Numerical calculation of the stress-strain state of piezoelectric layered polymer composite materials equipped with a control piezoactuator

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Abstract. A mathematical model is formulated for calculating the stress-strain state (SSS) of piezoelectric layered polymer composite materials (PCM) equipped with a control piezoactuator. The model implements the principle of thermo-piezoelectric analogy, which allows to move from the coupled boundary value problem of electroelasticity to the boundary value problem of thermoelasticity. A comparison of the deformation fields for model layered samples from PCM with a different arrangement of piezoactuators was carried out. Verification of the developed mathematical models was carried out. The main patterns and mechanisms are revealed for controlling the geometry of a layered PCM using embedded piezo-actuators.

1. Introduction
There is a recent trend of using SMART structures with controlled geometry in aviation technologies. SMART structures make it possible to significantly improve the aircraft performance by reducing the vibroacoustic loading of highly loaded elements, such as an airplane wing or helicopter blades [1]. The development of SMART structures made of PCM equipped with piezoelements on the one hand will reduce the weight of the structure, and on the other hand will eliminate mechanical drives and skew automatons, which will significantly reduce the cost of maintaining structures with controlled geometry and fuel consumption of an aircraft.

SMART designs can be most efficient while developing new aviation wings with variable geometry, which will allow optimal adaptation to the aerodynamic parameters of the air flow [2]. The application of SMART technologies to damping oscillations of helicopter blades is now being considered as well. The possibility of using SMART-technologies in the creation of fan blades and an air engine rectifier apparatus is also under consideration [3].

There are some scientific works describing the theoretical foundations of creating SMART structures from PCM with controlled geometry [4,5]. Numerical models of the blade cross section have been developed, and to identify its optimal configuration, parametric studies of the relative rigidity of the controlled structure equipped with AFC actuators have been carried out.

In [6], numerical results are presented for a four-bladed, bearingless model of a rotor with self-adjusting piezoelectric actuators located at an angle of 45° on the upper and lower surfaces of the composite blade. To analyze the dynamic loads on the main rotor with composite blades, a complex model was developed. To simulate the behavior of a layered structure from a smart composite, an approach based on the theory of high-order deformation was applied [7].

In [8–10], a series of works was carried out related to the analysis of the structural and aerodynamic parameters of rotor blades in order to better understand the effect of active twisting. The influence of...
the active twisting of the blade, the required power, the load on the blade and the hub were evaluated in relation to changes in stiffness (torsional and flexural in the swing and rotation planes), sectional mass and torsion inertia, center of gravity and location of the elastic axis of the blade.

However, the scientific problem limiting the creation of SMART structures (introduction into the practice of engineering design) is the lack of scientifically based approaches and proven methods for their design. To solve this problem, it is necessary to carry out a complex of simulation and experimental studies aimed at developing new mathematical models of composite materials with piezoelectric structural elements (piezocomposites), and developing methods for solving related boundary problems of electro-magnetic elasticity for inhomogeneous mediums with piezoelectric structure elements. There is a need to develop methods for calculating structures, choosing the control elements, developing concepts, calculation methods, creating methods for designing structures, testing models and methods using prototypes for SMART structures, developing experimental technologies for their manufacture from modern materials, testing these prototypes.

In this study, new mathematical models of composite materials with piezoelectric structural elements (piezo composites) have been developed. In the simulation experiments, the dependences of the effective electromagnetic coupling factors of piezocomposites on the structural geometric and physicomemrical parameters of the composite and the parameters of the electric field (control voltage) are investigated.

2. Numerical model

To carry out a numerical simulation of the SSS of a layered sample of PCM, equipped with piezoelectric actuators, a three-dimensional computer model was developed with an explicit description of the layered structure. The model consists of 8 anisotropic layers and a rectangular piezo actuator. Geometrical characteristics of layers: 400×45×0.21 mm, piezo-actuator: 102×35×0.3 mm. The solution of the problem was carried out by the finite element method (FEM) using the ANSYS Workbench software package.

The sample under study was made of carbon-fiber plastic based on equal-strength woven filler with a reinforcement scheme [0°/90°/45°/-45°/45°] 90°/90°. Technical elastic constants of CFRP and micro fiber piezoactuator (MFC) are taken from [13, 14]:

\[
 \begin{align*}
 G_{xx} &= 63.9 \text{ GPa}, \\
 E_x &= 20 \text{ GPa}, \\
 G_{xy} &= 19.5 \text{ GPa}, \\
 G_{yx} &= G_{zx} = 2.7 \text{ GPa}, \\
 \nu_{xy} &= 0.04, \\
 \nu_{yz} &= \nu_{xz} = 0.3; \\
 MFC - E_x &= 30 \text{ GPa}, \\
 E_y &= E_z = 15.5 \text{ GPa}, \\
 G_{xz} &= G_{zx} = 10.7 \text{ GPa}, \\
 G_{yz} &= 5.7 \text{ GPa}, \\
 \nu_{xy} &= \nu_{xz} = 0.44, \\
 \nu_{yz} &= 0.35, \\
 d_{31} &= d_{32} = -1.98 \times 10^{-10} \text{ m/V}, \\
 d_{33} &= 4.18 \times 10^{10} \text{ m/V}.
\end{align*}
\]

Similarly, in the course of numerical simulation, convergence was studied. For better convergence of the solution and to reduce the errors in the results obtained, a computational grid was generated, the cells of which have a prismatic shape. The maximum element size was set to 0.9 mm., the minimum – to 0.21 mm.

Also, in the numerical simulation the mechanical behavior of the samples was investigated at various levels of the control voltage applied to the piezo-actuator. To describe the inverse piezoelectric effect in the framework of the developed mathematical model, it is proposed to use a thermo-piezoelectric analogy between piezoelectric and thermostimulated deformations.

The analogy makes it possible to move from the need to solve a coupled boundary value problem of electroelasticity to the solution of a substantially simpler unrelated boundary value problem of thermoelasticity and is based on the hypothesis of invariance of an electrically inhomogeneous field in a piezoelectric element (arising from the action of a control electrical voltage on its electrodes) to deformation fields inside the volume of the piezoelectric element and, as a result, to the deformation of the structure, on the surface of which the piezoelectric element (piezoelectric actuator) is fixed. That is to say, the hypothesis suggests the presence of the “inverse piezoelectric effect” in the piezoelectric element in the absence of the “direct piezoelectric effect”. Such an assumption is admissible for the cases when the “controlling” component of the electric field in the piezoelectric element, caused by the action of the control electric voltage $U_{synp}$ on the electrodes of the piezoelectric element, is significantly greater than the “deformation” component of the electric field caused by the “direct piezoelectric effect” from the influence of the essentially non-uniform deformation that appear and initially unknown fields in the
piezoelectric element. Note that the non-uniformity of the deformation fields in the volume of the piezoelectric element is caused by many factors, in particular, the shape and location of the electrodes on the surfaces of the piezoelectric element, the characteristics of the control electrical voltage on the electrodes and the attachment of the piezoelectric element to the surface of the structure [11, 12].

The formulation of a coupled boundary problem of electroelasticity for a structurally inhomogeneous body with an outer boundary, in particular, for a fragment of an elastic composite structure with a piezoelectric actuator fixed on the surface in the form of a composite plate with piezoelectric and piezopassive phases and electrodes consists of several equilibrium equations

\[ \sigma_{ij,j}(\mathbf{r}) = 0, \]  
continuity equation
\[ \mathbf{D}_{i,j}(\mathbf{r}) = 0, \]  
equilibrium equation of defining relations
\[ \sigma_{ij}(\mathbf{r}) = C_{ijmn}(\mathbf{r})\epsilon_{mn}(\mathbf{r}) - \epsilon_{nij}(\mathbf{r})\mathbf{E}_n(\mathbf{r}) \]  
equilibrium equation for Cauchy relations of small elastic deformations
\[ \epsilon_{ij}(\mathbf{r}) = [u_{i,j}(\mathbf{r}) + u_{j,i}(\mathbf{r})]/2, \]  
equilibrium equation for coupling the stress
\[ \mathbf{E}_i(\mathbf{r}) = \phi_j(\mathbf{r}), \]  
with the field of electric field potentials \( \phi(\mathbf{r}) \) and boundary conditions, for example, in the form of given non-zero values of electric potentials on the electrodes of the piezoelectric actuator, mechanical fixation conditions (for displacement vector) or loading (for voltage vector) and, in addition, potentials or inductions (normal components to the border areas \( a \)) electric field for all parts of the outer boundary \( \Gamma \) of the area \( V \) with the fulfillment of the conditions of ideal contact on the interphase surfaces of continuity of the vectors of displacements, voltages and inductions the electric field in the area \( V \) where \( \sigma(\mathbf{r}), \epsilon(\mathbf{r}) \) and \( u(\mathbf{r}) \) are the fields of stresses, strains and displacements, \( \mathbf{D}(\mathbf{r}), \mathbf{E}(\mathbf{r}), \phi(\mathbf{r}) \) are the fields of inductions, strengths and potentials of the electric field. As a result, the statement of the coupled boundary value problem of electroelasticity in the area \( V \) with an external boundary \( \Gamma \) will take the form:

\[ [C_{ijmn}(\mathbf{r})u_{m,n}(\mathbf{r})]_j + [\epsilon_{nij}(\mathbf{r})\phi_n(\mathbf{r})]_j = 0 \]  
\[ [\epsilon_{ijmn}(\mathbf{r})u_{m,n}(\mathbf{r})]_j - [\lambda_{ijn}(\mathbf{r})\phi_n(\mathbf{r})]_j = 0 \]  
for finding the displacement fields \( u(\mathbf{r}) \) and electric field potentials \( \phi(\mathbf{r}) \) taking into account the given boundary conditions, the continuity conditions of the electroelastic fields at the internal interphase boundaries and the conditions for loading the area \( V \) with an electric voltage between the actuator electrodes.

For the case of the composite areas \( V_{1,2} \) of the structure fragment and the piezoelectric actuator in the area \( V \) with the corresponding homogeneous regions with effective, in general, anisotropic electroelastic properties, we have

\[ \left(\begin{array}{c} \mathbf{C}(\mathbf{r}) \\ \mathbf{e}(\mathbf{r}) \\ \mathbf{\lambda}(\mathbf{r}) \end{array}\right) = \sum_{f=1}^{2} \left(\begin{array}{c} \mathbf{C}^*_f \\ \mathbf{e}^*_f \\ \mathbf{\lambda}^*_f \end{array}\right) i_f(\mathbf{r}), \quad i_f(\mathbf{r}) = \begin{cases} 1, & \mathbf{r} \in V_f \\ 0, & \mathbf{r} \not\in V_f \end{cases} \]  
(7)
for the coefficients of differential operators (6), where the indicator functions i_{1,2}(r) for the areas V_1 of the actuator and the fragment of the structure V_2 = V/V_1, the tensors of the effective elastic C^*, piezoelectric e^*, (in particular e^*_{1,2} = 0) and dielectric λ^*_{1,2} properties of the areas V_{1,2} [12].

The use of "thermo-piezoelectric analogy" (2), (3) allows you to move from the system of differential equations (6) to a simpler

\[ [C_{ijmm}(r)u_{m,n}(r)]_{j} - \beta^{*}_{ij,j}(r)\Delta T = 0, \]  

where the field of temperature coefficients

\[ \beta^{*}_{ij,j}(r) = \left\{ \begin{array}{ll} e^{*}_{(1)ij}(r)\zeta^{*}_{(1)n}(r), & r \in V_1 \\ 0, & r \notin V_1 \end{array} \right. \]

calculated through a known field \( \zeta(r) \) with components

\[ \zeta^{*}_{(1)n}(r) = \tilde{E}^{*}_{(1)n}(r) / U_{yp}, \]

identifying the magnitude of the control voltage \( U_{cont} \) on the electrodes of the piezoelectric element with the reduced heating temperature \( \Delta \hat{T} = U_{cont} \). Note that in some approximation, the inhomogeneous field \( \zeta(r) \) in the area \( V_1 \) can be calculated on the basis of solving a model boundary value problem of electroelasticity for an unrelated piezo-actuator design. While the tensor of the coefficients of the linear thermal expansion is determined by the following ratio:

\[ a^{*}_{i}(r) = \left\{ \begin{array}{ll} a^{*}_{i}(r), & r \in V_1 \\ 0, & r \notin V_1 \end{array} \right. \]

where the components \( \tilde{a}^{*}_{(1)ij} = C^{*} - 1_{ijmm}e^{*}_{(1)pmn}(r)\zeta^{*}_{(1)p}(r) \) are calculated through the known tensors of the effective elastic \( C^{*} \) and piezoelectric \( e^{*} \) properties of the actuator and the field of coefficients \( \zeta^{*}(r) \) that take into account the peculiarities of the geometric shape and arrangement of the electrodes of the piezo actuator.

When implementing the connection \( \Delta T = \Delta U \) in the framework of the PC finite element analysis, you can use the simplified relation of the following form:

\[ a_{3t} = \frac{a_{3t}}{d_{3t}} (t = 1,2,3), \]

where \( \Delta_{el} \) is the effective distance between the electrodes, the piezoelectric strain factor \( d_{3t} \) characterizes the electromechanical properties of the piezoelectric material and describes the deformation of the material depending on the orientation of the electric field. The first index indicates the direction of the electric field, and the second index indicates the direction of deformation.

Knowing the values of the three piezoelectric strain coefficients \( d_{31}, d_{32} \) and \( d_{33} \) is enough to fully characterize the electromechanical properties of the piezoelectric material.

3. Results of numerical simulation

According to the results of model layered PCM samples mechanical behavior numerical simulation, stress and displacement fields (Fig. 1,2) were obtained for samples with different locations of piezoactuators.

An analysis of the axial displacements fields \( U_z \) revealed that when the bending piezoactuator is moved away from the fixing point, the normal displacements \( U_z \) decrease. When comparing the results obtained for a sample with a bottom and upper arrangement of a piezoactuator, a decrease in normal displacements of \( U_z \) by 5.07 times was registered.
Figure 1. Fields of distribution of normal displacements $U_z$ at the applied control voltage 120V: a - bottom location; b - middle location; c - upper location.

When comparing the results obtained for samples equipped with two (bottom and middle location) and three (bottom, middle and upper location) flexural piezoactuators with results obtained for a sample with one (bottom location) piezoactuator (at the embedment), an increase in normal displacements $U_z$ was 1.66 and 2 times respectively.

Figure 2. Fields of distribution of normal displacements $U_z$ at the applied control voltage 120V: a - bottom/middle location; b - bottom/middle/upper location.

According to the results of the research, we can formulate the following recommendations: when designing controlled structures made of PCM that work in bending, when the piezoactuator location is distant from sealing, the bending effect decreases, therefore, it is recommended that the piezoactuators should be placed as close as possible to the mounting location of the structure.
4. Comparison of numerical simulation results and laboratory tests data

As part of the verification of the developed mathematical models, a comparison was made of the results of a numerical simulation of the model layered samples of PCM equipped with piezoactuators mechanical behavior with the results of laboratory tests. A general view of a laboratory setup with a sample of carbon fiber is presented in Figure 3. According to the results of laboratory bending tests for samples with a reinforcement scheme: $[0^\circ/90^\circ/45^\circ/-45^\circ/45^\circ/90^\circ/0^\circ]$, the values of normal displacements in the control design points at various levels of electrical control voltage were determined.

On the basis of the obtained results, graphs of the sample displacement at the control points from the control voltage applied to the piezoactuator are plotted and are shown in Figure 4.

![Figure 3. General view of the laboratory setup with a sample of carbon fiber.](image1.png)

![Figure 4. Graph of sample movement at the control points depending on the control voltage applied to the piezoactuator.](image2.png)

The analysis of dependencies revealed that the results of numerical simulation are within the range of the bend test results. The error in comparison with the average values for numerical models with the obtained results for laboratory tests on bending does not exceed 8.32%.

5. Conclusion

Thus, according to the results of the research, a mathematical model was developed for calculating the SSS of piezoelectric layered PCMs equipped with a control piezoactuator. The model implements the principle of thermo-piezoelectric analogy, which allows moving from the coupled boundary problem of electroelasticity to the boundary problem of thermoelasticity. A study of the convergence of finite element models of the samples was carried out. The mechanical behavior of layered PCM samples equipped with piezoactuators at different levels of control voltage was simulated. The main regularities and mechanisms of geometry management (deformation) of layered PCM with the built-in piezoactuators are revealed. Recommendations for the location of the bending piezo-actuators in the design of structures SMART structures of PCM were formulated. As part of the verification of the developed mathematical models, a comparison was made of the mechanical behavior of samples equipped with piezoactuators numerical simulation results with the results of laboratory tests. The error in comparing the results of numerical experiments with laboratory tests was no more than 9%.

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