A Design Method of Two DOF Joint at the Base of Heavy-load Manipulator Based on Linear Driving

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Abstract. In this paper, we propose a two DOF joint mechanical design scheme for heavy-load manipulator based on linear driving, the key structural parameters that directly influencing on the linear driving force is determined. Based on the multi-body kinematics and the principle of virtual work, we derive the relationship between the torque of joint space at the base of the heavy-load manipulator and the force of linear driving, then by analyzing the relationship, we obtain the design method of key structural parameters. Finally, using simulation analysis we validate this design method.

1. Introduction

Since 1913, the Ford Motor Co. has been using automatic production line instead of manual production for more than 100 years[1-6]. Automatic production line has been developed rapidly in America, Europe, Japan and other developed countries. the most widely used in automatic production is the serial multi-joint manipulator, which is usually set with two rotational joint at the base. The two rotational joint consist of pitch rotation and yaw rotation, which are usually drove by motor and gear reducer.

In order to meet the requirement of larger workspace, the mechanical arms are often longer. Moreover, it needs a very large pitch torque to act on the base of mechanical for lifting a heavy loader. Thus the base of manipulator structure is usually designed bulky, as well as the ratio of payload-weight to self-weight is very low. In view of mentioned above, we propose a design scheme of two DOF joint at the base of heavy-load manipulator based on linear driving. It has the advantage of large output torque but with light self-weight, which sharply increase the ratio of payload-weight to self-weight.

2. Design Scheme

The design scheme for the two DOF joint at base of heavy-load manipulator based on linear driving is mainly composed of main arm yaw joint(Fig.1-(1)), main arm pitch joint(Fig.1-(2)), main arm(Fig.1-(3)), main arm left/right end joint(Fig.1-(4/5)), left/right link (Fig.1-(6/7)), left/right slider end joint(Fig.1-(8/9)), left/right slider(Fig.1-(10/11)), left/right slideway(Fig.1-(12/13)), left/right linear driving mechanism(Fig.1-(14/15)), etc. In Figure 1, the main arm’s yaw/pitch joint is the single DOF joint, the main arm’s left/right end joint and the left/right slider’s end joint both are three DOF joint,
the left/right linear driving mechanism can be driven by hydraulic cylinder, electric cylinder or nut screw. When driving the left (right) slider of linear motion along the left (right) slideway, the yaw and pitch rotating of the main arm can be realized.

3. Design Method of Key Structure Parameters

3.1. Analysis of kinematics
In this design scheme, the key structural parameters of yaw and pitch torque are determined by the following parameters: the common perpendicular distance a1 is between the main arm yaw joint and pitch joint, the horizontal distance d2 is between the main arm yaw joint and left/right end joint, the vertical distance a2 is between the main arm pitch joint and left/right end joint, the L is the length of left/right link, the horizontal distance a3 is between the main arm yaw joint and left/right slider end joint, the vertical distance d1 is between the main arm pitch joint and left/right slider end joint (see Fig. 2).

A coordinate system for the two DOF joint at the base of heavy-load manipulator based on linear driving is established by analyzing multi-body kinematics. In Fig.3, the O0 is the origin of base coordinate frame, the O1 is the origin of coordinate frame for main arm yaw joint, Z1 is the rotation axis of yaw joint, O2 is the origin of coordinate frame for main arm pitch joint, Z2 is the rotation axis of pitch joint, O3 is the origin of coordinate frame for main arm right end joint.
3.2. Principle of virtual work

According to reference[7], we use homogeneous coordinate transformation method to obtain the motion trajectory equation of the main arm left/right end joint.

\[
\begin{align*}
(x_L &= a_1\cos\theta_1 - d_2\sin\theta_1 + a_2\cos\theta_1 \cos\theta_2) \\
y_L &= a_1\sin\theta_1 + d_2\cos\theta_1 + a_2\sin\theta_1 \cos\theta_2 \\
z_L &= a_2\sin\theta_2 \\
\end{align*}
\]

(1)

\[
\begin{align*}
(x_R &= a_1\cos\theta_1 + d_2\sin\theta_1 + a_2\cos\theta_1 \cos\theta_2) \\
y_R &= a_1\sin\theta_1 - d_2\cos\theta_1 + a_2\sin\theta_1 \cos\theta_2 \\
z_L &= a_2\sin\theta_2 \\
\end{align*}
\]

(2)

In Formula (1) and (2), \(X_L/X_R, Y_L/Y_R, Z_L/Z_R\) isthe X/Y/Z position of main arm left/right end joint in the base coordinate frame, respectively. \(\theta_1\) is the angle of main arm yaw rotation, \(\theta_2\) is the angle of main arm pitch rotation.

Given that the designing feasibility for the joint structure layout, we assumption that \(-90^\circ \leq \theta_1 \leq 90^\circ, 0^\circ \leq \theta_2 \leq 90^\circ\). In Fig. 2, we can get the equation from the geometric relationships as following:

\[L^2=(d_{3L}X_L)^2+(d_{3L}Y_L)^2+(d_{3L}L)^2\]  

(3)

\[L^2=(d_{3R}X_R)^2+(d_{3R}Y_R)^2+(d_{3R}L)^2\]

(4)

In Formula (3) and (4), \(d_{3L}/d_{3R}\) is respectively the displacement of left/right slider end joint moving along left/right sliderway.

Substituting the Formula (1) into (3) and Formula (2) into (4), we can get the relationships as following:

\[d_{3L}=-A+ a_1\cos\theta_1+ a_2\sin\theta_1 \cos\theta_2\]

(5)

\[d_{3R}=-B+ a_1\cos\theta_1+ a_2\sin\theta_1 \cos\theta_2\]

(6)

In Formula (5) and (6),

\[A = \sqrt{L^2 - (a_3 - a_1\sin\theta_1 - d_2\cos\theta_1 - a_2\sin\theta_1 \cos\theta_2)^2} - (d_1 + a_2\sin\theta_2)^2}\]

\[B = \sqrt{L^2 - (-a_3 - a_1\sin\theta_1 + d_2\cos\theta_1 - a_2\sin\theta_1 \cos\theta_2)^2} - (d_1 + a_2\sin\theta_2)^2}\]

When \(\theta_1 = 0^\circ, \theta_2 = 0^\circ, d_{3L} = d_{3R}\), from Formula (5) and (6), we can get the relationships as following:

\[L^2=(d_{3L}a_1a_2)^2+(a_3d_2)^2+d_1^2\]

(7)
When \( \theta_1 = 0^\circ, \theta_2 = -90^\circ, d_{3L} = d_{3R} \), from Formula (5) and (6), we can get the relationships as following:

\[
L^2 = (d_{3L} \cdot a_1)^2 + (a_3 \cdot d_2)^2 + (d_1 + a_2)^2
\]  
(8)

When \( \theta_1 = 90^\circ, \theta_2 = 0^\circ, d_{3L} = d_{3R} \), from Formula (5) and (6), we can get the relationships as following:

\[
L^2 = (d_{3L} + d_2)^2 + (a_3 \cdot a_1)^2 + d_1^2
\]  
(9)

\[
L^2 = (d_{3R} - d_2)^2 + (a_3 + a_1)^2 + d_1^2
\]  
(10)

When \( \theta_1 = 90^\circ, \theta_2 = -90^\circ, d_{3L} = d_{3R} \), from Formula (5) and (6), we can get the relationships as following:

\[
L^2 = (d_{3L} + d_2)^2 + (a_3 \cdot a_1)^2 + (d_1 + a_2)^2
\]  
(11)

\[
L^2 = (d_{3R} - d_2)^2 + (a_3 + a_1)^2 + (d_1 + a)^2
\]  
(12)

From Formula (7) to (12), when \( (a_3 + a_1) \geq d_1 \), to ensure the base yaw joint angle can reach \(-90^\circ \leq \theta_1 \leq 90^\circ\), and the base pitch joint angle can reach \(0^\circ \leq \theta_2 \leq 90^\circ\), the length \( L \) of left/right link must meet the following requirements:

\[
L^2 \geq (a_3 + a_1 + a_2)^2 + d_1^2
\]  
(13)

In order to make the manipulator more compact and lighter, we can set \( L \) as following:

\[
L = \sqrt{(a_3 + a_1 + a_2)^2 + d_1^2}
\]  
(14)

According to the reference[8], we use the principle of virtual work. Under some assumptions, there are two linear forces \( F_L \) and \( F_R \) respectively act on the left and right slider end joint, which can cause two small differential displacement \( \delta d_{3L} \) and \( \delta d_{3R} \). The virtual work is as following:

\[
\partial W = F^T \partial d_3
\]  
(15)

In Formula (15), \( F^T = [F_L \ F_R] \), \( \partial d_3 = [\partial d_{3L} \ \partial d_{3R}]^T \).

At the same time, it also causes the output torque \( T_1 \) and \( T_2 \) with rotating small differential angle \( \theta_1 \) and \( \theta_2 \) of main arm yaw and pitch joint, respectively. The virtual work is:

\[
\partial W = T^T \partial \theta
\]  
(16)

In Formula (16), \( T^T = [T_1 \ T_2] \), \( \partial \theta = [\partial \theta_1 \ \partial \theta_2]^T \).

From principle of virtual work, the Formula (15) equals (16) as following:

\[
F^T \partial d_3 = T^T \partial \theta
\]  
(17)

From Formula (5) and (6), we can simply express the relationship \( d_3 = d_3(\theta) \), then we can establish the velocity relationship:

\[
\dot{d}_3 = J(\theta) \dot{\theta}
\]  
(18)

In Formula (18), \( \dot{d}_3 = [d_{3L} \ d_{3R}]^T \), \( \dot{\theta} = [\dot{\theta}_1 \ \dot{\theta}_2]^T \). \( J(\theta) \) is the velocity Jacobian matrix as following:

\[
J_{ij}(\theta) = \frac{\partial d_{3i}(\theta)}{\partial \theta_j}
\]  
(19)

From Formula (18), we can get:

\[
\partial d_3 = J(\theta) \partial \theta
\]  
(20)
Solving the simultaneous equations (17) and (20), we can get:

\[ T = J(\theta)^T F \]  

From Formula (19) and (21), we can get:

\[
\begin{align*}
T_1 &= \frac{\partial d_{3L}}{\partial \theta_1} F_L + \frac{\partial d_{3R}}{\partial \theta_1} F_R \\
T_2 &= \frac{\partial d_{3L}}{\partial \theta_2} F_L + \frac{\partial d_{3R}}{\partial \theta_2} F_R 
\end{align*}
\]  

In Formula (22), \( T_i \) is the yaw torque of main arm, which is used for overcoming the friction and rotation inertia moment, and \( T_i \) is much smaller than \( T_2 \) when the yaw rotation acceleration is low. \( T_2 \) is the pitch torque of main arm, which is used for lifting heavy load, and it importantly measures the load capacity of the manipulator. So we emphatically analyze the key structure parameters that how to influence on the \( T_2 \). For simplicity, we make \( \theta_1 = 0^\circ, 90^\circ \) and \( \theta_2 = 0^\circ, 90^\circ \) to formulate the base pitch torque of main arm at four limit positions.

### 3.3. Analysis of parameters

When \( \theta_1 = 0^\circ, \theta_2 = 0^\circ \), substituting \( F_L = F_R = F_1 \) into Formula (21), we can get the base pitch torque \( T_2 \) of main arm:

\[ T_{21} = \frac{2d_1 F_1}{\sqrt{e^2 + 3e + 2eg + (2fg - f^2) + 2g + 1}} \]  

In Formula (23), \( e = a_1/a_3, f = d_1/a_3, g = a_2/a_3 \). Given the same input force \( F_1 \), in order to output a higher torque \( T_2 \), we should make \( d_1 \) as large as possible, and make \( e, g, 2fg,f^2 \) as small as possible. So the parameters should as following:

\[
\begin{align*}
a_2 &\gg a_1 \\
a_2 &\gg a_3 \\
a_3 &= d_2 \\
d_1 &\gg 0
\end{align*}
\]  

When \( \theta_1 = 0^\circ, \theta_2 = 90^\circ \), substituting \( F_L = F_R = F_2 \) into Formula (21), we can get the base pitch torque \( T_2 \) of main arm:

\[ T_{22} = -2d_2 F_2 \]  

Given the same input force \( F_2 \), in order to output a higher torque \( T_2 \), we should make \( a_2 \) as large as possible. So the parameters should as following:

\[ a_2 >> 0 \]  

When \( \theta_1 = 90^\circ, \theta_2 = 0^\circ \), from Formula (21), we can get the base pitch torque \( T_2 \) of main arm:

\[
T_{23} = \frac{d_1 a_2}{\sqrt{L^2 - (a_3 - a_1 - a_2)^2 - d_1^2}} F_L + \frac{d_1 a_2}{\sqrt{L^2 - (a_3 + a_1 + a_2)^2 - d_1^2}} F_R 
\]  

Substituting Formula (14) into Formula (27), we can get:

\[
\frac{d_1 a_2}{\sqrt{L^2 - (a_3 - a_1 - a_2)^2 - d_1^2}} F_L = \frac{d_1 F_L}{\sqrt{4eg + 4g}} 
\]  

The right second expression of Formula (27) is infinite, so it is a singularity position for this manipulator. When \( \theta_1 = 90^\circ, \theta_2 = 90^\circ \), from Formula (21), we can get the base pitch torque \( T_2 \) of main arm:

\[
T_{24} = \frac{a_3 - a_1}{\sqrt{4eg + 2e + 2g - 2w}} F_L + \frac{-(a_3 + a_2)}{\sqrt{2e + 2g - 2w}} F_R 
\]
If \( F_L < 0 \) and given the same input force \( F_L \) and \( F_R \), in order to output a higher torque \( T_2 \), we should make parameters as following:

\[
\begin{align*}
  a_2 &> a_1 \\
  a_2 &> a_3 \\
  a_3 &> a_1 \\
  d_1 &\leq a_1 + a_3
\end{align*}
\]

Taking into consideration of Formula (24), (26) and (30), when the manipulator working in the range \(-90^\circ \leq \theta_1 \leq 90^\circ, \ 0^\circ \leq \theta_2 \leq 90\), if getting more driving torque, it should be constrained by the following parameters:

\[
\begin{align*}
  a_2 &> a_1 \\
  a_2 &> a_3 \\
  a_3 &> a_1 \\
  a_3 &= d_2 \\
  d_1 &= a_1 + a_3
\end{align*}
\]

4. Simulation Results

Using Maple/Sim software[9], we build a two DOF mode with five different key structural parameters for the linear driving manipulator (see Table 1), after that we make a simulation for the kinematical and dynamical model.

| Model | \( a_1 \) (m) | \( a_2 \) (m) | \( a_3 \) (m) | \( d_1 \) (m) | \( d_2 \) (m) | \( L_x \) (m) |
|-------|---------------|---------------|---------------|---------------|---------------|----------------|
| P1    | 0.12          | 0.41          | 0.38          | 0.5           | 0.26          | 0.9021         |
| P2    | 0.17          | 0.41          | 0.38          | 0.5           | 0.26          | 0.9525         |
| P3    | 0.17          | 0.41          | 0.38          | 0.55          | 0.38          | 0.9600         |
| P4    | 0.17          | 0.55          | 0.38          | 0.55          | 0.38          | 1.1000         |
| P5    | 0.17          | 0.65          | 0.38          | 0.55          | 0.38          | 1.2            |

Note*: \( L_x = \sqrt{(a_3 + a_1 + a_2)^2 - (a_3 - d_2)^2} \)

To input the same constant yaw torque \( T_1 = \text{1000Nm} \) and constant pitch torque \( T_2 = \text{34000Nm} \) on each model, meanwhile setting the base pitch angle as \( 0^\circ \), \( 15^\circ \), \( 30^\circ \), \( 45^\circ \), \( 60^\circ \), \( 75^\circ \), \( 90^\circ \), also setting the base yaw angle from \(-90^\circ \) to \( 0^\circ \), we plot the changing curves of \( F_L \) and \( F_R \) with simulation from the seven different models.

**Figure 4.** Changing curves of \( F_L (\theta_2=0^\circ) \)

**Figure 5.** Changing curves of \( F_R (\theta_2=0^\circ) \).
Figure 6. Changing curves of $F_L$ ($\theta_2=15^\circ$).

Figure 7. Changing curves of $F_R$ ($\theta_2=15^\circ$).

Figure 8. Changing curves of $F_L$ ($\theta_2=30^\circ$).

Figure 9. Changing curves of $F_R$ ($\theta_2=30^\circ$).

Figure 10. Changing curves of $F_L$ ($\theta_2=45^\circ$).

Figure 11. Changing curves of $F_R$ ($\theta_2=45^\circ$).

Figure 12. Changing curves of $F_L$ ($\theta_2=60^\circ$).

Figure 13. Changing curves of $F_R$ ($\theta_2=60^\circ$).
Figure 14. Changing curves of $F_L (\theta_2 = 75^\circ)$.

Figure 15. Changing curves of $F_R (\theta_2 = 75^\circ)$.

Figure 16. Changing curves of $F_L (\theta_2 = 90^\circ)$.

Figure 17. Changing curves of $F_R (\theta_2 = 90^\circ)$.

It can be seen from Figure 4 to Figure 17, when the manipulator working in the range $-90^\circ \leq \theta_1 \leq 90^\circ$, $0^\circ \leq \theta_2 \leq 90^\circ$, the bigger $a_j$ is, the bigger yaw or pitch driving torque is. When the smaller $a_j$ is and the closer $d_1$ to $a_1 + a_3$ is and the closer value $d_2$ to $a_3$ is, thus the bigger yaw or pitch driving torque is. The simulation results are consistent with Formula (31).

5. Conclusions
In this paper, a two DOF joint mechanical design scheme for heavy-load manipulator based on linear driving is proposed, which has the advantages of high driving capacity, simple structure and light weight. Through theoretical analysis and simulation, we come to a conclusion that when setting the same loader on the base yaw and pitch joint, in order to input lower linear driving force, the parameter $a_2$ should be as maximum as possible, $a_1$ should be as minimum as possible, $d_1$ should be as close to $a_1 + a_3$ as possible, $d_2$ should be as close to $a_3$ as possible. Compared with simulation results, the conclusions are also validated to be reasonable.

6. References
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