Comparison of optimal reactive power flow model in the polar form and the mixed polar form

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Abstract. This paper establishes the optimal reactive power flow (ORPF) model in the polar form and the mixed polar form. The analysis shows that the ORPF model in the polar form not only has less calculation amount of equality constraints than the ORPF model in the mixed polar form, but also has more concise expressions, which makes it easier to understand, analyze, and remember. The test results of IEEE30, 118, 300, 1047 bus systems show that the ORPF model in the polar form obtains the same optimal value and iteration number as the ORPF model in the mixed polar form, but its calculation time is shorter, and the solving efficiency is higher. With the increasing scale of modern power system, the calculation time of ORPF problem rises, so it is necessary to improve its solving efficiency. The ORPF model in the polar form is more suitable for power system analysis and calculation because of its high solving efficiency.

1. Introduction
Optimal reactive power flow (ORPF) is an essential work for improving the operational security and economy of power systems. It determines all kinds of controllable variables, such as reactive power output of generators and shunt capacitors, turn ratios of on-load tap changers (OLTC), etc., and minimizes transmission losses or other objective functions, while satisfying a given set of physical and operating constraints.

The rectangular form [1-3], the polar form [4-6], and the mixed polar form [7-9] widely apply to optimal power flow (OPF) and ORPF problems of power systems. Papers [10, 11] compare power flow calculation in the polar form and the rectangular form. There are many advantages to express bus voltages in the polar form. Firstly, in the actual power system, only the voltage magnitude and phase can be measured, but not the real and imaginary parts of the voltage. Secondly, voltage in the polar form can also reflect physical significance: the difference of voltage magnitude is related to the transmission of reactive power, and the difference of voltage phase is related to the transmission of active power. Thirdly, the power flow expression in the polar form is more concise than that in the rectangular form, which is convenient for people to understand, analyze and remember. Expressing bus voltage in the polar form and the mixed polar form both have the advantages mentioned above. However, there is no literature comparing models in the mixed polar form and the polar form. Therefore this paper establishes the ORPF models in the mixed polar form and the polar form, and compares the solving efficiency of the two models in order to find which one more suitable for power systems. Test results indicate that the ORPF model in the polar form has higher solving efficiency than the ORPF model in the mixed polar form.

The rest of this paper is organized as follows. Section 2 describes the ORPF model in the mixed polar form. Section 3 introduces the ORPF model in the polar form. Section 4 analyzes the difference
between the two models. Numerical tests are performed and discussed in section 5. Conclusions are drawn in section 6.

2. The ORPF model in the mixed polar form

2.1. The power equations in the mixed polar form of the on-load tap changer

\[
i P_{xy} + jQ_{xy} \quad k_{xy}y_{xy} \quad P_{yji} + jQ_{yji} \quad j
\]

\[
(k^2 - k^2) y_{xy} \quad (1 - k_{xy}) y_{xy}
\]

**Figure 1.** The transformer equivalent Π circuit.

This paper considers the voltage magnitude changing of the OLTC branch, ignoring its phase changing. In the ORPF model, the OLTC branch is represented by equivalent Π circuit, as shown in Figure 1. The OLTC branch is between bus \(i\) and bus \(j\), and the ratio of voltage magnitude at bus \(i\) to that at bus \(j\) is \(1:k_{ij}\). The admittance of the OLTC branch, the voltage at bus \(i\) and bus \(j\), were expressed in the mixed polar form as follows:

\[
y_{xy} = G_{xy} + jB_{xy} \quad (1)
\]

\[
\bar{v}_{i} = V_i e^{j\delta_i} \quad (2)
\]

\[
\bar{v}_{j} = V_j e^{j\delta_j} \quad (3)
\]

Where \(G_{xy}\) and \(B_{xy}\) are the conductance and susceptance of the OLTC branch between bus \(i\) and bus \(j\), while \(V_i, V_j\) and \(\delta_i, \delta_j\) are the voltage magnitude and phase at bus \(i\) and bus \(j\).

According to the power equation and the law of Kirchhoff, the active and reactive power flowing through the OLTC branch is:

\[
P_{xy} = V_i^2 k_{ij}^2 G_{xy} - V_j k_{ij} (G_{xy} \cos(\delta_i - \delta_j) + B_{xy} \sin(\delta_i - \delta_j)) \quad (4)
\]

\[
Q_{xy} = -V_i^2 k_{ij}^2 B_{xy} - V_j k_{ij} (G_{xy} \sin(\delta_i - \delta_j) - B_{xy} \cos(\delta_i - \delta_j)) \quad (5)
\]

\[
P_{yji} = V_j^2 k_{ij}^2 G_{xy} - V_i k_{ij} (G_{xy} \cos(\delta_j - \delta_i) + B_{xy} \sin(\delta_j - \delta_i)) \quad (6)
\]

\[
Q_{yji} = -V_j^2 k_{ij}^2 B_{xy} - V_i k_{ij} (G_{xy} \sin(\delta_j - \delta_i) - B_{xy} \cos(\delta_j - \delta_i)) \quad (7)
\]

2.2. The ORPF model in the mixed polar form

This paper mainly considers the regulating of transformer taps and the reactive power output of generators. Its mathematical model is made up of the objective function, power flow equality constraints, variables inequality constraints, etc.

The ORPF problem generally takes the minimum active power losses of the system as the objective function:

\[
\min P_{loss} = \sum_{i \in S_G} P_{gi} - \sum_{i \in S_D} P_{di}
\]

Here, \(P_{loss}\) is the active power losses of the system; \(P_{gi}\) and \(P_{di}\) are the active power output of generators and the active load at bus \(i\); \(S_G\) is the set of generators; \(S_D\) is the set of loads.

The power flow equality constraints can be expressed as:

\[
P_{gi} - P_{di} = V_i \sum_{j \in S_L} V_j (G_{ij} \cos \delta_j + B_{ij} \sin \delta_j) + V_i^2 \sum_{j \in S_L} k_{ij}^2 G_{ij} - V_i \sum_{j \in S_L} V_j k_{ij} (G_{ij} \cos \delta_j + B_{ij} \sin \delta_j) = 0
\]
\[ Q_{gi} - Q_{di} - V_i \sum_{j \in S_i} V_j (G_{Lij} \sin \delta_j - B_{Lij} \cos \delta_j) - V_i^2 \sum_{j \in S_i} k_{ij}^2 B_{Tij} = 0 \]
\[ -V_i \sum_{j \in S_i} V_j k_{ij} (G_{Tij} \sin \delta_j - B_{Tij} \cos \delta_j) - V_i^2 \sum_{j \in S_i} B_{Tij} - V_i \sum_{j \in S_i} V_j k_{ij} (G_{Tij} \sin \delta_j - B_{Tij} \cos \delta_j) = 0 \]

Where, \( Q_{gi} \) and \( Q_{di} \) are the reactive power output of generators and the reactive load at bus \( i \); \( G_{Lij} \) and \( B_{Lij} \) are the conductance and susceptance of the transmission line between bus \( i \) and bus \( j \); \( S_i \) is the set of transmission lines; \( S_q \) is the set of buses at the \( k \) side of the OLTC branch; \( S_{Tq} \) is the set of buses at the other side of the OLTC branch; \( \delta_j \) is the difference of phase between bus \( i \) and bus \( j \), \( \delta_j = \delta_i - \delta_j \).

The upper and lower inequality constraints of the variables are as follows:
\[ P_i \leq \bar{P}_i \leq \underline{P}_i \tag{11} \]
\[ Q_i \leq \bar{Q}_i \leq \underline{Q}_i \tag{12} \]
\[ V_i \leq V_i \leq \bar{V}_i \tag{13} \]
\[ k_{ij} \leq k_{ij} \leq \bar{k}_{ij} \tag{14} \]

Where, \( \bar{\text{ }} \) and \( \underline{\text{ }} \) indicate the lower and upper limits of variables.

3. The ORPF model in the polar form

3.1. The power equations in the polar form of the on-load tap changer

According to the power equation and the law of Kirchhoff, the active and reactive power in the polar form flowing through the OLTC branch can be expressed as:

\[ P_{Tij} = V_i^2 k_{ij} Y_{Tij} \cos \alpha_{Tij} - V_i V_j k_{ij} Y_{Tij} \cos(\delta_j - \delta_i - \alpha_{Tij}) \tag{15} \]

\[ Q_{Tij} = -V_i^2 k_{ij}^2 Y_{Tij} \sin \alpha_{Tij} - V_i V_j k_{ij}^2 Y_{Tij} \sin(\delta_j - \delta_i - \alpha_{Tij}) \tag{16} \]

\[ P_{Tij} = V_j^2 Y_{Tij} \cos(\delta_j - \delta_i - \alpha_{Tij}) \tag{17} \]

\[ Q_{Tij} = -V_j^2 Y_{Tij} \sin(\delta_j - \delta_i - \alpha_{Tij}) \tag{18} \]

Where \( Y_{Tij} \) and \( \alpha_{Tij} \) are the admittance magnitude and phase of the OLTC branch between bus \( i \) and bus \( j \).

3.2. The ORPF model in the polar form

The objective function of this model is also equation (8). The constraints include power flow equality equations (19) - (20), upper and lower limits of variables inequality equations (11) - (14).

The power flow equality constraints can be expressed as:

\[ P_{gi} - P_{di} - V_i \sum_{j \in S_i} V_j Y_{Lij} \cos(\delta_j - \alpha_{Lij}) + V_i^2 \sum_{j \in S_i} k_{ij}^2 Y_{Tij} \cos \alpha_{Tij} \]
\[ -V_i^2 \sum_{j \in S_i} V_j k_{ij} Y_{Lij} \cos(\delta_j - \alpha_{Lij}) + V_j \sum_{j \in S_i} Y_{Lij} \cos \alpha_{Tij} - \sum_{j \in S_i} V_j Y_{Tij} \cos \alpha_{Tij} = 0 \]  

\[ Q_{gi} - Q_{di} - V_i \sum_{j \in S_i} V_j Y_{Lij} \sin(\delta_j - \alpha_{Lij}) + V_j \sum_{j \in S_i} k_{ij}^2 Y_{Tij} \cos \alpha_{Tij} \]
\[ -V_i^2 \sum_{j \in S_i} V_j k_{ij} Y_{Lij} \sin(\delta_j - \alpha_{Lij}) - V_j \sum_{j \in S_i} Y_{Lij} \cos \alpha_{Tij} - \sum_{j \in S_i} V_j k_{ij} Y_{Tij} \sin \alpha_{Tij} = 0 \]

Where \( Y_{Lij} \) and \( \alpha_{Lij} \) are the admittance magnitude and phase of transmission line between bus \( i \) and bus \( j \).
4. Comparison of the two ORPF models

The objective function and inequality constraints are identical in the two ORPF models, which do not affect the solution efficiency. The difference in the equality constraints is the critical factor that affects the solving efficiency of problem. Comparing the equality constraints of the two ORPF models, equations (9) - (10) and equations (19) - (20) could deduce each other. Therefore the two models are mathematically equivalent, and the optimal reactive power flow model in the polar form has the following advantages:

The calculation amount of power flow equations in the polar form is less than that in the mixed polar form. Take the active power flow equation as an example, as shown in Table 1. From the third item to the seventh item of the equation (9) are different from that of equation (19), which is the main factor affecting the calculation amount. If there is a transmission line between bus $i$ and bus $j$, the third term is included in the active power flow equation. The third term of equation (9) is obtained by combining of two quartic polynomials, while the third term of equation (19) only has one quartic polynomial, so the calculation amount of equation (19) is less than that of equation (9). If an OLTC branch is between bus $i$ and bus $j$, in the fourth and fifth items are added to the active power flow equation. Compared with equation (9), the calculation amount of the fifth term of equation (19) reduces, and the fourth item has an additional trigonometric function multiplier, which could increase the calculation amount. However, the trigonometric function multiplier of the fourth item is a fixed value for a certain OLTC, so it needs to be computed only once through the whole iteration process, and little impact on the entire amount of calculation. Overall calculation amount of the fourth and fifth items of the equation (19) is still less than that of the equation (9). For bus $j$, the sixth and seventh items add to the active power flow equation, and similarly, the calculation amount of the sixth and seventh items of equation (19) is less than that of equation (9).

In a word, the calculation amount of the active power flow equation in the polar form is less than that of the mixed polar form, and the calculation amount of the reactive power flow equation is similar. Also, the expressions in the polar coordinates are more concise and easier to understand, analyze, and remember.

Table 1. Comparison of active power flow equation of the two ORPF models

|                        | The model in the mixed polar form: Equation(9) | The model in the polar form: Equation(19) |
|------------------------|-----------------------------------------------|------------------------------------------|
| The first item         | $P_{ci}$                                      | $P_{ci}$                                  |
| The second item        | $-P_{di}$                                     | $-P_{di}$                                 |
| The third item         | $-V_{j} \sum_{s \in S} V_j (G_{ls} \cos \delta_{lj} + B_{ls} \sin \delta_{lj})$ | $-V_{j} \sum_{s \in S} V_j Y_{lj} \cos(\delta_{lj} - \alpha_{lj})$ |
| The fourth item        | $V_{j}^2 \sum_{s \in S_{lj}} k_i^2 G_{lj}$    | $V_{j}^2 \sum_{s \in S_{lj}} k_i^2 Y_{lj} \cos \alpha_{lj}$ |
| The fifth item         | $-V_{j} \sum_{s \in S_{lj}} V_j (G_{lj} \cos \delta_{lj} + B_{lj} \sin \delta_{lj})$ | $-V_{j}^2 \sum_{s \in S_{lj}} V_j k_j Y_{lj} \cos(\delta_{lj} - \alpha_{lj})$ |
| The sixth item         | $V_{j}^2 \sum_{s \in S_{lj}} G_{lj}$          | $V_{j}^2 \sum_{s \in S_{lj}} Y_{lj} \cos \alpha_{lj}$ |
| The seventh item       | $-V_{j} \sum_{s \in S_{lj}} V_j k_j (G_{lj} \cos \delta_{lj} + B_{lj} \sin \delta_{lj})$ | $-V_{j}^2 \sum_{s \in S_{lj}} V_j k_j Y_{lj} \cos(\delta_{lj} - \alpha_{lj})$ |

5. Numerical test and discussion

5.1. Test systems and environment

This paper takes IEEE30, 118, 300, 1047 bus systems [12] as examples to discuss the performance of the two ORPF models. The characteristics of bus systems are as shown in Table 2. All the test systems...
calculate in the form of per-unit value, and the convergence precision is $10^{-6}$. The voltage magnitude range is set between 0.9 and 1.1, and the adjustment range of the turn ratios are all 0.9-1.1 identically. All the test systems simulate in an HP PC-compatible computer. The computer’s CPU is an Intel Core i5-6500, 3.20GHz, four cores, and its RAM is 8GB. All models were tested using the commercial software GAMS (General Algebraic Modeling System). The two ORPF models are modeled as a nonlinear programming problem, and the interior point method solver is used to solve them.

### Table 2. The characteristics of bus systems.

|          | Number of buses | Number of transmission lines | Number of transformers | Number of active power resources | Number of reactive power resources |
|----------|-----------------|-----------------------------|------------------------|----------------------------------|-----------------------------------|
| IEEE30   | 30              | 37                          | 4                      | 6                                | 6                                 |
| IEEE118  | 118             | 168                         | 11                     | 16                               | 54                                |
| IEEE300  | 300             | 302                         | 107                    | 21                               | 69                                |
| IEEE1047 | 1047            | 1018                        | 164                    | 68                               | 152                               |

5.2. The test results analysis of the two ORPF models

Take IEEE30 bus system as an example, the optimal value (per-unit) of active power losses (see Table 3), turn ratios of the OLTC branches (see Table 4), output of active and reactive power resources, voltage magnitude and phase of buses (see Table 5) calculated by the two models are same with each other, indicating that the two models are mathematically equivalent.

### Table 3. The test results of the two ORPF models.

|          | Optimization by the ORPF model in the mixed polar form | Optimization by the ORPF model in the polar form | Before optimization |
|----------|--------------------------------------------------------|-------------------------------------------------|---------------------|
|          | Optimal value(per-unit) | Calculation time(s) | Iteration number | Optimal value(per-unit) | Calculation time(s) | Iteration number | Active power losses(per-unit) |
| IEEE30   | 0.0122 | 0.125 | 12 | 0.0122 | 0.063 | 12 | 0.0126 |
| IEEE118  | 0.7086 | 0.156 | 17 | 0.7086 | 0.125 | 17 | 0.713 |
| IEEE300  | 3.2196 | 0.203 | 21 | 3.2196 | 0.156 | 21 | 3.5531 |
| IEEE1047 | 2.4517 | 0.469 | 28 | 2.4517 | 0.437 | 28 | 2.5379 |

Table 3 shows that the optimal value (per-unit) of IEEE30, 118, 300, 1047 bus systems by using the two ORPF models are same. Assuming that turn ratios of each test system are all equal to 1.0 before optimization, the active power losses of each test system is reduced to some extent after optimization by using the two ORPF models. It illustrates that the two ORPF models both can effectively reduce the active power losses of the power system in the precondition of ensuring the voltage quality. In addition, the model in the polar form has the same iteration number as the model in the mixed polar form. However, its calculation time is shorter, and the solving efficiency is higher than the model in the mixed polar form.

### Table 4. For IEEE30 bus system, the turn ratios of transformers computed by different models.

|          | The model in the mixed polar form | The model in the polar form |
|----------|-----------------------------------|-----------------------------|
| $k_{6,10}$ | 1.100                            | 1.100                       |
| $k_{9,6}$  | 1.050                            | 1.050                       |
| $k_{12,4}$ | 0.988                            | 0.988                       |
| $k_{28,27}$| 1.031                            | 1.031                       |
Table 5. For IEEE30 bus system, the output of active and reactive power resources, voltage magnitude and phase of buses computed by different models.

| Bus number | The ORPF model in the mixed polar form | The ORPF model in the polar form |
|------------|---------------------------------------|---------------------------------|
|            | Voltage magnitude | Voltage phase | Active power resources | Reactive power resources | Voltage magnitude | Voltage phase | Active power resources | Reactive power resources |
| 1          | 1.100             | 0.000          | 0.100                   | -0.049                  | 1.100             | 0.000          | 0.100                   | -0.049                  |
| 2          | 1.099             | -0.003         | 0.225                   | 0.059                   | 1.099             | -0.003         | 0.225                   | 0.059                   |
| 3          | 1.096             | -0.007         | -                       | -                       | 1.096             | -0.007         | -                       | -                       |
| 4          | 1.095             | -0.007         | -                       | -                       | 1.095             | -0.007         | -                       | -                       |
| 5          | 1.099             | -0.003         | 1.042                   | 0.199                   | 1.099             | -0.003         | 1.042                   | 0.199                   |
| 6          | 1.096             | -0.004         | -                       | -                       | 1.096             | -0.004         | -                       | -                       |
| 7          | 1.090             | -0.012         | -                       | -                       | 1.090             | -0.012         | -                       | -                       |
| 8          | 1.100             | -0.001         | 0.462                   | 0.364                   | 1.100             | -0.001         | 0.462                   | 0.364                   |
| 9          | 1.080             | 0.032          | -                       | -                       | 1.080             | 0.032          | -                       | -                       |
| 10         | 1.097             | -0.019         | -                       | -                       | 1.097             | -0.019         | -                       | -                       |
| 11         | 1.100             | 0.164          | 0.754                   | 0.156                   | 1.100             | 0.164          | 0.754                   | 0.156                   |
| 12         | 1.091             | -0.023         | -                       | -                       | 1.091             | -0.023         | -                       | -                       |
| 13         | 1.100             | 0.008          | 0.263                   | 0.073                   | 1.100             | 0.008          | 0.263                   | 0.073                   |
| 14         | 1.079             | -0.036         | -                       | -                       | 1.079             | -0.036         | -                       | -                       |
| 15         | 1.078             | -0.037         | -                       | -                       | 1.078             | -0.037         | -                       | -                       |
| 16         | 1.082             | -0.027         | -                       | -                       | 1.082             | -0.027         | -                       | -                       |
| 17         | 1.087             | -0.025         | -                       | -                       | 1.087             | -0.025         | -                       | -                       |
| 18         | 1.073             | -0.041         | -                       | -                       | 1.073             | -0.041         | -                       | -                       |
| 19         | 1.073             | -0.041         | -                       | -                       | 1.073             | -0.041         | -                       | -                       |
| 20         | 1.078             | -0.036         | -                       | -                       | 1.078             | -0.036         | -                       | -                       |
| 21         | 1.086             | -0.028         | -                       | -                       | 1.086             | -0.028         | -                       | -                       |
| 22         | 1.086             | -0.028         | -                       | -                       | 1.086             | -0.028         | -                       | -                       |
| 23         | 1.073             | -0.042         | -                       | -                       | 1.073             | -0.042         | -                       | -                       |
| 24         | 1.075             | -0.044         | -                       | -                       | 1.075             | -0.044         | -                       | -                       |
| 25         | 1.081             | -0.055         | -                       | -                       | 1.081             | -0.055         | -                       | -                       |
| 26         | 1.065             | -0.061         | -                       | -                       | 1.065             | -0.061         | -                       | -                       |
| 27         | 1.093             | -0.057         | -                       | -                       | 1.093             | -0.057         | -                       | -                       |
| 28         | 1.092             | -0.009         | -                       | -                       | 1.092             | -0.009         | -                       | -                       |
| 29         | 1.075             | -0.075         | -                       | -                       | 1.075             | -0.075         | -                       | -                       |
| 30         | 1.064             | -0.089         | -                       | -                       | 1.064             | -0.089         | -                       | -                       |

6. Conclusion
This paper establishes the optimal reactive power flow model in the mixed polar form and the polar form, then compares and analyzes them detailedly. The objective function and inequality constraints of the two models are identical, while the equality constraints are different. The optimal reactive power flow model in the polar form, not only has less calculation amount of equation constraints than
the model in the mixed polar form but also has more concise expressions, which makes it easier to understand, analyze and remember. The test results of IEEE30, 118, 300, 1047 bus systems shows that the optimal solution and the optimal value of active power losses obtained by the two models are the same with each other, which indicates the two models are equivalent in mathematics. The optimal reactive power flow model in the polar form has the same iteration number as the reactive power flow model in the mixed polar form, but has shorter calculation time and higher solving efficiency, and more suitable for the calculation of various problems of power systems.

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Acknowledgments
The work is supported by the National Nature Science Foundation of China (NSFC) (51667003).