Higgs Triplets, Decoupling, and Precision Measurements

Mu-Chun Chen\textsuperscript{a} \textsuperscript{*}, Sally Dawson\textsuperscript{b} \textsuperscript{†} and C. B. Jackson\textsuperscript{c} \textsuperscript{‡}

\textsuperscript{a}Department of Physics and Astronomy
University of California, Irvine, CA 92697, USA
\textsuperscript{b}Department of Physics, Brookhaven National Laboratory, Upton, NY 11973, USA
\textsuperscript{c}HEP Division, Argonne National Laboratory,
9700 Cass Ave. Argonne, IL 60439

(Dated: September 24, 2008)

Abstract

Electroweak precision data has been extensively used to constrain models containing physics beyond that of the Standard Model. When the model contains Higgs scalars in representations other than SU(2) singlets or doublets, and hence $\rho \neq 1$ at tree level, a correct renormalization scheme requires more inputs than the three needed for the Standard Model. We discuss the connection between the renormalization of models with Higgs triplets and the decoupling properties of the models as the mass scale for the scalar triplet field becomes much larger than the electroweak scale. The requirements of perturbativity of the couplings and agreement with electroweak data place strong restrictions on models with Higgs triplets. Our results have important implications for Little Higgs type models and other models with $\rho \neq 1$ at tree level.

\textsuperscript{*}muchunc@uci.edu
\textsuperscript{†}dawson@bnl.gov
\textsuperscript{‡}jackson@hep.anl.gov
I. INTRODUCTION

The Standard Model of electroweak physics is remarkably successful at explaining experimental data. From a theoretical standpoint, however, the theory has many failings. Attempts to address these perceived inadequacies have led to the construction of models which reduce to the Standard Model at energy scales below about 1 TeV, but which differ at higher energies. Models with physics beyond that of the Standard Model (SM), however, are severely constrained by precision electroweak data[1, 2]. If the mass scale of the new physics is near the TeV scale, it is often possible to learn about the parameters of the model by performing global fits to precision measurements. The simplest example is the prediction of the W boson mass, $M_W$. In the Standard Model, $M_W$ can be predicted in terms of other parameters of the theory and requiring agreement with the measured W mass therefore restricts the possibilities for new TeV scale physics.

In this paper, we introduce a Higgs triplet at the electroweak scale and consider the effect on $M_W^2$. We are motivated by Little Higgs models, which include a scalar triplet as a necessary ingredient, although our results are very general[3, 4, 5, 6]. In a model with Higgs particles in representations other than $SU(2)$ doublets and singlets, there are more parameters in the gauge/Higgs sector than in the Standard Model (SM). The SM tree level relation, $\rho = M_W^2/(M_Z^2 c_\theta^2) = 1$, no longer holds and when the theory is renormalized an extra input parameter is required[3, 4, 5, 6, 14, 15, 16, 17, 18]. We discuss two possible renormalization schemes for the triplet model: one where the extra parameter is chosen to be a low energy observable[3, 4, 5], and one where the extra parameter is taken to be the running vacuum expectation value (VEV) of the triplet scalar[19, 20]. Models with $\rho = M_W^2/(M_Z^2 c_\theta^2) \neq 1$ can be consistent with experimental data with the inclusion of certain types of new physics[21, 22], of which a Higgs triplet is a specific example.

In Section II we describe a model which contains a real Higgs triplet in addition to the Higgs doublet of the Standard Model. This example is a simplified version of the Higgs sector in Little Higgs Models and is the simplest example of a model with $\rho \neq 1$ at tree level. In Section III, we discuss the restrictions on models with scalar triplets at the electroweak scale from requiring perturbativity of the parameters of the scalar potential. We turn in Section IV to a discussion of the renormalization prescription and the role of the triplet vacuum expectation value (VEV). The role of the scalar particles is emphasized in obtaining predictions for $M_W$ in the triplet model[23]. Whereas in the SM, the Higgs scalar contributes logarithmically to the prediction for $M_W$, in the triplet model there are contributions which grow with the scalar masses- squared[24, 25, 26, 27]. We close in Section V with a discussion of the decoupling of Higgs triplet effects for large mass scales or alternatively in the limit that the triplet VEV goes to zero and we draw some general conclusions about the renormalization scheme dependence in models with $\rho \neq 1$ at tree limit.

II. THE MODEL

We consider a model with a real Higgs doublet, $H$, and a real, isospin $Y = 0$ triplet, $\Phi$. We assume that the scalar potential is such that the neutral components of both the doublet and the triplet receive VEVs, breaking the electroweak symmetry. The scalars are
conventionally written as,

$$H = \left( \frac{1}{\sqrt{2}} (v + h^0 + i\chi^0) \right), \quad \Phi = \begin{pmatrix} \eta^+ \\ v' + \eta^0 \\ -\eta^- \end{pmatrix}. \quad (1)$$

The kinetic part of the Lagrangian is

$$L = |D_\mu H|^2 + \frac{1}{2} |D_\mu \Phi|^2, \quad (2)$$

where,

$$D_\mu H = \left( \partial_\mu + i \frac{g}{2} \sigma^a W^a + i g' Y B_\mu \right) H$$

$$D_\mu \Phi = \left( \partial_\mu + ig_\mu W^a \right) \Phi, \quad (3)$$

$$\sigma^a (a = 1, 2, 3)$$ are the Pauli matrices, and

$$t_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad t_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (4)$$

Spontaneous symmetry breaking generates masses for the $W$ and $Z$ bosons,

$$M^2_W = \frac{g^2}{4} \left( v^2 + 4v'{}^2 \right)$$

$$M^2_Z = \frac{g^2}{4c_\theta v^2}, \quad (5)$$

where $c_\theta \equiv \cos \theta = g/\sqrt{(g')^2 + (g)^2}$ and $\sin \theta \equiv s_\theta$. At tree level, all definitions of $c_\theta$ are equivalent and the VEVs are related to the SM VEV by $v^2_{SM} = (246 \text{ GeV})^2 = v^2 + 4v'{}^2$.

The model violates custodial symmetry at tree level,

$$\rho = \frac{M^2_W}{M^2_Z c_\theta} = 1 + 4 \frac{v'{}^2}{v^2}. \quad (6)$$

Since experimentally $\rho \sim 1$, $v'$ will be restricted to be small. Neglecting scalar loops, Refs. [1, 2] found $v' < 12 \text{ GeV}$.

The most general $SU(2) \times U(1)$ scalar potential with a Higgs doublet and a real scalar triplet is given by,

$$V = \mu_1^2 |H|^2 + \mu_2^2 |\Phi|^2 + \lambda_1 |H|^4 + \frac{\lambda_2}{4} |\Phi|^4$$

$$+ \lambda_3 \frac{1}{2} |H|^2 |\Phi|^2 + \lambda_4 \Phi \sigma^a H \Phi^a. \quad (7)$$

We note that the coefficient $\lambda_3$ has dimensions of mass, which implies that its effects may be important even for large mass scales since the decoupling theorem is not applicable to interactions which are proportional to mass [3, 28, 29].
After spontaneous symmetry breaking, there are two physical neutral eigenstates, $H^0, K^0$,

$$
\begin{pmatrix}
H^0 \\
K^0
\end{pmatrix}
= 
\begin{pmatrix}
c_\gamma & s_\gamma \\
-s_\gamma & c_\gamma
\end{pmatrix}
\begin{pmatrix}
h^0 \\
\eta^0
\end{pmatrix}.
$$

(8)

The physical charged eigenstates are $H^\pm$ and the charged Goldstone bosons which become the longitudinal components of the $W^\pm$ bosons are $G^\pm$,

$$
\begin{pmatrix}
G^+ \\
H^+
\end{pmatrix}
= 
\begin{pmatrix}
c_\delta & s_\delta \\
-s_\delta & c_\delta
\end{pmatrix}
\begin{pmatrix}
\phi^+ \\
\eta^+
\end{pmatrix},
$$

(9)

where $\tan \delta = 2v'/v$.

Minimizing the potential gives the tree-level relations:

$$
0 = \mu_1^2 + \lambda_1 v^2 + \frac{\lambda_3}{2} v'-^2 - \lambda_4 v'
$$

(10)

$$
0 = v' \left( \mu_2^2 + \lambda_2 v'^2 + \frac{\lambda_3}{2} v^2 \right) - \lambda_4 \frac{v^2}{2}.
$$

(11)

For $v' = 0$, the only consistent solution to the minimization of the potential has $M_{K^0} = M_{H^+}$ and $\lambda_4 = \sin \delta = \sin \gamma = 0$. In this limit the custodial symmetry is restored and $\rho = 1$ at tree level.

We assume that $v' \neq 0$ and $\gamma \neq 0$. Utilizing Eqs. [10] and [11] the scalar mass eigenstates are [30],

$$
M_{H^+}^2 = \frac{\lambda_4 v}{c_\delta s_\delta},
$$

$$
M_{H^0}^2 = 2\lambda_1 v^2 + \tan \gamma (\lambda_3 v v' - \lambda_4 v),
$$

$$
M_{K^0}^2 = 2\lambda_1 v^2 - \cot \gamma (\lambda_3 v v' - \lambda_4 v).
$$

(12)

We take as our 6 input parameters in the scalar sector, $M_{H^+}, M_{H^0}, M_{K^0}, \gamma, \delta, v$. Inverting Eq. [12],

$$
\lambda_1 = \frac{1}{2v^2} \left( c_\gamma^2 M_{H^0}^2 + s_\gamma^2 M_{K^0}^2 \right),
$$

$$
\lambda_2 = \frac{2}{v^2} \cot^2 \delta \left[ s_\gamma^2 M_{H^0}^2 + c_\gamma^2 M_{K^0}^2 - c_\delta^2 M_{H^+}^2 \right],
$$

$$
\lambda_3 = \frac{1}{v^2 \tan \delta} \left[ (M_{H^0}^2 - M_{K^0}^2) \sin(2\gamma) + M_{H^+}^2 \sin(2\delta) \right],
$$

$$
\lambda_4 = c_\delta s_\delta \frac{M_{H^+}^2}{v},
$$

$$
\mu_1^2 = -\frac{M_{H^0}^2}{2} \left( c_\gamma^2 + \frac{s_\gamma c_\gamma}{2} \tan \delta \right) - \frac{M_{K^0}^2}{2} \left( s_\gamma^2 - \frac{s_\gamma c_\gamma}{2} \tan \delta \right) + \frac{M_{H^+}^2}{4} s_\delta^2,
$$

$$
\mu_2^2 = \frac{c_\gamma^2}{2} M_{H^+}^2 - \frac{M_{H^0}^2}{2} \left( s_\gamma^2 + \sin(2\gamma) \cot \delta \right) - \frac{M_{K^0}^2}{2} \left( c_\gamma^2 - \sin(2\gamma) \cot \delta \right).
$$

(13)
FIG. 1: Restriction on the mass difference, $M_{K^0} - M_{H^+}$ from the requirement that the scalar coupling, $\lambda_2$, be perturbative, $\lambda_2 \lesssim (4\pi)^2$. For $\gamma = 0$, there is no dependence on $M_{H^0}$.

FIG. 2: Restriction on the charged scalar mass from the requirement that the scalar coupling, $\lambda_4$, be perturbative, $\frac{\lambda_4}{\Lambda} \lesssim (4\pi)^2$.

III. PERTURBATIVITY OF THE SCALAR COUPLINGS

A priori, the input parameters, $M_{H^+}, M_{H^0}, M_{K^0}, \gamma, \delta, v$, are arbitrary. The requirement that the tree level contributions to the scalar self interactions be larger than the one loop contributions can be loosely interpreted as the restriction $\lambda_{1,2,3} < (4\pi)^2$, and $\frac{\lambda_4}{\Lambda} < (4\pi)^2$, (the scale $\Lambda$ is arbitrary.)

From Eq. 13, approximate bounds on the scalar masses can be derived\(^1\). The most

\(^1\)Ref. [30] derived the renormalization group improved bounds on the scalar masses. For our purposes the
restrictive bound on $M_{H^0}$ is found from $\lambda_1 \lesssim (4\pi)^2$ for $c_\gamma \sim 1,$

$$M_{H^0} \lesssim 4.4 \text{ TeV}.$$  

An interesting limit on the mass difference between the charged scalar, $H^+$, and the heavier of the neutral scalars, $K^0$, comes from Eq. 13 and the requirement that $\lambda_2 \lesssim (4\pi)^2$. This restriction is illustrated in Fig. 1. As the mass of $M_{H^+}$ becomes large, perturbativity requires that the mass difference between $M_{K^0}$ and $M_{H^+}$ be small, regardless of the mixing parameters. Similarly, assuming a scale $\Lambda \sim v$, the perturbativity limits on $M_{H^+}$ from $\lambda_4$ are shown in Fig. 2. These results are in agreement with those found in Refs. [30] and [28].

IV. RENORMALIZATION

A. Standard Model

In this section, we discuss the differences between renormalization in the SM and in a model with a scalar triplet. We begin with a brief overview of Standard Model renormalization in order to set the framework [31, 32, 33, 34]. The electroweak sector of the SM has four fundamental parameters, the $SU(2)_L \times U(1)_Y$ gauge coupling constants, $g$ and $g'$, the vacuum expectation value (VEV) of the neutral component of the Higgs doublet, $v$, and the physical Higgs boson mass, along with the fermion masses. Once these parameters are fixed, all other physical quantities in the gauge sector can be derived. The usual choice of input parameters is the muon decay constant, $G_\mu$, the $Z$-boson mass, $M_Z$, the fine structure constant, $\alpha$, and the unknown Higgs boson mass, $M_{h,SM}$. Experimentally, the measured values for these input parameters are [35],

$$G_\mu = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$$

$$M_Z = 91.1876(21) \text{ GeV}$$

$$\alpha^{-1} = 137.035999679(94) .$$

Tree level objects are denoted with a subscript 0 and satisfy the relationship,

$$M_{W0}^2 = \frac{\pi \alpha_0}{\sqrt{2} G_\mu s_\theta^2}. \quad (18)$$

and the SM satisfies $\rho_0 = 1$ at tree level,

$$\rho_0 \equiv \frac{M_{W0}^2}{M_{Z0}^2 s_\theta^2} = 1 . \quad (19)$$

The 1-loop renormalized quantities are defined$^2$:

$$M_{V0}^2 \equiv M_V^2 \left(1 + \frac{\delta M_V^2}{M_V^2} \right) = M_V^2 \left(1 + \Pi_{VV} (M_V^2) \right)$$

$^2$ Eq. 20 implicitly defines our sign conventions for $\Pi_{XY}$. We decompose the two-point functions as $\Pi_{\mu \nu}^X(k^2) = g_\mu \Pi_{XY}(k^2) + k^\mu k^\nu B_{XY}(k^2)$ and label the SM contributions as $\Pi_{XY,SM}$.
\begin{align*}
G_{\mu 0} & \equiv G_{\mu} \left(1 + \frac{\delta G_{\mu}}{G_{\mu}}\right) = G_{\mu} \left(1 - \frac{\Pi_{WW}(0)}{M_W^2}\right) \\
\sin^2 \theta_{00} & = \sin^2 \theta \left(1 + \frac{\delta \sin^2 \theta}{\sin^2 \theta}\right) \\
\alpha_0 & = \alpha \left(1 + \frac{\delta \alpha}{\alpha}\right) = \alpha \left(1 + \Pi_{\gamma \gamma}(0) + 2 \frac{\sin \theta \Pi_{\gamma Z}(0)}{c_{\theta} M_Z^2}\right),
\end{align*}

where \(\Pi_{\gamma \gamma}(0) = (d \Pi_{\gamma \gamma}(p^2)/dp^2)|_{p^2=0} \sim \frac{\Pi_{\gamma \gamma}(M_Z^2)}{M_Z^2}\). The box and vertex corrections are small and we neglect their finite contributions (although it is necessary to include the poles in order to achieve a finite result).

The \(W\)-boson mass is predicted at 1-loop,

\[M_W^2 = \frac{\pi \alpha}{\sqrt{2} G_{\mu} s_{\theta}^2} \left[1 + \Delta r_{SM}\right],\]

where \(\Delta r_{SM}\) summarizes the radiative corrections,

\[\Delta r_{SM} = -\frac{\delta G_{\mu}}{G_{\mu}} - \frac{\delta M_W^2}{M_W^2} + \frac{\delta \alpha}{\alpha} - \frac{\delta s_{\theta}^2}{s_{\theta}^2}\]

\[= \frac{\Pi_{WW,SM}(0) - \Pi_{WW,SM}(M_W^2)}{M_W^2} + \Pi_{\gamma \gamma,SM}(0) + 2 \frac{\sin \theta \Pi_{\gamma Z,SM}(0)}{c_{\theta} M_Z^2} - \frac{\delta s_{\theta}^2}{s_{\theta}^2}.\]

1. **On-Shell Definition of \(\sin \theta_W\)**

In the on-shell scheme, the weak mixing angle, \(s_W\), is not a free parameter, but is derived from

\[\rho \equiv \frac{M_W^2}{M_Z^2 c_{\theta}^2}.\]

The counter term for \(s_W^2\) can be derived from Eq. 23

\[\frac{\delta s_W^2}{s_W^2} = \frac{c_{\theta}^2}{s_W^2} \left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right] = \frac{c_{\theta}^2}{s_W^2} \left[\frac{\Pi_{ZZ,SM}(M_Z^2)}{M_Z^2} - \frac{\Pi_{WW,SM}(M_W^2)}{M_W^2}\right].\]

2. **Effective Mixing Angle Definition of \(\sin^2 \theta_{eff}\)**

One could take as input parameters, \(G_{\mu}, \alpha\), and the effective weak mixing angle, \(\sin \theta_{eff} \equiv s_{\theta,eff}\). The effective weak mixing angle is defined in terms of the electron coupling to the \(Z\):

\[L = \frac{e}{2 \cos \theta_{eff} \sin \theta_{eff}} \bar{e} \gamma_{\mu} \left(v_e - a_e \gamma_5\right) e Z^\mu\]

\[= \frac{e}{2 \cos \theta_{eff} \sin \theta_{eff}} \bar{e} \gamma_{\mu} \left(v_e - a_e \gamma_5\right) e Z^\mu\]
where,
\[ v_e = -\frac{1}{2} + 2\sin^2 \theta_{\text{eff}} \quad a_e = -\frac{1}{2}. \] (26)

In this scheme,
\[ \frac{\delta s_{\theta, \text{eff}}^2}{s_{\theta, \text{eff}}^2} = \left( \frac{c_{\theta, \text{eff}}}{s_{\theta, \text{eff}}} \right) \frac{\Pi_{\gamma Z, \text{SM}}(M_Z^2)}{M_Z^2} + O(m_e^2). \] (27)

This scheme is useful for comparing with the predictions of the triplet model using the renormalization scheme of Refs. [4, 5].

3. “\( M_Z \)” Definition of \( \sin \theta_Z \)

A third scheme for renormalizing the SM takes as inputs \( \alpha(M_Z), G_\mu, \) and \( M_Z \) and defines \( \sin \theta_Z \equiv s_Z, \cos \theta_Z \equiv c_Z \) in terms of,
\[ \sin(2\theta_Z) \equiv \frac{4\pi \alpha(M_Z)}{\sqrt{2}G_\mu M_Z^2}. \] (28)

where
\[ \frac{\delta s_Z^2}{s_Z^2} = \frac{c_Z^2}{c_Z^2 - s_Z^2} \left( \frac{\Pi_{\gamma Z, \text{SM}}(M_Z^2)}{M_Z^2} + \frac{2s_Z \Pi_{\gamma Z, \text{SM}}(M_Z^2)}{M_Z^2} - \frac{\Pi_{ZZ, \text{SM}}(M_Z^2)}{M_Z^2} + \frac{\Pi_{WW, \text{SM}}(M_Z^2)}{M_W^2} \right). \] (29)

The \( s_Z \) scheme is useful for comparing with the predictions of the triplet model using the renormalization scheme advocated in Ref. [19].

B. \( Y = 0 \) Triplet

In this section, we consider the 1–loop renormalization of the triplet model. We are particularly interested in the approach of the triplet model to the SM in different limits and in the scheme dependence of our results. Since \( \rho \neq 1 \) at tree level in the triplet model, 4 input parameters (along with the Higgs mass) are required for the electroweak renormalization [1, 3, 6, 14]. We will consider two possible renormalization schemes. The first scheme uses 4 measured low energy parameters as inputs, while the second employs 3 low energy parameters plus a running triplet VEV, \( v'(\mu) \):

- **Scheme 1**: Input \( \alpha, M_Z, G_\mu, \) and \( \sin^2 \theta_{\text{eff}} \)

- **Scheme 2**: Input \( \alpha, M_Z, G_\mu, \) and \( v'(\mu) \).

In both schemes, the \( W \) boson mass is a predicted quantity. Below we discuss the dependence of the prediction for the \( W \) mass on the renormalization scheme and focus on the approach to the SM limit as the triplet VEV becomes small, \( v' \rightarrow 0 \), or alternatively as \( M_{K^0} \) and \( M_{H^+} \rightarrow \infty \).
1. Triplet Model, Scheme 1

The renormalization of the triplet model in Scheme 1 has been discussed in Refs. [3, 4, 5]. The input parameters, $\alpha$, $M_Z$, and $G_\mu$ are given in Eq. (17), and

$$\sin^2 \theta_{\text{eff}} = 0.2324 \pm 0.0012.$$  \hfill (30)

The relation,

$$\rho = \frac{1}{\Delta s^2_{\theta,\text{eff}}} = \frac{M_W^2}{M_Z^2 c^2_{\theta,\text{eff}}},$$  \hfill (31)

implies that $\tan \delta = 2v'/v$ is not a free parameter in this scheme, but is fixed by the input parameters.

In this scheme, the $W$ mass is given by,

$$M_W^2 = \frac{\alpha \pi}{\sqrt{2} s_{\theta,\text{eff}} G_\mu} \left(1 + \Delta r_{\text{triplet}}(\text{Scheme 1})\right)$$  \hfill (32)

and

$$\Delta r_{\text{triplet}}(\text{Scheme 1}) = \frac{\Pi_{WW}(0) - \Pi_{WW}(M_W^2)}{M_W^2} + \Pi'_{\gamma\gamma}(0) + 2 \frac{s_{\theta,\text{eff}} \Pi_{\gamma Z}(0)}{c_{\theta,\text{eff}} M_Z^2} - \frac{\delta s^2_{\theta,\text{eff}}}{s^2_{\theta,\text{eff}}}$$  \hfill (33)

where

$$\frac{\delta s^2_{\theta,\text{eff}}}{s^2_{\theta,\text{eff}}} = \frac{s_{\theta,\text{eff}} \Pi_{\gamma Z}(M_Z^2)}{s_{\theta,\text{eff}} M_Z^2}.$$  \hfill (34)

Analytic formulae for the scalar, gauge boson, and Goldstone boson contributions to the two-point functions are given in Appendix 1 for arbitrary values of the mixing parameters $\sin \delta$ and $\sin \gamma$. The contributions from non-zero values of $\sin \gamma$ have not appeared elsewhere.

There are three types of contributions to the two-point functions. There are contributions from the $W$, $Z$, and $\gamma$ gauge bosons, the electroweak ghosts, the Goldstone bosons and the lightest neutral Higgs boson where the couplings have SM strength. These are labelled as $\Pi_{XY,\text{SM}}$ in Appendix 1. It is important to remember that these are not numerically equal to the results in the SM since the relationship between the $W$ and $Z$ masses is different in the SM and in the triplet model. The remaining contributions, which we label $\Delta \Pi_{XY}$, are of two types. There are contributions from the SM particles with couplings proportional to $\sin \delta$ or $\sin \gamma$ which vanish for $\delta, \gamma \to 0$, and there are contributions from the new particles of the triplet model, $K^0$ and $H^+$, which do not necessarily vanish for $\delta, \gamma \to 0$.

Eq. (33) has a form analogous to the SM results obtained using the sin $\theta_{\text{eff}}$ scheme, ($\Delta r_{\text{SM}}^{\text{eff}}$), except that in Eq. (33) there are additional contributions to the two-point functions from $K^0$ and $H^+$, and the SM-like gauge boson, Goldstone boson, and $H^0$ contributions are weighted by factors of $\cos \delta$ and $\cos \gamma$. At tree level, the SM and triplet predictions for $M_W$ are the

---

3 We neglect the finite contribution from vertex and box diagrams, although the pole contributions are included in order to make our result finite and gauge invariant. The vertex and box corrections in the triplet model can be found in Ref. [3].
\[ \Delta r_{\text{triplet}}(\text{Scheme 1}) = \tilde{\Delta} r_{\text{SM}}^{\text{eff}} + \Delta r,1. \tag{35} \]

The function \( \tilde{\Delta} r_{\text{SM}}^{\text{eff}} \) has the same functional form as \( \Delta r_{\text{SM}}^{\text{eff}} \) with the important difference that in calculating \( \tilde{\Delta} r_{\text{SM}}^{\text{eff}} \), \( M_Z \) must be taken as an input in the triplet scheme 1, while \( M_Z \) is a prediction in the \( \sin \theta_{\text{eff}} \) scheme of the SM. In the limit of small mixing (\( \delta \sim 0, \gamma \sim 0 \)),

\[ \Delta r,1 \rightarrow \frac{\alpha}{\pi \sin^2 \theta_{\text{eff}}} \left[ \frac{F_{22}(M_W^2, M_{K^0}, M_{H^+})}{M_W^2} - \frac{F_{22}(M_Z^2, M_{H^+}, M_{H^+})}{M_Z^2} \right] \tag{36} \]

where

\[ F_{22}(k^2, m_1, m_2) = B_{22}(k^2, m_1, m_2) - B_{22}(0, m_1, m_2). \tag{37} \]

The small mixing limit further simplifies in the limit that \( M_{K^0}, M_{H^+} >> M_W, M_Z \) and \( |M_{K^0} - M_{H^+}| << M_{K^0}^2 \)

\[ \Delta r,1 \rightarrow \frac{\alpha}{24 \pi \sin^2 \theta_{\text{eff}}} \left\{ \frac{M_{K^0}^2 - M_{H^+}^2}{M_{H^+}^2} \right\} + \ldots. \tag{38} \]

Eq. 38 is independent of the light Higgs boson mass and can be either positive or negative depending on the sign of \( M_{K^0} - M_{H^+} \).

In Fig. 3, we show the approach of the triplet model to the one-loop SM prediction (in the \( \sin^2 \theta_{\text{eff}} \) renormalization scheme) as \( M_{H^+} \) becomes large. The SM prediction for \( M_W \) is calculated using Eqs. 22 and 27, while the triplet prediction for \( M_W \) is calculated using Eqs. 33 and 34. For small mixing, and \( M_{K^0} = M_{H^+} \), the one loop prediction for \( M_W \) in the triplet model differs negligibly from the SM prediction. As the mass splitting, \( |M_{K^0} - M_{H^+}| \) is increased, significant differences from the SM prediction are seen at small \( M_{H^+} \). The remainder term, \( \Delta r,1 \), never goes exactly to zero, because the triplet model has
$M_Z$ as an input, while the SM computes $M_Z$ in the $\sin\theta_{eff}$ scheme. We recall from Section II that in the limit $M_{K^0} = M_{H^+}$, the only consistent solution to the minimization of the potential is $v' = 0$ and $c_\delta = c_\gamma = 1$. In this limit, $\rho_0 = 1$ and the only difference between the prediction of triplet model and the SM arises from the different input values of $M_Z$.

2. Triplet Model, Scheme 2

In Scheme 2, the input parameters are $\alpha$, $M_Z$, $G_\mu$ and a running $v'(\mu)$. This scheme has been advocated in Ref. [19] as being more natural than Scheme 1, in that it has 3 measured input parameters as does the SM, while $v'$ is unknown. We will treat $v'$ as a running $\overline{MS}$ parameter. Of particular interest is the $v' \to 0$ limit and the approach to the SM as $M_{K^0}$ and $M_{H^+} \to \infty$.

As usual, the $W$ boson mass is defined by

$$M^2_W = \frac{\alpha \pi}{\sqrt{2} s_0^2 G_\mu} \left(1 + \Delta r_{\text{triplet}}(\text{Scheme 2})\right)$$

(39)

where

$$\Delta r_{\text{triplet}}(\text{Scheme 2}) = \frac{\Pi_{WW}(0) - \Pi_{WW}(M^2_W)}{M^2_W} + \frac{\Pi_{\gamma\gamma}(M^2_Z)}{M^2_Z} + \frac{2 \hat{s}_Z \Pi_{\gamma Z}(0)}{\hat{c}_Z M^2_Z} - \frac{\delta s^2_Z}{\hat{s}_Z^2}.$$  

(40)

At tree level,

$$G_{\mu 0} = \frac{1}{\sqrt{2}(v_0^2 + 4v'_0^2)},$$

(41)

where $v_0$ and $v'_0$ are the tree level VEVs. Using

$$M^2_{Z_0} = \frac{e_0^2}{4s^2_{Z_0} v_0^2}$$

(42)

and Eq. (41) we find the weak mixing angle,

$$\hat{s}^2_{Z_0} = \frac{\pi \alpha_0 v_0^2}{M^2_{Z_0}} = \frac{\pi \alpha_0}{M^2_{Z_0}} \left[\frac{1}{\sqrt{2} G_{\mu 0}} - 4v_0^2\right].$$

(43)

This scheme is similar to the $M_Z$ scheme for the SM described in Eq. (28) in the limit $v'_0 \to 0$. At tree level $\hat{s}_{Z_0}$ is defined in terms of the input parameters as

$$\hat{s}^2_{Z_0} = \frac{1}{2} \left(1 - \frac{4\pi \alpha_0}{\sqrt{2} M^2_{Z_0} G_{\mu 0}} (1 - 4v_0^2)\right),$$

(44)

\footnote{Again, we neglect the small finite contributions from the vertex and box diagrams, although the pole contributions are included in order to make our result finite and gauge independent.}
and the 1-loop corrected value for the mixing angle is

$$\frac{\delta s_Z^2}{s_Z^2} = \frac{c_Z^2}{c_Z^2 - s_Z^2} \left\{ \frac{\Pi'_{\gamma\gamma}(0)}{\hat{c}_Z M_Z^2} + \frac{\Pi_{\gamma\gamma}(0)}{\hat{c}_Z M_Z^2} - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{1}{1 - 4\sqrt{2}v'G\mu} \left[ \frac{\Pi_{WW}(0)}{M_W^2} - 4\sqrt{2}G\mu v'^2 \delta v'^2 \right] \right\}. \quad (45)$$

Finally, we need to define the renormalized triplet vev:

$$v_0^2 = v'^2(1 + \frac{\delta v'^2}{v'^2}). \quad (46)$$

There is no compelling physical definition for the $v'$ counterterm and we simply utilize an $\overline{MS}$ definition and retain only the poles necessary to cancel the divergences.

In Fig. 4 we compare the one-loop corrected prediction for $M_W$ in the $M_Z$ scheme of the SM, with the one-loop corrected value for $M_W$ in the triplet model, Scheme 2, with $\gamma = 0$ and $v' = 0$. For $\gamma = 0$ and $v' = 0$, the only consistent solution to the minimization of the potential is $M_{H^+} = M_{K^0}$ and for these parameters, the contribution of the triplet model quickly decouples as $M_{H^+}$ becomes large and the SM result is exactly recovered.

The situation is quite different for non-zero $v'$ as shown in Figs. 5 and 6. The large effects can be understood from Eq. (45)

$$\frac{\delta s_Z^2}{s_Z^2} \sim \left\{ \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{1}{1 - 4\sqrt{2}v'^2G\mu} \frac{\Pi_{WW}(0)}{M_W^2} \right\}. \quad (47)$$

Both $\frac{\Pi_ZZ(M_Z^2)}{M_Z^2}$ and $\frac{\Pi_{WW}(0)}{M_W^2}$ have contributions which grow with $M_{H^+}^2$ and $M_{K^0}^2$ which cancel in Eq. (47) when $v' = 0$. The cancellation is spoiled for non-zero $v'$ leading to the large effects observed in Figs. 5 and 6. Figs. 5 and 6 show a modest sensitivity of our results to non-zero mixing in the neutral sector ($\gamma \neq 0$).
FIG. 5: Difference between the one-loop corrected values $M_W$ (Triplet, Scheme 2) and $M_W$ (SM, $M_Z$ Scheme) as a function of the charged Higgs mass, $M_{H^+}$, for no mixing in the neutral sector ($\gamma = 0.0$) and for $v' = 3$ GeV and $v' = 6.8$ GeV. Tadpoles are not included in this plot.

FIG. 6: Difference between the one-loop corrected values $M_W$ (Triplet, Scheme 2) and $M_W$ (SM, $M_Z$ Scheme) as a function of the charged Higgs mass, $M_{H^+}$, for small mixing in the neutral sector ($\gamma = 0.1$) and for $v' = 3$ GeV and $v' = 6.8$ GeV. Tadpoles are not included in this plot.

Eq. (47) makes it apparent that tadpole diagrams (shown in Fig. 7) do not cancel for non-zero $v'$ and make a contribution,

$$\Delta r_{\text{triplet}}(\text{Scheme 2})^{\text{tadpole}} = -\frac{c_Z^2}{c_Z^2 - s_Z^2} \left\{ -\frac{\Pi_{ZZ}^{\text{tadpole}}}{M_Z^2} + \frac{1}{1 - 4\sqrt{2} v'^2 G_{\mu}} \frac{\Pi_{WW}^{\text{tadpole}}}{M_W^2} \right\}, \quad (48)$$
FIG. 7: Tadpole contribution to the gauge boson two-point function.

where the tadpole contributions are,

$$
\Pi_{WW}^{tadpole} = -gM_W \left\{ (c_\delta c_\gamma + 2s_\delta s_\gamma) \frac{T_H}{M_{H^0}^2} + (-c_\delta s_\gamma + 2s_\delta c_\gamma) \frac{T_K}{M_{K^0}^2} \right\}
$$

$$
\Pi_{ZZ}^{tadpole} = -gM_Z \left\{ c_\gamma \frac{T_H}{M_{H^0}^2} - s_\gamma \frac{T_K}{M_{K^0}^2} \right\}.
$$

The scalar self couplings are given in Appendix 3 and lead to,

$$
T_H = -\frac{1}{16\pi^2} \left\{ \frac{g_{HHH}}{2} A_0(M_{H^0}) + \frac{g_{HKK}}{2} A_0(M_{K^0}) + \frac{g_{HG^0G^0}}{2} A_0(M_Z) + g_{H^0G^0} A_0(M_W) + g_{K^0H^0} A_0(M_{H^+}) - gM_W (c_\delta c_\gamma + 2s_\delta s_\gamma)(4 - 2\epsilon)A_0(M_W) + gM_Z \frac{c_\gamma}{c_9} (4 - 2\epsilon)A_0(M_Z) \right\}
$$

$$
T_K = -\frac{1}{16\pi^2} \left\{ \frac{g_{KHH}}{2} A_0(M_{H^0}) + \frac{g_{KKK}}{2} A_0(M_{K^0}) + \frac{g_{KG^0G^0}}{2} A_0(M_Z) + g_{K^0G^0} A_0(M_W) + g_{K^0H^0} A_0(M_{H^+}) - gM_W (-c_\delta s_\gamma + 2s_\delta c_\gamma)(4 - 2\epsilon)A_0(M_W) + gM_Z \frac{s_\gamma}{c_9} (4 - 2\epsilon)A_0(M_Z) \right\}.
$$

The tadpole diagrams generate terms which grow with mass-squared and the contribution of the tadpole diagrams in Scheme 2 to $M_W$ are shown in Figs. 8 and 9. The large size of the tadpole contributions makes it clear that $\delta v'$ must be defined in such a manner as to cancel the contributions from the tadpole diagrams in order to have a sensible theory. The tadpole contributions grow with $v'^2$ as expected and have a large dependence on $\gamma$.

V. DECOUPLING AND CONCLUSIONS

We have considered the simplest possible model with $\rho \neq 1$ at tree level: a model with a real scalar $SU(2)$ triplet in addition to the SM Higgs doublet and have presented results for the one-loop prediction for the $W$ mass in two different renormalization schemes. Our results are shown as differences from the SM predictions. A correct renormalization scheme
in the triplet model requires four input parameters, in contrast to the three required in the electroweak sector of the SM.

In the first scheme, four low energy measured parameters are used as inputs and the theory is renormalized as a low energy theory. The effects of the scalar loops are negligible for large triplet scalar masses, when the mass difference between the scalar masses associated with the triplets is small ($|M_{K^0} - M_{H^+}| \ll M_{K^0}$). This renormalization scheme fixes the triplet $v'$ in terms of the input parameters and so the limit $v' \to 0$ cannot be taken. In the second scheme, three low energy parameters and a running triplet VEV are used as inputs. The non-zero
FIG. 10: Contributions from Goldstone Bosons and $H^0$, $K^0$ and $H^+$ to the gauge boson two-point functions.

triplet VEV generates large contributions from tadpole diagrams which must be cancelled by hand by an appropriate definition of the triplet VEV renormalization condition. This fine tuning implies a lack of predictivity for the model. Neither renormalization scheme is entirely satisfactory, although our results clearly demonstrate the importance of scalar loops in theories with $\rho \neq 1$ at tree level.

Acknowledgements

The work of S.D. (C.J.) is supported by the U.S. Department of Energy under grant DE-AC02-98CH10886 (DE-AC02-06CH11357). The work of M.-C. C. is supported, in part, by the National Science Foundation under grant PHY-0709742. S.D. thanks the SLAC theory group for their hospitality, where this work was begun.

Appendix 1: 2-Point contributions

In general, we write

$$\Pi_{XY}(k^2) = \Pi_{XY,SM}(k^2) + \Delta \Pi_{XY}(k^2).$$

(51)

The contributions labelled $\Pi_{XY,SM}$ have the same functional form as the SM contributions from gauge and Goldstone bosons, ghosts, and the lightest neutral Higgs, $H^0$, in the $\rho_0 = 1$ limit. We remind the reader yet again that the $\Pi_{XY,SM}$ terms utilize different relations between $M_Z$ and $M_W$ in the triplet and SM and hence are not in general numerically equal. The remainder, $\Delta \Pi_{XY}$, contains terms which vanish in the limit $\sin \delta, \sin \gamma \to 0$, and also the contributions of $K^0$ and $H^+$, which need not vanish in the $\sin \delta = \sin \gamma = 0$ limit. The SM-like contributions agree with those found in Ref. [34] and the triplet contributions for $\sin \gamma = 0$ agree with Ref. [3]. The contributions for non-zero $\gamma$ are new.

From Fig. 10, the Standard Model-like contributions in Feynman gauge are:

$$\Pi_{WW,SM}^1(k^2) = \frac{\alpha}{16\pi s_\theta^2} \left\{ A_0(M_{H^0}) + A_0(M_Z) + 2A_0(M_W) \right\}$$

$$\Pi_{ZZ,SM}^1(k^2) = \frac{\alpha}{16\pi s_\theta c_\theta} \left\{ A_0(M_{H^0}) + A_0(M_Z) + 2(c_\theta^2 - s_\theta^2)A_0(M_W) \right\}$$

$$\Pi_{\gamma\gamma,SM}^1(k^2) = \frac{\alpha}{2\pi} \left[ A_0(M_W) \right]$$

$$\Pi_{\gamma Z,SM}^1(k^2) = \frac{\alpha}{4\pi s_\theta c_\theta} \left[ (s_\theta^2 - c_\theta^2)A_0(M_W) \right].$$

(52)
FIG. 11: Contribution to the gauge boson 2-point function with one internal scalar and one internal vector boson.

The non-Standard Model contributions from Fig. 11 are:

\[
\Delta \Pi_{WW}^1(k^2) = \frac{\alpha}{16\pi s_\theta^2} \left\{-3s_\theta^2 \left[A_0(M_{K^0}) - A_0(M_{H^0})\right] + 2s_\theta^2 \left[A_0(M_W) - A_0(M_{H^+})\right] + 4 \left[A_0(M_{K^0}) + A_0(M_{H^+})\right]\right\}
\]

\[
\Delta \Pi_{ZZ}^1(k^2) = \frac{\alpha}{16\pi s_\theta^2 c_\theta^2} \left\{s_\theta^2 \left[A_0(M_{K^0}) - A_0(M_{H^0})\right] + 2s_\theta^2 (1 - 4c_\theta^2) \left[A_0(M_{H^+}) - A_0(M_W)\right] + 8c_\theta^4 A_0(M_{H^+})\right\}
\]

\[
\Delta \Pi_{\gamma\gamma}^1(k^2) = \frac{\alpha}{2\pi} A_0(M_{H^+})
\]

\[
\Delta \Pi_{\gamma Z}^1(k^2) = \frac{\alpha}{4\pi s_\theta c_\theta} \left\{s_\theta^2 \left[A_0(M_{H^+}) - A_0(M_W)\right] - 2c_\theta^2 A_0(M_{H^+})\right\}
\]

From Fig. 11 the SM-like contributions are,

\[
\Pi_{WW,SM}^2(k^2) = \frac{\alpha}{4\pi} \frac{M_W^2}{s_\theta^2} \left\{\frac{s_\theta^4}{c_\theta^2} B_0(k^2, M_Z, M_W) + s_\theta^2 B_0(k^2, 0, M_W) + B_0(k^2, M_{H^0}, M_W)\right\}
\]

\[
\Pi_{ZZ,SM}^2(k^2) = \frac{\alpha}{4\pi} \frac{M_Z^2}{s_\theta^2} \left\{\frac{1}{c_\theta^2} B_0(k^2, M_{H^0}, M_Z) + 2s_\theta^4 B_0(k^2, M_W, M_W)\right\}
\]

\[
\Pi_{\gamma\gamma,SM}^2(k^2) = \frac{\alpha}{2\pi} M_W^2 B_0(k^2, M_W, M_W)
\]

\[
\Pi_{\gamma Z,SM}^2(k^2) = \frac{\alpha}{2\pi} M_W^2 c_\theta s_\theta B_0(k^2, M_W, M_W).
\]

The non-Standard Model contributions from Fig. 11 are:

\[
\Delta \Pi_{WW}^2(k^2) = \frac{\alpha}{4\pi} \frac{M_W^2}{s_\theta^2} \left\{\frac{c_\theta^2 s_\theta^2}{c_\theta^2} B_0(k^2, M_Z, M_{H^+}) - B_0(k^2, M_Z, M_W)\right\}
\]

\[
+ \left(4s_\theta c_\theta s_\gamma c_\gamma - s_\gamma^2 + 5s_\theta^2 s_\gamma^2\right) \left[B_0(k^2, M_{H^0}, M_W) - B_0(k^2, M_{K^0}, M_W)\right]
\]

\[
+ s_\theta^2 \left[\frac{c_\theta^2 - s_\theta^2}{c_\theta^2} B_0(k^2, M_Z, M_W) - B_0(k^2, M_{H^0}, M_W) + 4B_0(k^2, M_{K^0}, M_W)\right]\right\}
\]

\[
\Delta \Pi_{ZZ}^2(k^2) = \frac{\alpha}{4\pi} \frac{M_Z^2}{s_\theta^2} \left\{\frac{s_\theta^2}{c_\theta^2} \left[B_0(k^2, M_{K^0}, M_Z) - B_0(k^2, M_{H^0}, M_Z)\right]\right\}
\]

\[
+ 2s_\theta^2 \left[B_0(k^2, M_{H^+}, M_W) - B_0(k^2, M_W, M_W)\right]
\]
FIG. 12: Contribution to the gauge boson 2-point function from Goldstone boson and scalar loops.

\[ +2 \frac{s_\delta^2}{c_\delta^4} B_0(k^2, M_W, M_W) \] \]

\[ \Delta \Pi_{\gamma\gamma}^2(k^2) = 0 \]

\[ \Delta \Pi_{\gamma Z}^2(k^2) = -\frac{\alpha}{2\pi} M_W^2 \frac{s_\delta^2}{s_\theta c_\theta} B_0(k^2, M_W, M_W) . \] (55)

From Fig. 12 the SM-like contributions are

\[ \Pi_{WW,SM}^3(k^2) = -\frac{\alpha}{4\pi s_\theta^2} \left\{ B_{22}(k^2, M_{H^0}, M_W) + B_{22}(k^2, M_Z, M_W) \right\} \]

\[ \Pi_{ZZ,SM}^3(k^2) = -\frac{\alpha}{4\pi s_\theta^2 c_\theta^2} \left\{ B_{22}(k^2, M_{H^0}, M_Z) + (c_\theta^2 - s_\theta^2)^2 B_{22}(k^2, M_W, M_W) \right\} \]

\[ \Pi_{\gamma\gamma,SM}^3(k^2) = -\frac{\alpha}{\pi} B_{22}(k^2, M_W, M_W) \]

\[ \Pi_{\gamma Z,SM}^3(k^2) = \frac{\alpha}{2\pi c_\theta s_\theta} (c_\theta^2 - s_\theta^2) B_{22}(k^2, M_W, M_W) . \] (56)

From Fig. 12 the non-SM contributions are,

\[ \Delta \Pi_{WW}^3(k^2) = -\frac{\alpha}{4\pi s_\theta^2} \left\{ 4s_\gamma c_\gamma s_\delta c_\delta \left( B_{22}(k^2, M_{K^0}, M_{H^0}) - B_{22}(k^2, M_{H^0}, M_{H^0}) \right) + B_{22}(k^2, M_{H^0}, M_W) - B_{22}(k^2, M_{K^0}, M_W) \right\} \]

\[ +4s_\gamma^2 s_\delta^2 \left( B_{22}(k^2, M_{H^0}, M_W) - B_{22}(k^2, M_{H^0}, M_{H^0}) \right) + s_\gamma^2 s_\delta^2 \left( B_{22}(k^2, M_{K^0}, M_{H^0}) - B_{22}(k^2, M_{H^0}, M_{H^0}) \right) + 4c_\gamma^2 s_\delta^2 \left( B_{22}(k^2, M_{K^0}, M_W) - B_{22}(k^2, M_{K^0}, M_{H^0}) \right) + s_\gamma^2 c_\delta^2 \left( B_{22}(k^2, M_{K^0}, M_W) - B_{22}(k^2, M_{H^0}, M_W) \right) + s_\delta^2 \left( B_{22}(k^2, M_Z, M_{H^0}) - B_{22}(k^2, M_Z, M_{H^0}) \right) + B_{22}(k^2, M_{H^0}, M_{H^0}) - B_{22}(k^2, M_{H^0}, M_{H^0}) \right\} \]

\[ +4s_\gamma^2 \left( B_{22}(k^2, M_{H^0}, M_{H^0}) - B_{22}(k^2, M_{K^0}, M_{H^0}) \right) + 4B_{22}(k^2, M_{K^0}, M_{H^0}) \right\} \]
\[ \Delta \Pi_{ZZ}^3(k^2) = -\frac{\alpha}{4\pi s_\theta^2 c_\theta} \left\{ s^2_\theta \left[ B_{22}(k^2, M_{K^0}, M_Z) - B_{22}(k^2, M_{H^0}, M_Z) \right] \\
+ s^2_\theta \left[ 4 c^2_\theta \left( B_{22}(k^2, M_W, M_W) - B_{22}(k^2, M_{H^+}, M_{H^+}) \right) \right] \\
+ s^2_\theta \left( B_{22}(k^2, M_{H^+}, M_{H^+}) - B_{22}(k^2, M_{H^+}, M_W) \right) \\
+ B_{22}(k^2, M_W, M_W) - B_{22}(k^2, M_{H^+}, M_W) \right] \\
+ 2 \left( B_{22}(k^2, M_{H^+}, M_W) - B_{22}(k^2, M_W, M_W) \right) \right\} \\
+ 4 c^4_\theta B_{22}(k^2, M_{H^+}, M_{H^+}) \right\} \\
\Delta \Pi_{\gamma\gamma}^3(k^2) = -\frac{\alpha}{\pi} B_{22}(k^2, M_{H^+}, M_{H^+}) \\
\Delta \Pi_{\gamma Z}^3(k^2) = \frac{\alpha}{2\pi c_\theta s_\theta} \left\{ s^2_\theta \left[ B_{22}(k^2, M_W, M_W) - B_{22}(k^2, M_{H^+}, M_{H^+}) \right] \\
+ 2 c^2_\theta B_{22}(k^2, M_{H^+}, M_{H^+}) \right\}. \quad (57) \]

There are additional contributions which only contribute to \( \Pi_{SM} \), which we list for completeness. From Fig. [13]

\[ \Pi_{WW,SM}^4(k^2) = \frac{\alpha}{4\pi s_\theta^2} (3 - 2\epsilon) \left[ A_0(M_W) + c^2_\theta A_0(M_Z) \right] \]

\[ \Pi_{ZZ,SM}^4(k^2) = \frac{\alpha c^2_\theta}{2\pi s_\theta^2} (3 - 2\epsilon) A_0(M_W) \]

\[ \Pi_{\gamma\gamma,SM}^4(k^2) = \frac{\alpha}{2\pi} (3 - 2\epsilon) A_0(M_W) \]

\[ \Pi_{\gamma Z,SM}^4(k^2) = -\frac{\alpha c_\theta}{2\pi s_\theta} (3 - 2\epsilon) A_0(M_W). \quad (58) \]

From Fig. [14]

\[ \Pi_{WW,SM}^5(k^2) = \frac{\alpha}{4\pi s_\theta^2} \left[ s^2_\theta A_1(k^2, 0, M_W) + c^2_\theta A_1(k^2, M_Z, M_W) \right] \]
FIG. 15: Contribution to gauge boson two-point function from ghosts in loop.

\[ \Pi_{Z\bar{Z},SM}(k^2) = \frac{\alpha_c^2}{4\pi s_\theta} A_1(k^2, M_W, M_W) \]
\[ \Pi_{\gamma\gamma,SM}(k^2) = \frac{\alpha}{4\pi} A_1(k^2, M_W, M_W) \]
\[ \Pi_{\gamma Z,SM}(k^2) = -\frac{\alpha_c^4}{4\pi s_\theta} A_1(k^2, M_W, M_W). \]

(59)

where,
\[ A_1(k^2, m_1, m_2) = -A_0(m_1) - A_0(m_2) - \left( m_1^2 + m_2^2 + 4k^2 \right) B_0(k^2, m_1, m_1) \]
\[ -10B_{22}(k^2, m_1, m_2) + 2 \left( m_1^2 + m_2^2 - \frac{k^2}{3} \right). \]

(60)

From Fig. 15
\[ \Pi_{WW,SM}(k^2) = \frac{\alpha}{2\pi s_\theta} \left[ s_\theta^2 B_{22}(k^2, 0, M_W) + c_\theta^2 B_{22}(k^2, M_Z, M_W) \right] \]
\[ \Pi_{Z\bar{Z},SM}(k^2) = \frac{\alpha c_\theta^2}{2\pi s_\theta} B_{22}(k^2, M_W, M_W) \]
\[ \Pi_{\gamma\gamma,SM}(k^2) = \frac{\alpha}{2\pi} B_{22}(k^2, M_W, M_W) \]
\[ \Pi_{\gamma Z,SM}(k^2) = -\frac{\alpha c_\theta}{2\pi s_\theta} B_{22}(k^2, M_W, M_W). \]

(61)

Appendix 2: Scalar Integrals

The scalar integrals in \( n = 4 - 2\epsilon \) dimensions are defined as,
\[ \frac{i}{16\pi^2} A_0(m) = \int \frac{d^n q}{(2\pi)^n} \frac{1}{q^2 - m^2} \]
The tensor integral is,
\[
\frac{i}{16\pi^2} B_0(k^2, m_1, m_2) = \int \frac{d^n q}{(2\pi)^n} \frac{1}{[q^2 - m_1^2][(q + k)^2 - m_2^2]} .
\]  

(62)

The tensor integral is,
\[
\frac{i}{16\pi^2} \left\{ g^{\mu\nu} B_{22}(k^2, m_1, m_2) + k^\mu k^\nu B_{12}(k^2, m_1, m_2) \right\} = \int \frac{d^n q}{(2\pi)^n} \frac{q^\mu q^\nu}{[q^2 - m_1^2][(q + k)^2 - m_2^2]} .
\]  

(63)

Appendix 3: Scalar Self-Couplings

\[
g_{HHH} = 6 \left( v c_\gamma^3 \lambda_1 + v' s_\gamma^3 \lambda_2 + \frac{c_\gamma s_\gamma}{2} (s_\gamma v + c_\gamma v') \lambda_3 - \frac{s_\gamma c_\gamma^2}{2} \lambda_4 \right)
\]

\[
g_{HHK} = 2 \left( -3 v s_\gamma c_\gamma^2 \lambda_1 + 3 s_\gamma^2 c_\gamma v' \lambda_2 + \frac{\lambda_3}{2} \left[ -v s_\gamma (1 - 3 c_\gamma^2) + v' c_\gamma (1 - 3 s_\gamma^2) \right] - \frac{c_\gamma}{2} (1 - 3 s_\gamma^2) \lambda_4 \right)
\]

\[
g_{HKK} = 2 \left( 3 v \lambda_1 c_\gamma s_\gamma^2 + 3 v' \lambda_2 c_\gamma s_\gamma + \frac{\lambda_3}{2} \left[ v c_\gamma (1 - 3 s_\gamma^2) + v' s_\gamma (1 - 3 c_\gamma^2) \right] - \frac{s_\gamma}{2} (1 - 3 c_\gamma^2) \lambda_4 \right)
\]

\[
g_{KKK} = 6 \left( -\lambda_1 v s_\gamma^3 + \lambda_2 v' c_\gamma^3 + \frac{c_\gamma s_\gamma}{2} (-c_\gamma v + s_\gamma v') \lambda_3 - \frac{c_\gamma c_\gamma^2}{2} \lambda_4 \right)
\]

\[
g_{HG + G^-} = \left( 2 v \lambda_1 c_\gamma c_\delta^2 + 2 v' \lambda_2 s_\gamma s_\delta^2 + \lambda_3 (v c_\gamma s_\delta^2 + v' s_\gamma c_\delta^2) \right)
\]

\[
+ c_\delta \lambda_4 (2 c_\gamma s_\delta + s_\gamma c_\delta) \right)
\]

\[
g_{HH + H^-} = \left( 2 v \lambda_1 c_\gamma s_\delta^2 + 2 v' \lambda_2 s_\gamma c_\delta^2 + \lambda_3 (v c_\gamma s_\delta^2 + v' s_\gamma c_\delta^2) \right)
\]

\[
- s_\delta \lambda_4 (2 c_\gamma s_\delta + s_\gamma c_\delta) \right)
\]

\[
g_{KG + G^-} = \left( -2 v \lambda_1 s_\gamma c_\delta^2 + 2 v' \lambda_2 c_\gamma c_\delta^2 + \lambda_3 (-v s_\gamma s_\delta^2 + v' c_\gamma c_\delta^2) \right)
\]

\[
+ c_\delta \lambda_4 (-2 s_\gamma s_\delta + c_\gamma c_\delta) \right)
\]

\[
g_{KH + H^-} = \left( -2 v \lambda_1 s_\gamma s_\delta^2 + 2 v' \lambda_2 c_\gamma c_\delta^2 + \lambda_3 (-v s_\gamma c_\delta^2 + v' c_\gamma s_\delta^2) \right)
\]

\[
+ s_\delta \lambda_4 (2 s_\gamma c_\delta + c_\gamma s_\delta) \right)
\]

\[
g_{HG_{00}G^0} = \left( \lambda_1 c_\gamma v + \frac{\lambda_3}{2} v' s_\gamma - \frac{\lambda_4}{2} c_\gamma \right)
\]

\[
g_{KG_{00}G^0} = \left( -\lambda_1 s_\gamma v + \frac{\lambda_3}{2} v' c_\gamma + \frac{\lambda_4}{2} s_\gamma \right)
\]  

(64)
[1] J. Erler and P. Langacker (2004), hep-ph/0407097.
[2] J. Erler and P. Langacker (2008), 0807.3023.
[3] M.-C. Chen, S. Dawson, and T. Krupovnickas, Phys. Rev. D74, 035001 (2006), hep-ph/0604102.
[4] M.-C. Chen, S. Dawson, and T. Krupovnickas, Int. J. Mod. Phys. A21, 4045 (2006), hep-ph/0504286.
[5] T. Blank and W. Hollik, Nucl. Phys. B514, 113 (1998), hep-ph/9703392.
[6] B. W. Lynn and E. Nardi, Nucl. Phys. B381, 467 (1992).
[7] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Lett. B513, 232 (2001), hep-ph/0105239.
[8] N. Arkani-Hamed, A. G. Cohen, E. Katz, and A. E. Nelson, JHEP 07, 034 (2002), hep-ph/0206021.
[9] N. Arkani-Hamed et al., JHEP 08, 021 (2002), hep-ph/0206020.
[10] S. Chang and J. G. Wacker, Phys. Rev. D69, 035002 (2004), hep-ph/0303001.
[11] W. Skiba and J. Terning, Phys. Rev. D68, 075001 (2003), hep-ph/0305302.
[12] S. Chang, JHEP 12, 057 (2003), hep-ph/0306034.
[13] M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. 55, 229 (2005), hep-ph/0502182.
[14] M.-C. Chen, Mod. Phys. Lett. A21, 621 (2006), hep-ph/0601126.
[15] G. Passarino, Nucl. Phys. B361, 351 (1991).
[16] G. Passarino, Phys. Lett. B247, 587 (1990).
[17] M. Czakon, J. Gluza, F. Jegerlehner, and M. Zralek, Eur. Phys. J. C13, 275 (2000), hep-ph/9909242.
[18] M. Czakon, M. Zralek, and J. Gluza, Nucl. Phys. B573, 57 (2000), hep-ph/9906356.
[19] P. H. Chankowski, S. Pokorski, and J. Wagner, Eur. Phys. J. C50, 919 (2007), hep-ph/0605302.
[20] P. H. Chankowski and J. Wagner, Phys. Rev. D77, 025033 (2008), 0707.2323.
[21] M. E. Peskin and J. D. Wells, Phys. Rev. D64, 093003 (2001), hep-ph/0101342.
[22] R. S. Chivukula, C. Hoelbling, and N. J. Evans, Phys. Rev. Lett. 85, 511 (2000), hep-ph/0002022.
[23] J. L. Hewett, F. J. Petriello, and T. G. Rizzo, JHEP 10, 062 (2003), hep-ph/0211218.
[24] D. Toussaint, Phys. Rev. D18, 1626 (1978).
[25] G. Senjanovic and A. Sokorac, Phys. Rev. D18, 2708 (1978).
[26] M.-C. Chen and S. Dawson, Phys. Rev. D70, 015003 (2004), hep-ph/0311032.
[27] J. R. Forshaw, D. A. Ross, and B. E. White, JHEP 10, 007 (2001), hep-ph/0107232.
[28] R. Sekhar Chivukula, N. D. Christensen, and E. H. Simmons, Phys. Rev. D77, 035001 (2008), 0712.0546.
[29] T. Appelquist and J. Carazzone, Nucl. Phys. B120, 77 (1977).
[30] J. R. Forshaw, A. Sabio Vera, and B. E. White, JHEP 06, 059 (2003), hep-ph/0302256.
[31] A. Sirlin, Phys. Rev. D22, 971 (1980).
[32] A. Sirlin and W. J. Marciano, Nucl. Phys. B189, 442 (1981).
[33] F. Jegerlehner (1991), lectures given at the Theoretical Advanced Study Institute in Elementary Particle Physics, (TASI), Boulder, Colo., Jun 3-29, 1990.
[34] W. Hollik (1993), prepared for 7th Summer School Jorge Andre Swieca: Particles and Fields,
Sao Paulo, Brazil, 10-23 Jan 1993.

[35] C. Amsler et al. (Particle Data Group), Phys. Lett. B667, 1 (2008).
[36] M. E. Peskin and D. V. Schroeder (1995).
[37] LEP Electroweak Working Group, http://lepewwg.web.cern.ch/LEPEWWG/.