Dynamic performance of planar mechanism with joint clearance considering stochastic parameters

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Abstract. Due to the influence of working temperature and pressure, collision restitution coefficient, Poisson’s ratio, friction coefficient and other material parameters could range in different intervals. However, the uncertainties and variability of material parameters is usually ignored in most researches. Along with vibration and wear, joint clearance is inevitable and widely exists in mechanical devices, which also should be taken into consideration. In this paper, the kinematic and dynamic equations are developed based on continuous Lankarani-Nikravesh contact model and the modified Coulomb friction model using a typical crank-slider mechanism as the research object. The influence of stochastic parameters (obeying normal distribution) on the dynamic characteristics of crank-slider mechanism is studied. In addition, dynamic analysis of the crank-slider mechanism with cylindrical joint clearance of different sizes is carried out in MATLAB environment. Results show that parameter random uncertainty should also be involved in dynamic models.

1. Introduction

Due to manufacturing tolerances, defects, wear, and material deformation, joint clearance can decrease the dynamic performance of mechanical structures and is non-ignorable. Theoretical and experimental work about joint clearance has been conducted in [1, 2]. To investigate the influence of clearance on the dynamic performance, Bai et al. [3] carried out dynamic simulation of a four-bar linkage mechanism with clearances, and the simulation results can accurately predict the impact of motion pair clearance on the dynamic characteristics. Yang et al. [4] and Dupac et al. [5] considered the clearance with two rotating pairs of a crank-slider mechanism and analyzed the impact of joint clearance on its dynamic response. Chaker et al. [6] studied the relationship between the pose error of end effector of a spherical parallel robot and the manufacturing error as well as joint clearance of different connecting rods.

However, in the mentioned studies, the material properties of the system are accurately determined or can be measured. Actually, some material parameters of mechanical system, such as collision restitution coefficient $C_e$, Poisson’s ratio $\nu$ and friction coefficient $\mu$, could vary a lot in different environment and working conditions. The uncertainty of system mainly includes three major categories such as randomness, fuzziness and unpredictability [7], which has been studied by many researchers. Zhu and Ting [8] used a probability density function to investigate the performance uncertainty caused by joint clearance of robots based on a theoretical model. Yan et al. [9] analyzed the influence of parameter uncertainty on the motion accuracy of a four-bar mechanism with joint clearance using the probabilistic method. Zaman et al. [10] proposed a probabilistic method to deal
with parametric random and cognitive uncertainty. FAKER and JAMES [11] used Bayesian method to study the problem of random uncertainty of input model and input parameters. Xu et al. [12] used the concept of generalized quadratic stability to study the stability of continuous uncertain singular systems with parameter delays. Mark Martinez et al. [13] studied the use of quantum computing to accelerate the processing of computational problems related to probability, interval and fuzzy uncertainty. Li et al. [14] analyzed the influence of uncertain parameters on the dynamic characteristics of a mechanism by modeling a space expandable mechanism. Sun and Chen [15] proposed a general method for kinematic accuracy analysis of a clearance mechanism involving random and cognitive uncertainty, and studied its effect on the dynamic performance of a crank slider mechanism with clearance. To sum up, most these studies did not consider the influence of some random parameters including collision restitution coefficient $C_r$, Poisson’s ratio $\nu$ and friction coefficient $\mu$. The distribution of stochastic parameters they generally used is normal distribution.

Thus in this paper, the kinematic and dynamic equations are developed in section 2 based on continuous Lankarani-Nikravesh (L-N) contact model and the modified Coulomb friction model using a typical crank-slider mechanism as the research object. Then, the influence of stochastic parameters (obeying normal distribution) on the dynamic characteristics of crank-slider mechanism is studied in section 3. In addition, dynamic analysis of the crank-slider mechanism with cylindrical joint clearance of different sizes is carried out in MATLAB environment. In section 4, the conclusions of the dynamic analysis with joint clearance considering stochastic parameters are presented.

2. Modelling of the crank-slider mechanism with joint clearance

2.1. Contact force model of joint clearance

In an ideal joint, the center of bearing and journal is completely coincident. But in reality, joint clearance will inevitably appear after long-time wear, resulting in continuous separation and collision between two parts. During the process from separation to contact, a severe collision will occur when the transmission speed is too fast, which can seriously affect the accuracy and stability of multibody system. Currently, there are three assumptions for the description of the cylindrical joint clearance. The first assumption is that the bearing and the journal are in constant contact, and no other movement occurs. The second assumption is that the bearing and the journal are in a cycle of separation and contact, and a collision force will occur during contact. The third type considers that contact, separation, and collision simultaneously exist in an operating cycle. Considering the generality of the mechanism, the second model state is adopted when establishing the mathematical model in this work. The contact members are represented by bearing and journal, and the schematic diagram of the clearance between them is shown in Figure 1. $R_i$ represents the radius of bearing, $R_j$ represents the radius of journal, $c$ represents the clearance between bearing and journal, and $e$ represents the center distance between bearing and journal.

Due to the existence of clearance, when the situation of two relative moving parts changes from the separated state to the contact state, collision will occur at the contact point. The collision depth $\delta = e - c$, and apply an impact contact force perpendicular to the collision plane. The impact contact forces between the two components are the normal contact force $F_n$ and the tangential contact force $F_t$, respectively. Based on the Hertz contact law [16], the normal contact force can be expressed by:

$$F_n = \begin{cases} \frac{K\delta^n + D\dot{\delta}}{\delta \geq 0} \\ 0 \delta < 0 \end{cases}$$ (1)
where $K\delta^n$ represents the elastic force term and $D\delta\dot{\delta}$ accounts for the energy dissipation term, $\delta$ is the relative impact velocity, $K$ represents the generalized stiffness coefficient, and $D$ is the damping coefficient. The exponent $n$ is set to 1.5. These parameters can be calculated by:

\[
K_n = \frac{4}{3\pi(h_i + h_j)}\left[\frac{R_i R_j}{R_i + R_j}\right]^{1/2} \\
h_i = \frac{1 - \nu_i^2}{\pi E_i} \\
h_j = \frac{1 - \nu_j^2}{\pi E_j} \\
D = \frac{3K_n(1 - C_r^2)\delta^n}{4\delta^{n-1}}
\]

where $\nu$ and $E$ are the Poisson’s ratio and the Young’s Modulus, and $C_r$ are the restitution coefficient, and $\delta^{n-1}$ are the initial deformation velocity.

The tangential contact force can be expressed as a tangential friction force, which is expressed as

\[
F_t = \mu F_n \text{sign}(v_t)
\]

In this model, the friction coefficient is not a fixed value, but a number related to the tangential velocity. It can be calculated by[17]:

\[
\mu = \begin{cases} 
\mu_s & |v_t| > v_d \\
\mu_s \cdot \sin\left(\frac{\pi}{2} \frac{|v_t|}{v_s}\right) & |v_t| < v_s \\
\frac{\mu_s + \mu_d}{2} + \frac{1}{2} \left(\mu_s - \mu_d\right) \cdot \cos\left(\pi \frac{|v_t| - v_s}{v_d - v_s}\right) & v_s < |v_t| < v_d
\end{cases}
\]

where $v_t$ is the tangential relative velocity, $v_s$ and $v_d$ are stick-slip switch velocity and static-sliding friction switch velocity, respectively. $\mu_s$ and $\mu_d$ are the static and sliding friction coefficients, respectively.

2.2. Kinematic and dynamic equations of the crank-slider mechanism

Figure 2 shows a crank-slider mechanism where there is a clearance at the joint between the crank and the connecting rod, and the other two joints on base and slider are treated as ideal. The numbers 1, 2 and 3 correspond to the crank, rod and slider, respectively. In Figure 2, the position coordinates of bearing and journal at the clearance can be calculated as

\[
\text{Figure 1. Contact model of cylindrical joint with clearance.}
\]
\[
\begin{align*}
\begin{cases}
x_1 = L_1 \cos \theta_1 \\
y_1 = L_1 \sin \theta_1
\end{cases}
\end{align*}
\begin{align*}
\begin{cases}
x_2 = x_3 - L_2 \cos \theta_2 \\
y_2 = L_2 \sin \theta_2
\end{cases}
\end{align*}
\]

(7)

where \( \theta_1 \) and \( \theta_2 \) are the angle between the x-axis with the crank and the connecting rod, respectively, \( x_1 \) and \( y_1 \) are the displacements of the journal center, \( x_2 \) and \( y_2 \) are the displacements of the bearing center, and \( x_3 \) denotes the position of slider.

Set \( e_x \) and \( e_y \) to be the components of the center distance between the bearing and the journal in the \( x \) and \( y \) directions. \( \phi \) is the angle between the axis and the center of the bearing and the \( x \)-axis. From Figure 2, the expression of \( e_x \) and \( e_y \) can be derived:

\[
\begin{align*}
\begin{cases}
e_x &= x_1 - x_2 = L_1 \cos \theta_1 - (x_3 - L_2 \cos \theta_2) \\
e_y &= y_1 - y_2 = L_1 \sin \theta_1 - L_2 \sin \theta_2
\end{cases}
\end{align*}
\]

(8)

Then we can calculate the distance between the center of bearing and journal: \( e = \sqrt{e_x^2 + e_y^2} \). The angle \( \phi \) between the connecting line of bearing center and journal center and the \( x \)-axis direction can be expressed as

\[
\phi = \arctan \left( \frac{e_y}{e_x} \right)
\]

(9)

The impact depth between bearing and journal can be expressed as

\[
\delta = \begin{cases}
e - c & e > c \\
0 & e \leq c
\end{cases}
\]

(10)

The relative velocity of bearing and journal can be expressed as

\[
v = v_n \hat{n} + v_t \hat{t}
\]

(11)

where \( \hat{n} \) and \( \hat{t} \) represent the unit vectors of the normal impact force and tangential impact force, respectively. \( v_n \) and \( v_t \) are the normal and tangential velocity components which are expressed as

\[
\begin{align*}
\begin{cases}
v_n &= \dot{e}_x \cos \phi + \dot{e}_y \sin \phi \\
v_t &= \dot{e}_y \cos \phi - \dot{e}_x \sin \phi + \dot{\theta}_1 R_1 + \dot{\theta}_2 (R_1 + e)
\end{cases}
\end{align*}
\]

(12)

According to the above equations, the component of the impact force in the \( x \) and \( y \) directions can be obtained as:

\[
\begin{align*}
\begin{cases}
F_x = F_n \cos \phi - F_t \sin \phi \\
F_y = F_n \sin \phi + F_t \cos \phi
\end{cases}
\end{align*}
\]

(13)
As shown in Figure 3, according to the force diagram of each member, the dynamic equations of each part can be obtained.

\[ L_y F_x \sin \theta_1 - L_y F_y \cos \theta_1 - \frac{1}{2} m_1 g L_1 \cos \theta_1 = 0 \]  \hspace{1cm} (14)

For connecting rod:

\[ F_y + F_{3y} - m_2 g = m_2 \ddot{y}_{s2} \]  \hspace{1cm} (15)
\[ F_x + F_{3x} = m_2 \ddot{x}_{s2} \]  \hspace{1cm} (16)
\[ \frac{1}{2} L_2 (F_{3y} \cos \theta_2 + F_{3x} \sin \theta_2) - \frac{1}{2} L_2 (F_x \sin \theta_2 + F_y \cos \theta_2) \]
\[ + (R + e)(F_y \cos \phi - F_x \sin \phi) = J_2 \ddot{\theta}_2 \]  \hspace{1cm} (17)
\[ \ddot{x}_{s2} = \ddot{x}_3 + \frac{1}{2} \dddot{\theta}_2^2 L_2 \cos \theta_2 + \frac{1}{2} \dddot{\theta}_2^2 L_2 \sin \theta_2 \]  \hspace{1cm} (18)
\[ \ddot{y}_{s2} = \frac{1}{2} \dddot{\theta}_2 L_2 \cos \theta_2 - \frac{1}{2} \dddot{\theta}_2 L_2 \sin \theta_2 \]  \hspace{1cm} (19)

where \( \ddot{x}_{s2} \), \( \ddot{y}_{s2} \) and \( J_2 \) are the centroid acceleration of the connecting rod in \( x \) and \( y \) directions and the moment of inertia around the end point.

For slider:

\[ -F_{3y} + F_{dx} = m_3 \ddot{x}_3 \]  \hspace{1cm} (20)
\[ F_{dx} = f (m_3 g + F_{3y}) \text{sign}(\sin \theta_1) \]  \hspace{1cm} (21)

Where \( F_{dx} \) is the friction force come from ground.

The crank-slider system has three degrees of freedom (namely \( \theta_1 \), \( \theta_2 \), \( x_3 \)). The rotating velocity of crank \( \omega \) is set to be a constant value (1200 r/min). Therefore, \( \theta_1 \) can be solved by its relationship with time: \( \theta_1 = \omega t + \theta_{10} \) (\( \theta_{10} \) denotes the initial value of \( \theta_1 \)). After solving the dynamic equations (14) ~ (21) established above, the expression of the slider acceleration \( \ddot{x}_3 \) and the link angle \( \dot{\theta}_2 \) can be derived. Then initialize the values of \( x_3, \dot{x}_3, \theta_2, \dot{\theta}_2 \) based on equation(22). By Euler method or Runge-Kutta method for numerical solution, the motion parameters of the crank-slider mechanism with clearance and the mechanical parameters of the joint clearance can be derived.

![Figure 3. Schematic diagram of the force of the crank, connecting rod and slider.](image-url)
\[
\begin{align*}
\dot{x}_3 &= f_1(t, x_1, \dot{x}_1, \theta_2, \dot{\theta}_2) \\
\dot{\theta}_2 &= f_2(t, x_3, \dot{x}_3, \theta_2, \dot{\theta}_2)
\end{align*}
\] (22)

3. Dynamic analysis of planar mechanism with clearance and stochastic parameters

In the study, the value of each parameter [14,18-20] is shown in Tables 1 and 2. In this paper, Monte Carlo method is used to obtain the values of stochastic parameters. The specific implementation methods and steps are as follows:

1) Use a random number generator to generate a random number that obeys one distribution.
2) Solve the equation (22) by the generated random number.
3) Determine whether the obtained result is acceptable, and then return to step (1) to continue the next calculation until the accepted parameters is \(N\).

Table 1. Geometric parameters and material parameters of the crank slider mechanism.

| Parameters          | Notations and Units | Crank | Rod |
|---------------------|---------------------|-------|-----|
| Length              | \(L\) (m)           | 0.1   | 0.4 |
| Young’s modulus     | \(E\) (N/m\(^2\))   | 2.07e11 | 2.07e11 |
| Poisson’s ratio     | \(v\)               | N(0.4,(0.1/3) \(^2\)) | N(0.4,(0.1/3) \(^2\)) |
| Mass                | \(m\) (Kg)          | 0.45814 | 1.2972 |
| Moment of inertia   | \(J\) (Kg\(\cdot\)m\(^2\)) | -     | 0.0173 |

Table 2. Simulation parameters used in this paper.

| Parameters                          | Notations and Units | Values                      |
|-------------------------------------|---------------------|-----------------------------|
| Static coefficient of friction      | \(\mu_s\)           | 0.3                         |
| Dynamic coefficient of friction     | \(\mu_d\)           | N(0.15,0.05\(^2\))         |
| Journal radius                      | \(R_j\) (m)         | 9.8e-3                      |
| Bearing radius                      | \(R_i\) (m)         | 10e-3                       |
| Restitution coefficient              | \(C_r\)             | N(0.75,0.08\(^2\))         |
| Stick-slip switch velocity          | \(v_s\) (m/s)       | 0.01                        |
| Static-sliding friction switch velocity | \(v_d\) (m/s)   | 0.1                         |
| Friction coefficient of slider      | \(f\)               | 0.05                        |
| Initial impact velocity             | \(v_0\) (m/s)       | 0.05                        |
| Crank rotating velocity             | \(\omega\) (r/min)  | 1200                        |
| The initial angle of crank          | \(\theta_{i0}\) (rad) | 0                           |

3.1. Validation of the developed contact force model

In order to compare the results between numerical model and Adams simulation, the contact model in Adams software has been substituted with L-N model by modifying a library file named ‘cnfsub.c’. Therefore the contact model in numerical model and Adams simulation is the same. Then the results in Adams simulation can be used as a reference to judge the process of the theoretical analysis right or not. The displacement, velocity, acceleration of slider and the angle, angular velocity, and angular acceleration of the connecting rod are investigated. The comparison of simulation results is shown in Figures 4.

As can be seen from Figures 4(a), (b), (d) and (e), the slider displacement curve, velocity curve, and link angle and angular velocity curve calculated by MATLAB basically coincide with Adams
simulation curves. In Figure 4(c) and (f), although there are larger impulses in the results from Adams, the trend of slider acceleration curve and connecting rod angular acceleration curve are consistent with the results from numerical model. So the deriving process of the theoretical analysis could be considered correct.

**Figure 4.** Comparison of the simulation results in MATLAB and Adams: (a) the displacement of slider, (b) the velocity of slider, (c) the acceleration of slider, (d) the angle of connecting rod, (e) the angular velocity of connecting rod, and (f) the angular acceleration of connecting rod.

3.2. **The influence of stochastic parameters on mechanism dynamics**

The comparison of simulation results with stochastic and certain parameters is shown in Figures 5. Figures 5 (a), (b) and (c) show the motion trajectory of the slider when consider stochastic parameters and determinate parameters. It can be seen that when the parameters are determinate, the slider's displacement, velocity, and acceleration are all determined values. When consider the random parameters, we get a range of values that represents the range between the maximum and minimum
values of the results in all samples. Figures 5(d), (e) and (f) show the trajectory of the connecting rod when considering stochastic parameters and determinate parameters. Like the slider trajectory, when the parameters are determinate, the rotation angle, angular velocity and angular acceleration of the connecting rod are a certain value, and when considering stochastic parameters, we get a range of values, which also represent the range between the maximum and minimum values of the results in all samples. Therefore, when considering stochastic parameters, the kinematic accuracy of the mechanism will decrease.

![Diagram of mechanism motion when considering stochastic parameters](image)

**Figure 5.** Diagram of mechanism motion when considering stochastic parameters: (a) the displacement of slider, (b) the velocity of slider, (c) the acceleration of slider, (d) the angle of...
connecting rod, (e) the angular velocity of connecting rod, and (f) the angular acceleration of connecting rod.

Figure 6.

Figure 6 shows the normal probability density distribution curve of the slider velocity standard deviation when considering different clearance sizes. It can be seen from the Figure 6 that as the clearance size increases, the normal probability density curve tends to become lower and wider, which indicating that increasing the clearance size can reduce the standard deviation of the motion, but at the same time increases the sensitivity to stochastic parameters.

4. Conclusions

In order to study the influence of stochastic parameters on the dynamic performance of a mechanism with joint clearance, the kinematics model of the joint clearance is established using the geometric relationship between the clearance joint and the system components, then the normal contact velocity and the tangential contact velocity between the clearance joint elements is derived. Further, the normal and tangential contact forces between the clearance joint parts are established while using the Lankarani-Nikravesh contact force model and the modified Coulomb friction model. Besides, based on the multi-body system dynamics equations and the Monte Carlo method, a dynamic model of mechanical system was established. Finally, verified the accuracy of the Lankarani-Nikravesh contact force model by Adams. When considering stochastic parameters such as collision restitution coefficient $C_r$, Poisson’s ratio $\nu$ and friction coefficient $\mu$, many unexpected vibrations and shocks may arise, which can significantly decrease the kinematic accuracy and eventually result in a great impact on the dynamic performance of the system. Meanwhile, if a joint possesses a relatively large clearance, the system will be more sensitive to stochastic parameters.

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