21-cm power spectrum in interacting cubic Galileon model

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Abstract. We study detectability of the deviation in interacting and non-interacting cubic Galileon models from the \( \Lambda \text{CDM} \) model through the 21-cm power spectrum. We show that the interferometric observations like the upcoming SKA1-mid can detect the deviations in both interacting and non-interacting cubic Galileon model from the \( \Lambda \text{CDM} \) model depending on the parameter values.

Keywords. Dark energy—modification of gravity—cubic Galileon—21-cm cosmology—SKA1-mid.

1. Introduction

Recent cosmological observations (Riess et al. 1998; Perlmutter et al. 1999; Ade et al. 2016) unveiled that our present Universe is going through an accelerated expansion phase. But to date, we do not have any proper theoretical explanation for this accelerated expansion. The simple possible explanation could be an exotic matter having negative pressure known as dark energy (Sahni & Starobinsky 2000; Copeland et al. 2006; Linder 2008; Silvestri & Trodden 2009). The cosmological constant (\( \Lambda \)) is the simplest candidate for dark energy. Arguably, it is also the most favored cosmological model observationally. However, it is plagued with the fine-tuning problem (Martin 2012) and cosmic coincidence problem (Zlatev et al. 1999). Apart from these well-known problems, the concordance \( \Lambda \text{CDM} \) model is also in conflict with the string swampland conjectures (Vafa 2005; Andriot 2018; Obied et al. 2018) and the recent local measurement of the present value of the Hubble parameter \( H_0 \) (Riess et al. 2019; Pesce et al. 2020; Wong et al. 2020; Dinda 2022). While the swampland conjecture rules out any stable de Sitter solution in string theory, the local measurement of \( H_0 \) points towards a discrepancy of 5σ between \( H_0 \) constraint from the Planck observation of cosmic microwave background (CMB) with \( \Lambda \text{CDM} \) as the underlying model (Aghanim et al. 2020) and the recent model independent local measurement of \( H_0 \) by Riess et al. (2019).

To look for alternative theories of gravity, one way is to modify gravity at the large cosmological scale in such a way that it becomes repulsive at large scales, which gives rise to the accelerated expansion of the Universe (Clifton et al. 2012; de Rham 2012, 2014; Nojiri et al. 2017). Dvali, Gabadadze and Porrati (DGP) gave a scenario, where a 4D Minkowsky brane is located on an infinitely large extra dimension, and gravity is localized in the 4D Minkowsky brane (Dvali et al. 2000). This model is known as the DGP model, which gives rise to late-time acceleration, but its self-accelerating branch has a ghost (Luty et al. 2003; Nicolis & Rattazzi 2004). When we reduce the 5D DGP theory to 4D, at the decoupling limit, the theory gives rise to a Lagrangian of the form \((\nabla \phi)^2 \Box \phi \) (Luty et al. 2003), where \( \phi \) is the corresponding scalar field; \( \nabla \) and \( \Box \) correspond to the usual gradient and box operators, respectively. This Lagrangian, in the Minkowski background, possesses the Galilean shift symmetry \( \phi \to \phi + b_\mu x^\mu + c \), where \( c \) is a constant, \( b_\mu \) is a vector with constant components and \( x^\mu \) is the usual coordinate vector. This Galilean shift symmetry gives rise to a second-order equation of motion and hence, it is free from ghosts (Luty et al. 2003; Nicolis & Rattazzi 2004; Nicolis et al. 2009). Because of possessing the shift symmetry, the scalar
field is dubbed as the ‘Galileon’ (Nicolis et al. 2009). In the Minkowski background, we can construct two more Lagrangians containing higher derivatives and shift symmetry, which gives rise to a second-order equation of motion. Along with a linear potential term and standard canonical kinetic term, there exist five such terms, which can possess the above-mentioned shift symmetry and give a second-order equation of motion (Nicolis et al. 2009). All these five terms together form the Galileon Lagrangian (Nicolis et al. 2009). In the curved background, the non-linear completion of the Galileon Lagrangian includes non-minimal coupling to keep the equation of motion second order (Deffayet et al. 2009). The non-minimal terms may cause the fifth force effect, which can be evaded locally by implementing the Vainshtein mechanism (Vainshtein 1972).

The Galileon models are the sub-classes of the more general scalar-tensor theory known as Horndeski theory (Horndeski 1974; Kobayashi et al. 2011). Late-time cosmic acceleration is well-studied in the Galileon theory (Chow & Khoury 2009; Silva & Koyama 2009; Ali et al. 2010, 2012; Deffayet et al. 2010; De Felice & Tsujikawa 2010b; De Felice et al. 2010; Gannouji & Sami 2010; Kobayashi 2010; Kobayashi et al. 2010; Mota et al. 2010; de Rham et al. 2011; de Rham & Heisenberg 2011; Hossain & Sen 2012).

Horndeski theories are constrained by the detection of the event of binary neutron star merger GW170817, using both gravitational waves (GW) (Abbott et al. 2017a) and its electromagnetic counterpart (Abbott et al. 2017b, c), which rules out a large class of Horndeski theories that predict the speed of GW propagation different from that of the speed of light (Ezquiaga & Zumalacárregui 2017; Zumalacarregui 2020). The only higher derivative term that survives is \( G(\phi, X) \nabla^2 \phi \), where \( X = -(1/2)(\nabla \phi)^2 \) and \( G(\phi, X) \) is a function of \( \phi \) and \( X \), where \( G(\phi, X) \approx (\nabla \phi)^2 \) is the cubic Galileon term. This cubic term along with the usual kinetic term and a potential term forms the cubic Galileon model. Potentials other than the linear one break the shift symmetry, but the equation of motion is still a second order. These kinds of models are known as light mass Galileon models (Ali et al. 2012; Hossain & Sen 2012). Without the potential term, in the Cubic Galileon models, we cannot have stable late-time acceleration (Gannouji & Sami 2010). This model has been studied extensively in the context of late-time cosmology (Chow & Khoury 2009; Silva & Koyama 2009; Ali et al. 2012; Hossain & Sen 2012; Barreira et al. 2013; Bartolo et al. 2013; Bellini & Jimenez 2013; Hossain 2017; Dinda et al. 2018b; Brahma & Hossain 2019; Zhang et al. 2020).

In literature, there are several dark energy and modified gravity models, but a good dark energy or a modified gravity model should be consistent with different cosmological observations. Hence, it is important to study the cubic Galileon model in the context of cosmological observations. In literature, such efforts involving the Galileon model have been done earlier, for example, in the context of type Ia supernova observations (Brahma & Hossain 2021), cosmic microwave background, baryonic acoustic oscillations observations (Renk et al. 2017), etc. In the same spirit, the 21-cm cosmological observations like using the upcoming square kilometer array (SKA) telescopes will be promising to detect dark energy and modification of gravity models. For this purpose, the post-reionization epoch (redshift <6) is particularly important to constrain the dynamics of dark energy or the dynamics of cosmological geometry in the modified gravity models. In the post-reionization epoch, the Universe is assumed to be highly ionized, but neutral hydrogen atoms (HI) are still present and these HI are the biased tracers of the matter in the Universe. Thus, the HI energy density tracks the matter-energy density in the Universe and the HI power spectrum is related to the matter power spectrum. The power spectrum of the intensity mapping of the large-scale HI distribution (commonly called 21-cm power spectrum) is thus one kind of measurement of the large-scale structure formation (Loeb & Wyithe 2008; Wyithe & Loeb 2008).

This paper aims to study the effect of the cubic Galileon model in the 21-cm power spectrum and to check the detectability of the deviation due to this model from the \( \Lambda \)CDM in the context of the upcoming SKA1-mid telescope specification. The upcoming SKA1-mid telescope is specifically proposed and designed to study the structure formation in the low redshift regions (<3), which is useful to constrain dark energy and modified gravity model parameters (Bacon et al. 2020). According to the updated proposed design of SKA1-mid, it will have a total of 197 antennas. Among these, 64 antennas are from MeerKAT and 133 antennas are originally from SKA1-mid (Bacon et al. 2020). SKA1-mid is proposed to be detecting the redshifted 21-cm line signal at the redshift range from 0.5 to 3, which will have good accuracy (competitive or even with better accuracy than other observations related to galaxy clustering, etc.) to test the dynamics of the dark energy or modified gravity (Braun et al. 2019; Bacon et al. 2020).

This paper is organized as follows: In Section 2, we describe the background dynamics of the Universe in the presence of interacting cubic Galileon field;
in Section 3, we describe the evolution of the matter inhomogeneity and the corresponding matter power spectrum; in Section 4, we describe the 21-cm power spectrum; in Section 5, we show the detectability of interacting cubic Galileon model in the SKA1-mid telescope specifications; finally in Section 6, we present our conclusion.

2. Background evolution

We consider the following action in the Einstein frame with a potential \( V(\phi) \) given as (Ali et al. 2012):

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 \left( 1 + \frac{\alpha}{M^2} \right) \right] - V(\phi) + S_m[\Psi_m; e^{2\beta\phi/M_{\text{pl}}} g_{\mu\nu}],
\]

(1)

where the scalar field is non-minimally coupled with gravity in the Jordan frame with \( \beta \) as the coupling constant and \( e^{2\beta\phi/M_{\text{pl}}} \) is the conformal factor that relates the Jordan and Einstein frame metric tensors. \( g_{\mu\nu} \) is the corresponding metric and \( g \) is its determinant. \( d^4x \) is the usual four-dimensional volume element. In Equation (1), \( M \) is a constant of mass dimension one, \( M_{\text{pl}} = 1/\sqrt{8\pi G} \) is the reduced Planck mass, where \( G \) is the Newtonian gravitational constant. \( \alpha \) is a dimensionless constant and \( S_m \) corresponds to the matter action with \( \Psi_m \) as the matter field. Action (1) can be realized as a sub-class of Horndeski theories (Horndeski 1974; Kobayashi et al. 2011) and one can recover the usual quintessence models on taking \( \alpha \rightarrow 0 \). Throughout the paper, we have considered only the linear potential, because Galileon shift symmetry is preserved only in linear potential. All the equations are written in \( C = 1 \) unit, where \( C \) is the speed of light in a vacuum.

We consider flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric, given by \( ds^2 = -dt^2 + a^2(t) d\vec{r} \cdot d\vec{r} \), where \( t \) is the cosmic time, \( \vec{r} \) is the comoving coordinate vector and \( a \) is the cosmic scale factor. With this metric, the background cosmological equations can be obtained by varying action (1) regarding the metric tensor \( g_{\mu\nu} \) (Ali et al. 2012; Hossain & Sen 2012) given as:

\[
3M_{\text{pl}}^2 H^2 = \dot{\rho}_m + \frac{\dot{\phi}^2}{2} \left( 1 - 6 \frac{\alpha}{M^3} H \dot{\phi} \right) + V(\phi),
\]

(2)

\[
M_{\text{pl}}^2 (2\ddot{H} + 3H^2) = -\frac{\dot{\phi}^2}{2} \left( 1 + 2 \frac{\alpha}{M^3} \dot{\phi} \right) + V(\phi),
\]

(3)

where over-dot is the derivative regarding the cosmic time \( t \), \( H \) is the Hubble parameter and \( \dot{\rho}_m \) is the background matter-energy density.

The background equation of motion for the Galileon field \( \phi \) is given by (Ali et al. 2012):

\[
\ddot{\phi} + 3H \dot{\phi} - 3 \frac{\alpha}{M^3} \dot{\phi} (3H^2 \dot{\phi} + \ddot{H} \dot{\phi} + 2H \ddot{\phi}) + V_{\phi} = - \frac{\beta}{M_{\text{pl}}} \dot{\rho}_m,
\]

(4)

where subscript \( \phi \) is the derivative with respect to the field \( \phi \).

The continuity equations for matter and scalar field are given by:

\[
\dot{\rho}_m + 3H \rho_m = \frac{\beta}{M_{\text{pl}}} \dot{\phi} \dot{\rho}_m,
\]

(5)

\[
\dot{\rho}_\phi + 3H (1 + w_\phi) \rho_\phi = - \frac{\beta}{M_{\text{pl}}} \dot{\phi} \dot{\rho}_m,
\]

(6)

respectively, where \( \dot{\rho}_\phi \) is the background cubic Galileon field energy density and \( w_\phi \) is the equation of the state of the cubic Galileon field given as:

\[
w_\phi = \frac{\dot{\phi}^2}{\dot{\phi}^2 - (1 - 6 \frac{\alpha}{M^3} H \dot{\phi}) + V(\phi)}.
\]

(7)

To study the background evolution, we rewrite the above differential equations as an autonomous system of equations. To do this, we defined the following dimensionless variables (Ali et al. 2012; Hossain & Sen 2012):

\[
x = \frac{\dot{\phi}}{\sqrt{6HM_{\text{pl}}}}, \quad \frac{d\phi}{dN} = \sqrt{6M_{\text{pl}}},
\]

(8)

\[
y = \frac{\sqrt{V}}{\sqrt{3HM_{\text{pl}}}},
\]

(9)

\[
\epsilon = -6 \frac{\alpha}{M^3} H \dot{\phi} = -6 \frac{\alpha}{M^3} H^2 \left( \frac{d\phi}{dN} \right),
\]

(10)

\[
\lambda = -M_{\text{pl}} \frac{V_{\phi} V}{V^2},
\]

(11)

\[
\Gamma = \frac{V_{\phi} V}{V^2},
\]

(12)

where \( N = \ln a \).

Using the above-mentioned dimensionless variables, we can form the following autonomous system (Ali et al. 2012):

\[
\frac{dx}{dN} = x \left( \frac{\ddot{\phi}}{H \dot{\phi}} - \frac{\ddot{H}}{H^2} \right), \quad \frac{dy}{dN} = -y \left( \sqrt{\frac{3}{2} \lambda x + \frac{\dot{H}}{H^2}} \right),
\]

\[
\frac{d\epsilon}{dN} = \epsilon \left( \frac{\dot{\phi}}{H \dot{\phi}} + \frac{\dot{H}}{H^2} \right),
\]

\[
\frac{d\lambda}{dN} = \sqrt{6x \lambda^2 (1 - \Gamma)},
\]

(13)
where we have:
\[
\frac{\dot{H}}{H^2} = \frac{6(1 + \epsilon)(y^2 - 1) - 3x^2(2 + 4\epsilon + \epsilon^2)}{4 + 4\epsilon + x^2\epsilon^2} + \frac{\sqrt{6}\epsilon(\delta_\lambda - \beta\Omega_m)}{4 + 4\epsilon + x^2\epsilon^2}, \quad (14)
\]
\[
\frac{\ddot{\phi}}{H\dot{\phi}} = \frac{3x^3\epsilon - 3x(4 + \epsilon(1 + y^2))}{x(4 + 4\epsilon + x^2\epsilon^2)} + \frac{2\sqrt{6}(\delta_\lambda - \beta\Omega_m)}{x(4 + 4\epsilon + x^2\epsilon^2)}, \quad (15)
\]
where \(\Omega_m\) is the matter-energy density parameter. From Equation (2), we have the constraint equation \(\Omega_1 + \Omega_\phi = 1\), where \(\Omega_\phi\) is the cubic Galileon energy density parameter given as:
\[
\Omega_\phi = x^2(\epsilon + 1) + \delta^2. \quad (16)
\]

With the dimensionless quantities, the equation of the state of the cubic Galileon field can be rewritten as:
\[
w_\phi = \frac{x(3x(\epsilon(\epsilon + 8) + 4) - 2\sqrt{6}\beta x(2(\epsilon + 1) - 1))}{3(\epsilon(x^2\epsilon + 4) + 4)(x^2(\epsilon + 1) + y^2)} - \frac{2y^2(\epsilon(\sqrt{6}x(\beta + \lambda) + 6) + 6)}{3(\epsilon(x^2\epsilon + 4) + 4)(x^2(\epsilon + 1) + y^2)}. \quad (17)
\]

The effective equation of state, \(w_{\text{eff}}\) is given as:
\[
w_{\text{eff}} = \frac{w_{\phi}}{\Omega_\phi}. \quad (18)
\]

To solve the system of differential equations in Equation (13), we need initial conditions for \(x, y, \epsilon\) and \(\lambda\). We denote these by \(x_i, y_i, \epsilon_i\) and \(\lambda_i\), respectively. Throughout this paper, the subscript \(i\) to a quantity corresponds to the value of that quantity at the initial time. In literature, scalar field models are mainly two types, depending on the attractor or thawing behavior. Here, we consider the thawing type of behavior for the cubic Galileon field and accordingly choose the initial conditions. In the thawing kind of behavior, the equation of state of the scalar field is very close to \(-1\) at initial times, and at late times it thaws away from \(-1\). From Equation (17), we can see that \(w_{\phi} \approx -1\) when \(x \ll y\). We require this behavior at initial times hence, we fix \(x_i = 10^{-3} y_i\) throughout. We fix the initial conditions at redshift, \(z_i = 1000\).

Next, we fix the \(y_i\) parameter, for which \(\Omega_m0 = 0.3111\), where \(\Omega_m0\) is the present value of the matter-energy density parameter. This value corresponds to the Planck 2018 results (Aghanim et al. 2020).

So, we are left with three parameters, \(\epsilon_i, \lambda_i\) and \(\beta\) parameters. \(\epsilon_i\) represents the deviation from the quintessence model, \(\lambda_i\) represents the initial slope of the potential and \(\beta\) represents the interaction between matter and the Galileon field. We keep these three parameters free in our analysis.

With the above-mentioned initial conditions, we showed the background evolutions in the interacting cubic Galileon model, by plotting the equation of state and the Hubble parameter in Figure 1. The solid lines are for non-interacting cubic Galileon field, where \(\beta = 0\). Among these, the black and blue lines are for \(\epsilon_i = 0\) and \(\epsilon_i = 10\), respectively, with \(\lambda_i = 0.7\). Red and green lines are for \(\epsilon_i = 0\) and \(\epsilon_i = 10\), respectively, with \(\lambda_i = 0.9\). The dotted and dashed lines are for interacting cubic Galileon field. The dotted-blue and dashed-blue lines are for \(\beta = -0.1\) and \(\beta = -0.2\), respectively, for \(\epsilon_i = 10\) and \(\lambda_i = 0.7\). To plot the Hubble parameter, we have fixed \(h = 0.6766\), where \(h\) is defined as \(H_0 = 100h\ \text{km s}^{-1} \text{ Mpc}^{-1}\). This value corresponds to the Planck 2018 results (Aghanim et al. 2020).

From the left panel, we see that higher the \(\epsilon_i\) value, the lesser the deviation from \(\Lambda\)CDM. Higher the \(\lambda_i\) value, larger the deviation from \(\Lambda\)CDM. If the \(\beta\) value, is more negative larger the deviation from the \(\Lambda\)CDM. We see the same behavior in the deviation of Hubble parameter in the right panel.

3. Evolution of perturbations

In this work, we are functioning in the sub-Hubble limit. In this limit, we can use the quasi-static approximation i.e., we can consider Newtonian perturbation. With the Newtonian perturbation and the linear theory, the evolution equation of the growth function, \(D_+\) for the cubic Galileon model is given by (Bartolo et al. 2013; Hossain 2017):
\[
\frac{d^2 D_+}{dN^2} + \left[ -\frac{1}{2} 3(w_{\text{eff}} + 1) + \sqrt{6}\beta x + 2 \right] \frac{d D_+}{dN} - \frac{3}{2} \Omega_m G_{\text{eff}} \frac{G}{G} D_+ = 0. \quad (19)
\]

The growth function is defined as \(\delta_m(z) = D_+(z)\delta^i_m\), where \(\delta_m\) is the matter inhomogeneity and \(\delta^i_m\) is its initial value. In the interacting cubic Galileon model, the effective gravitational constant, \(G_{\text{eff}}\) is given as (Ali et al. 2012):
\[
\frac{G_{\text{eff}}}{G} = \frac{x(3x(\epsilon(\epsilon + 8) + 4) - 2\sqrt{6}\beta x(2(\epsilon + 1) - 1))}{3(\epsilon(x^2\epsilon + 4) + 4)} - \frac{2y^2(\epsilon(\sqrt{6}x(\beta + \lambda) + 6) + 6)}{3(\epsilon(x^2\epsilon + 4) + 4)}. \quad (20)
\]

To solve the differential equation of the growth function, we consider the fact that at the initial time, at
mature-dominated epoch (here at redshift, \(z_i = 1000\)), \(D_+ \propto a\). This corresponds to
\[ D_{+\mid i} = \frac{1}{1 + 1000} = \left. \frac{dD_+}{dN} \right|_{i}. \]

Once we have the solution for the growth function, we can compute the linear matter power spectrum, \(P_m\) given by:
\[ P_m(k, z) = A k^{n_s} T^2(k) \frac{D_+^2(z)}{D_+^2(z = 0)}, \]
where \(k\) corresponds to the amplitude of the wave vector, \(n_s\) is the scalar spectral index for the primordial power spectrum, \(T(k)\) is the transfer function and \(A\) is the normalization constant corresponds to the usual \(\sigma_8\) normalization. Here, we consider the Eisenstein–Hu transfer function (Eisenstein & Hu 1998). For the \(\sigma_8\) normalization, we fixed \(\Omega_{m0} = 0.3111\) and \(h = 0.6766\), which are same for the background evolution. For other parameters, we fixed \(\Omega_{b0} = 0.049\), \(n_s = 0.9665\) and \(\sigma_8 = 0.8102\), where \(\Omega_{b0}\) is the present value of the baryonic matter energy density parameter. All these values are best-fit values according to the Planck 2018 results (Aghanim et al. 2020).

4. 21-cm power spectrum

The matter distribution is not directly related to the observation. However, in this regard, 2-point correlation in excess brightness temperature corresponding to the neutral hydrogen (HI) field is useful. This is usually referred as the 21-cm power spectrum. So, we study the detectability of the cubic Galileon model over the \(\Lambda\)CDM model using the 21-cm power spectrum. The 21-cm power spectrum, \(P_{21}\) of the excess brightness temperature corresponding to the neutral hydrogen (HI) field is given by (Furlanetto et al. 2004; Bharadwaj & Ali 2005; Ali et al. 2006; Datta et al. 2007; Wyithe & Loeb 2009; Sarkar et al. 2011; Hussain et al. 2016; Liu & Parsons 2016):
\[ P_{21}(k, z, \mu) = C_T^2(1 + \beta_T \mu^2) P_m(k, z), \]
where \(\mu = \hat{n} \cdot \hat{k} = \cos \theta\), where \(\hat{n}\) is the line of sight (LOS) unit direction and \(\theta\) is the angle between LOS and the wave vector. \(\beta_T\) is defined as:
\[ \beta_T = \frac{f}{b}, \]
where \(f\) is the growth factor and it is defined as:
\[ f = \frac{d \ln D_+}{d \ln a}. \]

\(b\) is the linear bias that connects the HI distribution to the matter distribution. Throughout this paper, we consider the linear bias i.e., \(b = 1\). \(C_T\) is the mean HI excess brightness temperature given by (Datta et al. 2007; Sarkar et al. 2011):
\[ C_T(z) = b \bar{x}_{\text{HI}} \bar{T}(z), \]
where \(\bar{x}_{\text{HI}}\) is the neutral hydrogen fraction. \(\bar{T}\) is given by (Datta et al. 2007; Sarkar et al. 2011):
\[ \bar{T}(z) = 4.0 \text{ mK}(1 + z)^2 \left( \frac{\Omega_{b0} h^2}{0.02} \right) \left( \frac{0.7}{h} \right) \frac{H_0}{H(z)}. \]

The angle averaged (equivalently \(\mu\) averaged) 21-cm power spectrum is given as:
\[ P_{21}(k, z) = \int_0^1 d\mu P_{21}(k, z, \mu). \]
Figure 2. Percentage deviation in the angle averaged 21-cm power spectrum for the cubic Galileon model from $\Lambda$CDM model. Color combinations are same as in Figure 1. Top-left, top-right and bottom panels correspond to redshifts 0.5, 1.5 and 2.5, respectively.

Note that we keep the same notation, $P_{21}$ for the $\mu$ averaged 21-cm power spectrum. We show the deviations in $P_{21}(k, z)$ in Figure 2 for the cubic Galileon model from $\Lambda$CDM with the same combinations of parameter values as in Figure 1. Top-left, top-right and bottom panels correspond to redshifts 0.5, 1.5 and 2.5, respectively. We see that the deviations are up to 2–20% depending on the parameters.

In radio interferometric observations like in SKA observations, the observables are not measured in $k$, but rather in the baseline distribution, $U$. The conversion from $k$ to $U$ is fiducial cosmological model-dependent. If this fiducial model is different from the actual model, we need to consider the correction to the 21-cm power spectrum, defined in Equation (22). Throughout this paper, we consider $\Lambda$CDM as the fiducial model. Hence, for the cubic Galileon model, the observed 21-cm power spectrum would be (Ballinger et al. 1996; Sarkar et al. 2011; Bull et al. 2015; Raccanelli et al. 2015; Hussain et al. 2016):

$$P_{3D}^{21}(k, z, \mu) = \frac{1}{\alpha_{||}\alpha_{\perp}^2} C_T^2 \left[1 + \beta_T \frac{\mu^2 / F^2}{1 + (F^{-2}-1)\mu^2}\right]^2 \times P_m \left(\frac{k}{\alpha_{\perp} \sqrt{1 + (F^{-2}-1)\mu^2}}_{\perp}, z, \mu\right).$$  \hspace{1cm} (27)

where $\alpha_{||} = H_{\text{fid}} / H$, $\alpha_{\perp} = r_v / r_v^{\text{fid}}$ and $F = \alpha_{||} / \alpha_{\perp}$. The subscript ‘fid’ and superscript ‘fd’ correspond to the fiducial model. The above corrected 21-cm power spectrum is sometimes referred as the 3D 21-cm power spectrum and hence, we denote this by placing a superscript ‘3D’. Now, we compute the angle averaged (or $\mu$ averaged) 21-cm power spectrum as:

$$P_{21}^{3D}(k, z) = \int_0^1 d\mu P_{21}^{3D}(k, z, \mu).$$  \hspace{1cm} (28)

Note that, we use the same notation for angle averaged 3D 21-cm power spectrum.

In Figure 3, we show the deviation in the angle averaged observed (3D) 21-cm power spectrum for the cubic Galileon model from the $\Lambda$CDM model for the same parameter values as in Figure 1. Color combinations are same as in Figure 1. Top-left, top-right and bottom panels correspond to redshifts 0.5, 1.5 and 2.5, respectively. In the 3D 21-cm power, the deviations are similar as in Figure 2. Another important thing to notice here is that there is $k$ dependence in the deviations in the 3D power spectrum unlike in the standard 21-cm power spectrum. This is because the fiducial model and the actual model are different here.

5. Detectability with SKA1-mid telescope

Here, we are considering the detectability of the cubic Galileon model from the $\Lambda$CDM model with the SKA1-mid telescope. To achieve this, we need to consider the observational errors that arise in the observed 21-cm power spectrum. We only consider two types of errors here. One is the system noise and another one is the sample variance. The system noise arises from the observational instruments. The cosmic variance arises from the finite sampling of modes. We ignore other errors like astrophysical residual foregrounds, effects from the ionosphere, etc. So, we consider an ideal instrumental detection where these errors are completely removed and the only errors present are the system noise and the sample variance. We closely follow the approach of Villaescusa-Navarro et al. (2014) to estimate the noise.

We assume a circularly symmetric antenna distribution ($\rho_{\text{ant}}$), which is a function of $l$ only, where $l$ is the distance from the center of the antennae distributions. For the SKA1-mid telescope antenna distributions, we
follow the document https://astronomers.skatelescope.org/wp-content/uploads/2016/09/SKA-TEL-INSA-00-00537-SKA1_Mid_Physical_Configuration_Coordi
mates_Rev_2-signed.pdf. The SKA1-mid telescope specifications have 133 SKA antennas with 64 MEERKAT antennas. So, the total number of antennas is 197. Then, the 2D baseline distribution is given by (Villaescusa-Navarro et al. 2014):

$$\rho^{2D}(U, \nu_{21}) = B(\nu_{21}) \int_0^\pi \int_0^{2\pi} 2\pi l d\rho_{\text{ant}}(l)$$

$$\times \int_0^{2\pi} d\phi \rho_{\text{2D}}(|\vec{l} - \lambda_{21} \vec{U}|),$$

(29)

where $\vec{U}$ is the baseline vector given by $\vec{U} = (k \perp r_v) / (2\pi)$. $k \perp$ is the component of the wavevector, $k$ at the transverse direction. $r_v$ is the co-moving distance. $|\vec{l} - \lambda_{21} \vec{U}|$ is given as:

$$|\vec{l} - \lambda_{21} \vec{U}| = \sqrt{l^2 + \lambda_{21}^2 U^2 - 2\lambda_{21} l U \cos \phi},$$

where $\phi$ is the angle between $\vec{l}$ and $\vec{U}$. $l$ and $U$ are the magnitudes of the vectors, $\vec{l}$ and $\vec{U}$, respectively, $\nu_{21}$ and $\lambda_{21}$ are the observed frequency and the wavelength of the 21-cm signal, respectively. $B(\nu_{21})$ is computed by the normalization given by:

$$\int_0^\infty UdU \int_0^\pi d\phi \rho_{\text{2D}}(U, \nu_{21}) = 1.$$  

(30)

The 3D baseline distribution ($\rho^{3D}(k, \nu_{21})$) is defined as:

$$\rho^{3D}(k, \nu_{21}) = \left[ \int_0^{2\pi} d\mu \rho_{\text{2D}}^2 \left( \frac{r_v k}{2\pi} \sqrt{1 - \mu^2}, \nu_{21} \right) \right]^{\frac{1}{2}},$$

(31)

where $\mu = \cos \theta$, with $\theta$ being the angle between direction of $k$ and the line of sight direction.

The system noise in the 21-cm power spectrum is given by (McQuinn et al. 2006; Geil & Gaensler 2010; Villaescusa-Navarro et al. 2014; Sarkar & Datta 2015; Dinda et al. 2018a; Hotinli et al. 2021):

$$\delta P_N(k, \nu_{21}) = \frac{T_{\text{sys}}^2}{B_{l0}} \left( \frac{\lambda_{21}^2}{A_e} \right)^2$$

$$\times \frac{2r_v^2 L}{N_t(N_t - 1)\rho_{3D}(k, \nu_{21}) \sqrt{N_k(k)}},$$

(32)

where $N_t$ is the total number of antennas (here 197), $B$ is the bandwidth corresponding to the observed signal, $L$ is the corresponding co-moving length and $t_0$ is the observation time. Here, we are considering $t_0$ to be 1000 h. $A_e$ is the effective collecting area of an individual antenna. This is related to the physical collecting area, $A$ of an antenna given by $A_e = eA$, where $e$ is the efficiency of an antenna. We consider $e = 0.7$ and $A \approx 1256.6 \text{ m}^2$ for an SKA1-mid antenna. $T_{\text{sys}}$ is the system temperature. We are considering a typical value for $T_{\text{sys}}$ given as 40 K. $N_k$ is the total number of independent modes in the range between $k$ to $k + dk$ given by:

$$N_k(k) = \frac{2\pi k^2 dk}{V_{1\text{-mode}}},$$

where $V_{1\text{-mode}}$ is the volume occupied by a single independent mode given by:

$$V_{1\text{-mode}} = \frac{(2\pi)^3 A}{r_v^2 L \lambda_{21}^2}. $$

The sample variance in the 21-cm power spectrum is given by (McQuinn et al. 2006; Geil & Gaensler 2010; Villaescusa-Navarro et al. 2014; Sarkar & Datta 2015; Dinda et al. 2018a; Hotinli et al. 2021):

$$\delta P_{\text{SV}}(k, \nu_{21}) = \left[ \sum_{\theta} \frac{N_m(k, \theta)}{P_{21}(k, \theta)} \right]^{-\frac{1}{2}},$$

(33)
Figure 4. Detectability of interacting cubic Galileon model from the ΛCDM model. Here, we have plotted $\delta P/P$, where $P = P_{\text{fiducial}} = P_{21}(\Lambda CDM)$ and $\delta P = |P_{21}^{\text{3D}} - P_{21}(\Lambda CDM)|$. Color combinations are same as in Figure 1. Top-left, top-right and bottom panels correspond to redshifts 0.5, 1.5 and 2.5, respectively. The orange lines correspond to the total error (including sample variance and system noise). The top-dashed, middle-solid and bottom-dotted orange lines correspond to redshifts 0.5, 1.5 and 2.5, respectively. The orange lines correspond to the total power spectrum. The parameter values corresponding to the lower limit of Planck 2018 TT, TE, EE+lowE+lensing+BAO (Aghanim et al. 2020).

where $N_m$ is the total number of independent modes in the range of $k-k + dk$ and $\theta - \theta + d\theta$ given as:

$$N_m(k, \theta) = \frac{2\pi k^2 dk \sin \theta d\theta}{V_{\text{mode}}}.$$  

The total noise is given by:

$$\delta P_{\text{tot}}(k, \nu_{21}) = \sqrt{\delta P^2_N(k, \nu_{21}) + \delta P^2_{\text{SV}}(k, \nu_{21})}.  \quad (34)$$

In Figure 4, we have shown that the cubic Galileon model can be distinguished from the ΛCDM model with the 21-cm power spectrum. The parameter values and the color codes are the same as in Figure 1. The orange line is the total error in the 21-cm power spectrum with the SKA1-mid telescope specifications. The lines above this orange line are detectable by the SKA1-mid telescope. The top-dashed orange line corresponds to the fiducial model with $\Omega_{m0} = 0.3111 + 0.0056$, $h = 0.6766 + 0.0042$, $\Omega_{b0} = 0.049 + 0.004$, $n_s = 0.9665 + 0.0038$ and $\sigma_8 = 0.8102 + 0.006$ corresponding to the upper limit of Planck 2018 TT, TE, EE+lowE+lensing+BAO data (Aghanim et al. 2020). The middle-solid orange line corresponds to the fiducial model with $\Omega_{m0} = 0.3111$, $h = 0.6766$, $\Omega_{b0} = 0.049$, $n_s = 0.9665$ and $\sigma_8 = 0.8102$ corresponding to the best fit of Planck 2018 TT, TE, EE+lowE+lensing+BAO data (Aghanim et al. 2020). The bottom-dotted orange line corresponds to the fiducial model with $\Omega_{m0} = 0.3111-0.0056$, $h = 0.6766-0.0042$, $\Omega_{b0} = 0.049-0.004$, $n_s = 0.9665-0.0038$ and $\sigma_8 = 0.8102-0.006$ corresponding to the lower limit of Planck 2018 TT, TE, EE+lowE+lensing+BAO data (Aghanim et al. 2020).

6. Conclusion

In this work, we considered the cubic Galileon model, which is a special case of Horndeski gravity. Because of the Galileon shift symmetry, this model is ghost-free. We considered the thawing kind of behavior in this model using proper initial conditions. We considered both interacting and non-interacting cubic Galileon models. Interaction means the reaction between total matter and Galileon field. The potential is linear in this model, which naturally arises in this kind of model. We showed the deviations of this model from the ΛCDM, both through the background and the perturbative quantities. Our main focus here is to detect this model through the 21-cm observations. For this purpose, we considered interferometric observations like SKA1-mid observations. We considered system noise and the sample variance according to this observation and considering ΛCDM as the fiducial model. We ignored other errors in the 21-cm power spectrum by the assumption that we can completely remove them for an ideal observation. With this error, we showed the possibility of the detection of the cubic Galileon model from the ΛCDM model.

Note that, in this analysis, we have considered the ΛCDM model as the fiducial model for the SKA1-mid telescope specifications to study the detectability of the cubic Galileon model. We found that, for some particular values of the model parameters, there is a prospect to detect the deviation of the cubic Galileon model from the ΛCDM model. However, we should mention the consideration of choosing the ΛCDM model as the fiducial, could lead to a possible bias towards the ΛCDM model. To investigate thoroughly, the accurate estimation of the cubic Galileon model with SKA1-MID telescope specification or any other cosmological observations, one needs to do a thorough parameter estimation analysis, which we aim to do in a future study.

However, without doing the detailed parameter estimation with Monte Carlo Markov Chain (MCMC) or
any other methods, we can comment on the prospect of the detectability of the cubic Galileon model from the theoretical point of view as follows. From Figure 1, we can see that the $\epsilon_i = 0$ corresponds to the quintessence model, and as we increase the value of $\epsilon_i$, we get closer to $\Lambda$CDM. This fact would be reflected in the constraints on the cubic Galileon model and the constraints would be such that if most of the cosmological observations suggest a $\Lambda$CDM model as a best fit, the parameter space of larger $\epsilon_i$ would be less constrained and the parameter space of lower $\epsilon_i$ would be tightly constrained. Regarding Hubble and $S_8$ tensions, consideration of the cubic Galileon model, probably, would not lower any tension significantly. One of the main reasons is that the equation of state of the dark energy in the cubic Galileon model is always non-phantom similar to the quintessence and such behavior of dark energy neither solves nor minimizes Hubble and $S_8$ tensions.

In summary, our results show that with forthcoming SKA observations, we can put strong constraints on the modified gravity models like the interacting cubic Galileon model. Also with SKA observation at higher redshifts, there is a bright possibility to distinguish such kind of Galileon models from $\Lambda$CDM, which is surely encouraging.

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