Decoherence and Recoherence in Model Quantum Systems

Jen-Tsung Hsiang

Department of Physics,
National Dong Hwa University
Hualien, Taiwan, R.O.C.

L. H. Ford

Institute of Cosmology, Department of Physics and Astronomy
Tufts University, Medford, MA 02155 USA

Abstract

We discuss the various manifestations of quantum decoherence in the forms of dephasing, entanglement with the environment, and revelation of “which-path” information. As a specific example, we consider an electron interference experiment. The coupling of the coherent electrons to the quantized electromagnetic field illustrates all of these versions of decoherence. This decoherence has two equivalent interpretations, in terms of photon emission or in terms of Aharonov-Bohm phase fluctuations. We consider the case when the coherent electrons are coupled to photons in a squeezed vacuum state. The time-averaged result is increased decoherence. However, if only electrons which are emitted during selected periods are counted, the decoherence can be suppressed below the level for the photon vacuum. This is the phenomenon of recoherence. This effect is closely related to the quantum violations of the weak energy condition, and is restricted by similar inequalities. We give some estimates of the magnitude of the recoherence effect and discuss prospects for observing it in an electron interferometry experiment.
I. INTRODUCTION

Decoherence is a nearly universal effect in quantum systems, and one which plays a central role in the quantum to classical transition. However, the word “decoherence” is often used to denote several related, but distinct concepts. The first is dephasing, the loss of definite phase relations between different components in a superposition state. These phase relationships are essential for quantum interference, which can be regarded as the key phenomenon which distinguishes the quantum world from the classical world. In general, interaction of a quantum system with its environment can lead to dephasing. The second, closely related concept, is that of entanglement between the quantum system and its environment. Entanglement by itself need not lead to decoherence, but often the variables of the environment are too numerous or complex to be readily measured. It is when one gives up on any attempt to keep track of the environmental degrees of freedom that decoherence arises. The third concept associated with decoherence is the revelation of “which path” information. Any measurement in a double slit interference experiment which reveals the path taken by the particles will destroy the interference pattern.

Here we will consider a specific example of a quantum system, coherent electrons coupled to the quantized electromagnetic field. An electron interference experiment, such as that illustrated in Fig. 1 deals with one of the simplest possible quantum systems, but also one in which many beautiful experiments have been performed in recent years[1, 2, 3, 4, 5]. The coupling to the quantized electromagnetic field allows for photon emission by the electrons, leading to decoherence. This example illustrates all three of the versions of decoherence discussed above. As will be shown in Sect. II the coupling to the quantized electromagnetic field produces dephasing, or a loss of contrast in the interference pattern as a result of Aharonov-Bohm phase fluctuations. The emission of photons leads to entanglement between the quantum state of the electrons and that of the photons. Finally, photon emission is capable of revealing “which path” information. If the wavelength of an emitted photon is less than the separation of the two electron paths in Fig. 1 then detection of that photon reveals the path taken by a given electron. In the cleanest version of the electron interference experiment, the flux of electrons may be made so low that only one electron is in the interferometer at any one time.

II. AHARONOV-BOHM PHASE FLUCTUATIONS

Consider a double slit interference experiment in which coherent electrons can take either one of two paths, as illustrated in Fig. 1. First consider the case of no field fluctuations. If the amplitudes for the electrons to take path $C_1$ and $C_2$ are $\psi_1$ and $\psi_2$, respectively, to point $P$, then the mean number of electrons at $P$ will be proportional to

$$n(P) = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + 2\text{Re}(\psi_1 \psi_2^*) .$$  \hspace{1cm} (1)

In the presence of a classical, non-fluctuating electromagnetic field described by vector potential $A^\mu$, there will be an Aharonov-Bohm phase shift of the form[1],

$$\varphi_{AB} = e \int_C dx^\mu A_\mu ,$$  \hspace{1cm} (2)
FIG. 1: An electron interference experiment in which the electrons may take either one of two paths, $C_1$ or $C_2$, from the source to the point $P$ where the interference pattern is formed. The emission of photons by the electrons tends to cause decoherence. The detection of an emitted photon with wavelength smaller than the path separation can reveal which path a particular electron takes, and hence causes decoherence.

where the integral is taken around the closed path $C = C_1 - C_2$. Here $e$ is the electron’s charge. We will use Lorentz-Heaviside units with $\hbar = c = 1$. The phase shift alters the locations of the interference minima and maxima, but does not alter their relative amplitudes, the contrast.

If the electromagnetic field undergoes fluctuations, then the situation is different. In the case of Gaussian fluctuations, the fluctuating Aharonov-Bohm phase causes a change in the contrast by a factor of

$$\Gamma = e^W,$$

where we define the coherence functional by

$$W = -\frac{1}{2} \langle \varphi_{AB}^2 \rangle$$

with the angular brackets denoting averaging over the fluctuations. This functional can be expressed as

$$W = -2\pi \alpha \oint_C dx_\mu \oint_C dx'_\nu \, D^{\mu\nu}(x, x'),$$

where $\alpha$ is the fine-structure constant and

$$D^{\mu\nu}(x, x') = \frac{1}{2} \left\langle \{A^\mu(x), A^\nu(x')\} \right\rangle.$$

If the initial quantum state of the electromagnetic field is the vacuum, then $D^{\mu\nu}(x, x')$ is the vacuum Hadamard function. Note that the Hadamard function itself is gauge dependent, but that $W$ is gauge invariant.
In general, $W < 0$, leading to a reduction in contrast or dephasing. At the same time, the coherence functional reflects the effects of photon emission. If the electromagnetic field is initially in its vacuum state, $|0\rangle$, then after an electron traverses path $C_j$, it will be in the state
\[ e^{ie\int_{C_j} dx^\mu A_\mu} |0\rangle, \]
which is a superposition of photon number eigenstates. Thus the descriptions in terms of a fluctuating Aharonov-Bohm phase or in terms of photon emission are complementary viewpoints.

Various aspects of decoherence associated with fluctuating or time-dependent electromagnetic fields have been discussed by numerous authors in recent years \cite{7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23}. The connection between Aharonov-Bohm phase fluctuations and decoherence was discussed in by Stern \textit{et al} \cite{7}. The effects of boundaries, such as a plane mirror, were considered by Ford \cite{9, 11}. Some of these estimates for the magnitude of the effects were criticized by Mazzitelli \textit{et al} \cite{13} as being too large. The discrepancy is likely due to the sharp corners in the electron paths used \cite{8, 11}; when smooth paths are employed \cite{13}, the effects are much smaller. This can be understood because sharp boundaries cause large amounts of photon emission, which can in turn be modified by a perfectly reflecting boundary. A different effect was discussed by Anglin \textit{et al} \cite{12} and Machnikowski \cite{20}, who considered the effect of an imperfect conductor. Here the motion of electrons over the conductor can cause excitations inside the metal, resulting in decoherence. This type of decoherence was recently observed by Sonnentag and Hasselbach \cite{5}.

The initial quantum state of the electromagnetic field need not be the vacuum. The case of a thermal state, and the resulting increased decoherence, was discussed by Breuer and Petruccione \cite{14}. Another possibility is a time-dependent classical electromagnetic field \cite{16, 17}. Here one really has Aharonov-Bohm phase variations rather than fluctuations. However in most experiments, one would average over the emission time of the electrons and effectively lose the information carried by this time-dependent phase, leading to decoherence.

\section{Squeezed States and Recohherence}

Now we consider the situation when the initial state of the photon field is not the vacuum, but rather a squeezed state \cite{24, 25}. Such states have the property that they can temporarily suppress the quantum fluctuations in a given variable below the vacuum level. For example, the local energy density in such a state can be negative, although the total energy is always positive. The effects of squeezed states of photons on coherent electrons were recently analyzed by us \cite{23}. In this section, we will give a summary of the results found there.

Consider the special case where the quantized electromagnetic field is in a state in which one mode is excited to a squeezed vacuum state, and all other modes remain in the ground state. We take the excited mode to be a plane wave in a box with periodic boundary conditions, with wave vector $\vec{k}$ and polarization $\lambda$, so the quantum state may be denoted by $|\zeta_{\vec{k}}\rangle$. This is a one complex parameter family of states, labeled by $\zeta_{\vec{k}} = re^{i\theta}$. Take the plane wave to be travelling in the $y$-direction, with linear polarization in the $z$-direction. For electrons emitted at $t = t_0$, we take the paths $C_1$ and $C_2$ to be given by
\[ z(t) = \pm \frac{R}{T_1} [(t - t_0 - T)^2 - T^2]^{1/2}. \]
That is, the electrons start at $z = 0$ at $t = t_0$, reach their maximum separations where $z = \pm R$ at $t = t_0 + T$, and finally return to $z = 0$ at $t = t_0 + 2T$. Here $2T$ and $2R$ can be thought of as the effective flight time and path separation, respectively. Electrons which start from the source at different times will experience different fluctuations.

We are interested in the change in the coherence functional due to the non-trivial state of the photon field, so define a renormalized coherence functional $W_R = W - W_0$, where $W_0$ is the functional in the Minkowski vacuum state. In our case, one finds that

$$W_R = -\frac{8\pi\alpha}{V \bar{\omega}} M g(r, t_0),$$

(9)

where $\alpha$ is the fine structure constant, $V$ is the normalization volume, and $\bar{\omega}$ is the frequency of the excited photon mode. In addition,

$$M = \left(\frac{16R}{\omega^4 T^4}\right)^2 \left[(-3 + \omega^2 T^2) \sin \omega T + 3 \omega T \cos \omega T \right]^2,$$

(10)

which does not depend on the electron emission time $t_0$ and is always positive definite. Thus the sign of $W_R$ is solely determined by the quantity

$$g(r, t_0) = \eta [\mu \cos (\alpha + \beta t_0) + \eta],$$

(11)

where $\mu = \cosh r$, $\eta = \sinh r$, and

$$\alpha = \omega T - \theta, \quad \beta = 2\bar{\omega}.$$

(12)

The behavior of $g(r, t_0)$ as a function of $t_0$ for fixed $r$ is illustrated in the left part of Fig. 2. The key feature is that $g(r, t_0) < 0$ for $t_i < t_0 < t_f$. This means that for electrons emitted during this interval, we have $W_R > 0$, an increase of contrast compared to the photon vacuum state. This is the phenomenon of recoherence. The minimum value of $g$ in the interval $t_i < t_0 < t_f$ is $g_m(r)$, plotted in the right part of Fig. 2 for which we see that $g(r, t_0) > -\frac{1}{3}$.

Let $\tilde{g}(r)$ and $\tilde{W}_R$ denote the averages of $g(r, t_0)$ and of $W_R$, respectively, over the interval $t_i < t_0 < t_f$. The dependence of $\tilde{g}(r)$ upon $r$ is illustrated in Fig. 3 where we see that $\tilde{g}(r) > -\frac{1}{3}$. This bound limits the degree of recoherence, and is analogous to the quantum inequalities which limit the magnitude and duration of negative energy densities in quantum field theory [26, 27, 28, 29]. Marecki [30, 31] has recently derived variants of the quantum inequalities for limiting the amount of squeezing which might be observed in photodetection experiments in quantum optics. The bounds on $\tilde{g}(r)$ and on $\tilde{W}_R$ are sufficient to ensure that unitarity is always preserved, so that $W < 0$. Thus the recoherence effect can never be greater in magnitude that the decoherence in the vacuum state.

The bound given above may be used to make estimates of the maximum recoherence. If we assume that $\tilde{W}_R$ attains its maximum value, then for a single mode in a box, we find

$$\tilde{W}_R \approx 8 \times 10^{-4} \frac{\lambda^3}{V} \left(\frac{R}{T}\right)^2 \left(\frac{\lambda}{T}\right)^2,$$

(13)

where $\lambda = 2\pi/\bar{\omega}$ is the wavelength of the excited mode. This estimate was derived assuming non-relativistic motion and periodic boundary conditions. However, it should serve as a
FIG. 2: The left figure shows the behavior of $g(r,t)$ defined in Eq. (11), as a function of the emissions time $t_0$. The right figure shows how the minimum value of $g$ as a function of $t_0$, $g_m(r)$, depends on $r$.

FIG. 3: The left figure shows how $\bar{g}(r)$, the average of $g(r,t)$ over the interval when $g(r,t) < 0$, as a function of $r$. The right figure illustrates the width of this interval, $\Delta t$, also as a function of $r$.

rough estimate for more realistic cavities. For a rough estimate, let us take $V \approx \lambda^3$ and $R \approx \lambda$, corresponding to the lowest frequency mode in the cavity and a path separation of the order of the cavity size. This leads to

$$\widetilde{W}_R \approx 10^{-3} \left( \frac{R}{T} \right)^4 .$$

(14)

Non-relativistic motion requires $T \gg R$. If, for example, we take $R/T \approx 1/10$, we would get the estimate $\widetilde{W}_R \approx 10^{-7}$.

The case where a finite bandwidth of modes is excited is treated in our previous paper[23]. If $\Delta \omega$ is the bandwidth of excited modes and $\Delta \Omega$ is the solid angle within which they lie, then one finds the estimate

$$\widetilde{W}_R \approx 10^{-2} \left( \frac{R}{T} \right)^2 \frac{\Delta \omega}{\bar{\omega}} \Delta \Omega .$$

(15)
All of the factors in the above expression, $R/T$, $\Delta \omega / \bar{\omega}$, and $\Delta \Omega$, should be small compared to unity for our analysis to be strictly valid. If we take all three of these factors to be of order $10^{-1}$, then we would obtain $\tilde{W}_R \approx 10^{-6}$.

IV. SUMMARY

In this article, we have reviewed selected aspects of decoherence, using coherent electrons coupled to the quantized electromagnetic field as our model quantum system. This system exhibits all three forms of decoherence, dephasing, entanglement, and “which-path” information. The basic effect of the photon field can be described either as photon emission or as Aharonov-Bohm phase fluctuations. If the photon field is not in its vacuum state, then the decoherence is typically larger than in the vacuum. However, we have discussed how this can be reversed for selected quantum states, resulting in recoherence. The phenomenon of recoherence is a sub-vacuum phenomenon, similar to quantum violations of the weak energy condition. Both effects are limited in magnitude by quantum inequalities. Although the effect of recoherence is small, its eventual measurement is a possibility.

Acknowledgments

We would like to thank C.I. Kuo, D.S. Lee, K.W. Ng and C.H. Wu for useful discussions. This work was supported in part by the National Science Foundation under Grant PHY-0555754.

[1] A. Tonomura, et al, Phys. Rev. Lett. 48, 1443 (1982).
[2] A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki and H. Ezawa, Am. J. Phys. 57, 117 (1989).
[3] M. Nicklaus and F. Hasselbach, Phys. Rev. A 48, 152 (1993).
[4] A. Tonomura, Proc. Natl. Acad. Sci. USA 102, 14952 (2005).
[5] P. Sonnentag and F. Hasselbach, Phys. Rev. Lett. 98, 200402 (2007).
[6] Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959).
[7] A. Stern, Y. Aharonov, and Y. Imry, Phys. Rev. A 41, 3436 (1990).
[8] A. Stern, Y. Aharonov, and Y. Imry, in Quantum Coherence, edited by J. Anandan (World Scientific, Singapore, 1991).
[9] L.H. Ford, Phys. Rev. D 47, 5571 (1993).
[10] L.H. Ford, Annals N.Y. Acad. Sci., 755, 741 (1995).
[11] L.H. Ford, Phys. Rev. A 56, 1812 (1997).
[12] J. R. Anglin, J. P. Paz, and W. H. Zurek, Phys. Rev. A 58, 4041 (1997).
[13] A. Voudras and B.C. Sanders, Europhys. Lett. 43, 659 (1998).
[14] H.P. Breuer and F. Petruccione, Phys. Rev. A 63, 032102 (2001).
[15] F. D. Mazzitelli, J. P. Paz, and A. Villanueva, Phys. Rev. A 68, 062106 (2003).
[16] J.-T. Hsiang, Ph.D. Dissertation, Tufts University (2004).
[17] J.-T. Hsiang and L. H. Ford, Phys. Rev. Lett. 92, 250402 (2004).
[18] F.C. Lombardo, F.D. Mazzitelli, and P.I. Villar, Phys. Rev. A 72, 042111 (2005).
[19] J.-T. Hsiang and D.-S. Lee, Phys. Rev. D 73, 065022 (2006).
[20] P. Machnikowski, Phys. Rev. B 73, 155109 (2006).
[21] J.-T. Hsiang, T.-H. Wu, and D.-S. Lee, Phys. Rev. D 77, 105021 (2008).
[22] E. Alvarez and F.D. Mazzitelli, Phys. Rev. A 77, 032113 (2008).
[23] J.-T. Hsiang and L. H. Ford, Phys. Rev. D 78, 065012 (2008).
[24] D. Stoler, Phys. Rev. D 1, 3217 (1970); 4, 1925 (1971).
[25] C.M. Caves, Phys. Rev. D 23, 1693 (1981).
[26] L.H. Ford, Proc. R. Soc. London A364, 227 (1978).
[27] L.H. Ford, Phys. Rev. D 43, 3972 (1991).
[28] L.H. Ford and T.A. Roman, Phys. Rev. D 55, 2082 (1997).
[29] C.J. Fewster and S.P. Eveson, Phys. Rev. D 58, 084010 (1998).
[30] P. Marecki, Phys. Rev. A 66, 053801 (2002).
[31] P. Marecki, Phys. Rev. A 77, 012101 (2008).