspace{2em} \hspace{1em} u ← pick a random node from U
17: \hspace{2em} \hspace{1em} E(G) = E(G) ∪ (v, u)
18: \hspace{2em} \hspace{1em} E(G) = E(G) \{(x, y)|deg(x) = deg(y) = degree\}
19: \hspace{1em} end while
20: end function

Once genuine identities become mortal, it is no longer guaranteed that the mechanism will retrieve double the amount of coins minted by sybils, as every time a corrupt identity ceases to exist, any debt it did not yet pay is lost. We estimate that the life expectancy of a sybil identity in a real world community will be in the order of months (maybe few years). On the other hand, life expectancy of a genuine identity should be in the order of tenths of years (maybe about 20 years less than the life expectancy of a human being). We are therefore interested to see how our model behave when \(q \approx p/20\). For the second simulation we chose \(p = 0.034\) and \(q = 0.0017\), which gives a half life of 400 loops for a genuine identity.

The matlab simulation keeps the graph of identities as a matrix with 120 columns. Whenever an identity is removed from the graph, a new identity of the same type is created in the same slot. Figure 4 shows the fine property for some of these slots, hence every line represents several identities, where each new identity is added to the graph immediately when its predecessor is removed from the graph. The diagram shows some of the corrupt identities (lines 90 to 100) and some of the sybil identities (lines 101 to 112). The graph shows the fine for each identity per loop, after the simulation finished. Mostly the fine is 0, as most of the fine was paid. On the other hand, every time a corrupt identity ceased to exist, some of its fine was left without being paid. This is the noise-like signal on lines 90 to 100, which unintentionally also marks the point in time in which each identity ceased to exist.

Figure 5 shows the excess minting in this run. While the graph in figure 3 remained bounded under a constant, this graph continues to rise slowly, as the
Fig. 3. Excess money minting with sybil exposure probability 0.034 and genuine termination probability 0. The graph shows the amount of coins minted by sybils and not yet retrieved. That is inCirculation \(-\) expected, where expected = \(t \cdot (\# \text{ of genuine})\).

fine lost when a corrupt identity ceases to exist, is not recovered. However, the graph on the right shows that the excess money minted is negligible compared to the money collected as tax. Remember that the desired goal is to collect two coins in return for each coin minted by a sybil. The first coin retrieved is used to nullify the undesirable coin, while the second coin retrieved acts as a deterring punishment and as a source for budgeting sybil hunting. Figure 5 shows that this goal is not achievable with the chosen parameters, but the result is not far from it. By giving up on a small amount of the collected tax, the community can still make sure that 100\% of the undesirable coins (minted by sybils) are nullified.

The last run of the simulation tested several \(q\) values in the range 0 \(\leq q \leq p\). Figure 6 shows the ratio between the excess minting to the amount of tax collected. It shows that as long as the probability that a genuine identity will cease to exist is not bigger than the probability to expose a sybil, the community has enough tax collected to nullify all money minted by sybils.

7 Outlook

In the context of constructing a community, bottom up, from the grass roots, in a decentralized manner, we showed a possible starting point for the economy of the group, where money is being minted equally by all individuals. We showed a mechanism for money minting that is resilient to sybils in three accounts: (1) all money minted by sybils is eventually nullified; (2) supporting sybils is disincentivized; and (3) enough money is collected in the process to award sybil hunting. Our mechanism relies on a trust graph that defines who is a member of the
Fig. 4. The fine parameter of 11 corrupts (index 90 to 100) and 12 sybils (index 101 to 112). The lines are shifted by the identity’s index for visibility. Whenever an identity is removed from the graph, a new identity of the same type immediately takes its place in the same index.

Fig. 5. Excess money minting with sybil exposure probability 0.034 and genuine termination probability 0.0017. On the right the same graph, compared to the \( \text{taxCollected} \) parameter.

Fig. 6. The ratio of excess minting to tax collected for different \( q \) values.
community. In another work in progress we show how to construct a distributed ledger shared among the members of such a growing community. Together, these three pillars (the trust graph, the minting process, and the distributed ledger) form a basis for the community social construction. The dynamics of this construction, the social structure that follows and the community practices that it enables are the subject of further research.

A further future research direction would be to study our situation as a Stackelberg game between an attacker (a wealthy oligarch) and a defender (possibly, the community as a whole).

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