THE SCALE OF HOMOGENEITY IN THE LAS CAMPANAS REDSHIFT SURVEY

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ABSTRACT

We analyze the Las Campanas Redshift Survey using the integrated conditional density (or density of neighbors) in volume-limited subsamples up to unprecedented scales (200 Mpc $h^{-1}$) in order to determine without ambiguity the behavior of the density field. We find that the survey is well described by a fractal up to 20–30 Mpc $h^{-1}$ but flattens toward homogeneity at larger scales. Although the data are still insufficient to establish with high significance the expected homogeneous behavior, and therefore to rule out a fractal trend to larger scales, a fit with a cold dark matter–like spectrum with high normalization well represents the data.

Subject headings: galaxies: clusters; general — large-scale structure of universe

Following the seminal work of L. Pietronero and coworkers (see, e.g., Pietronero, Montuori, & Sylos Labini 1997), the possibility of a large-scale fractal distribution of the galaxies has been investigated by various authors. In the current literature, however, there are several conflicting estimates of the largest scale at which the galaxy distribution can be approximated by a fractal, ranging from a few megaparsecs (see, e.g., Peebles 1993) to 20 Mpc $h^{-1}$ (Davis 1997) to 40 or 50 Mpc $h^{-1}$ (Guzzo 1997; Cappi et al. 1998) up to more than 100 Mpc $h^{-1}$ (see, e.g., Pietronero et al. 1997). A fractal distribution with dimension $D$ is characterized by the property that the correlation function

$$g(r) = 1 + \xi(r) \quad (1)$$

decreases as a power law, $\sim r^{3-D}$. Consequently, the average density $\rho_{av}$ of galaxies at distance $r$ from another galaxy, or the conditional density, also decreases as $\sim r^{3-D}$ since, by the definition of a correlation function, $\rho_{av} = \rho_0[1 + \xi(r)]$, where $\rho_0$ is the cosmic average density.

Naturally, one can infer the scale at which fractality gives way to homogeneity from several other observations, like the cosmic backgrounds, although the conclusions are bound to be model dependent. The availability in recent years of deep redshift surveys finally allows us to study the matter distribution directly from its primary tracers, the galaxies. The deepest galaxy redshift survey so far published is the Las Campanas Redshift Survey (LCRS; Schectman et al. 1996). LCRS contains 23,697 galaxies with an average redshift of $z = 0.1$, distributed over six $125 \times 80'$ slices. In this Letter, we determine the behavior of $g(r)$, the volume integral of $g(r)$, and of the fractal dimension $D(r)$ in volume-limited (VL) subsamples of LCRS up to $r = 200$ Mpc $h^{-1}$, the largest scale so far investigated with such a statistic, by making use of purposely designed cells. This scale is more than 4 times the scale previously reached by Cappi et al. (1998) using the Southern Sky Redshift Survey 2 (SSRS2).

Let us begin by discussing why the statistic $g(r)$ is particularly convenient for our purposes. By far, the most popular two-point estimators that have been used to investigate the clustering of the galaxies are the correlation function $\xi(r)$ and the power spectrum $P(k)$, a Fourier conjugate pair. However, the simplest statistic to use in order to determine the fractal properties of a distribution is $g(r)$, or $g(r) = V^{-1} \int V g(r) dV$, in terms of which a fractal is defined. There is also another reason to use $g(r)$ to study the galaxy clustering. The spectrum of a finite survey is actually the convolution of the true power spectrum with the survey geometry. As a consequence, both the shape and the amplitude of the estimated spectrum are different from the true spectrum. In particular, the estimation of the average density from the sample itself forces the spectrum to vanish for $k \to 0$, so that the detection of a turnaround in the spectrum is often suspect (see, for instance, Sylos Labini & Amendola 1996). A similar problem occurs with the correlation function: the integral constraint (Peebles 1980) forces the correlation function to become negative at some scale, distorting its shape. In fact, using a subscript $s$ to denote quantities estimated in a finite sample of size $R_s$, and employing the fact that the density $\rho_s$ in a sample around an observer is a conditional density, the following relation holds:

$$\xi_s(r) = \frac{\xi(r) - \xi(R_s)}{1 + \xi(R_s)} \quad (2)$$

which shows explicitly the integral constraint $\int \xi_s dV = 0$. This problem is clearly absent from the statistic $g(r)$ because $g$ and $g_s$ are simply proportional, $g_s(r) = g(r) g_s(R_s)$. It follows that, contrary to what happens to $\xi$ and $P(k)$, the slope of $g_s$ is an unbiased estimator of the slope of the clustering trend, i.e., of its fractal dimension. In fact, we can evaluate the fractal dimension as

$$D = 3 + \frac{d \log \hat{g}}{d \log r} \quad (3)$$

and verify that $D_s = D$. Therefore, to test the claim of fractality, it is necessary to check whether or not $g(r)$ can be approximated by a power law, in which range of scales, and with which slope. The approach to homogeneity is then characterized by $D \to 3$, i.e., to a flattening of $g(r)$.

The statistic $g(r)$ is a differential quantity. When applied to surveys with a relatively small density, like the volume-limited slices of Las Campanas, it tends to be very noisy. In this case, one can smear out the noise by integrating over the cells, thus obtaining the integrated correlation $\hat{g}(r)$. Here an important problem arises. Let us consider a distribution of particles described by a statistically isotropic correlation function $\xi(r)$. When we evaluate the expected total number of neighbors...
within a distance \( r \), we are performing the integral

\[
\hat{\xi}(r) = \frac{3}{r^3} \int_0^r \xi(r')r'^2\,dr'.
\] (4)

If the cell is not spherical, the integral becomes

\[
\hat{\xi} = \int \xi(r')W(r', \theta, \phi)dV,
\]

where the window function \( W(r, \theta, \phi) \) is defined as constant inside the cell and zero outside and is normalized to unity. Of course, only the part of the cell completely contained within the survey has to be considered. However, if the cells are not spherically symmetric, the value of \( \hat{\xi}(r) \) depends on the exact form of \( W(r, \theta, \phi) \) and, in general, is different from the definition in equation (4). Moreover, \( W(r, \theta, \phi) \) may vary from cell to cell. The obvious solution to this problem is to restrict the analysis to spherical cells. However, this limits the scale to the largest sphere contained within the survey boundaries, which, for most surveys, can be very small (e.g., less than 10 Mpc \( h^{-1} \) by radius for LCRS). So far, as a matter of fact, all the works that used the \( \hat{g} \) statistic adopted spherical cells, thereby limiting the scales to less than 50 Mpc \( h^{-1} \). Since at this scale the fractal behavior is more or less within the standard description (e.g., cold dark matter [CDM]), it is crucial to extend the analysis to larger scales. The simplest way to do so is to consider radial cells, i.e., cells whose window function can be factorized in a radial and an angular function, both normalized to unity:

\[
W(r, \theta, \phi) = W_r(r)W_\theta(\theta, \phi).
\] (5)

In this case, in fact, the function \( \hat{\xi}(r) \) is the same as in spherical cells, since the angular factor can be integrated out. The advantage is that one can design a radial cell so as to maximize the scale \( r \), thus still fitting it within the survey. As elementary as it is, this method has never been used in the literature on galaxy clustering.

Before going to the data analysis, let us estimate the variance of \( \hat{g} \). Let us first assume that the three-point correlation function can be written as (Peebles 1980)

\[
\xi_{ijk} = \langle \hat{\xi}(\xi_i, \xi_j, \xi_k) \rangle = \xi_i \xi_j \xi_k + \xi_i \xi_j \xi_k + \xi_i \xi_j \xi_k,
\]

where \( \xi \) is independent of the spatial coordinates. Then the variance of \( \hat{g} \) evaluated in \( N_0 \) independent cells containing, on average, \( N_0 \) galaxies is (Peebles 1980; Amendola 1998)

\[
\sigma_g^2 = N_0^{-1} \left[ N_0^{-1}(1 + \hat{\xi}) + \sigma^2 - \xi^2 + Q(\xi^2 + 2K_2) \right],
\] (6)

where \( \sigma^2 = \int W_i dV W_j dV \xi_{ij} \xi_{ij} \). The first term in equation (6) is the Poisson noise. Inserting the power spectrum, we have

\[
\sigma^2 = (2\pi)^{-2} \int P(k)W(k)k^2\,dk,
\] (7)

where \( W(k) = W(k) \) for spherical cells but has to be evaluated numerically in the more general case that we study here (see Amendola 1998). The last term in equation (6) can be written as \( K_2 = \int W_i dV W_j dV \xi_{ij} \xi_{ij} \) (Peebles 1980). In the important case in which \( \hat{\xi} \) is a power law, it can be shown that a very good approximation is

\[
K_2 = \alpha^2 \hat{\xi}^2.
\]

For instance, if \( \hat{\xi} \sim r^{-1} \), it turns out that \( K_2 = 1.04\alpha^2 \hat{\xi}^2 \). In the following, we will always approximate \( K_2 \) in this way.

Another problem arises in practice, namely, that the cells we use are not independent, both because they are partially overlapping and because the clustering scale may be larger than the distance from cell to cell. The effect of the correlation is to reduce the number \( N_c \) at the denominator in equation (6). For instance, if the cells oversample the volume by a factor of 2, it means that one cell out of two is redundant, and the effective number of cells can be taken as \( N_c/2 \). In general, the number of effective independent cells may be approximated as

\[
N_c = \min(N_c, V/V_c),
\]

where \( V \) is the cell volume, although of course even this is an overestimation of the independent cells. Naturally, we could use \( N \)-body simulations to estimate the errors, including the cell-to-cell correlation, but then we should generate a different simulation for any model we want to compare it with; moreover, any finite \( N \)-body will inevitably cut the large-scale power, which, in the case of testing fractals, is a particularly severe limitation. Some comparisons with \( N \)-body simulations show that equation (6) underestimates the errors by only 30% at most.

In the case of an exact fractal, the expected value of \( \hat{g} \) inside a spherical region of radius \( r \) that is embedded in a larger box of size \( R_c \) is (Coleman & Pietronero 1992)

\[
\hat{g}(r) = (r/R_c)^{D-3}.
\] (8)

It is not difficult to show that, neglecting \( \sigma^2 \) and \( \hat{\xi} \) with respect to \( \xi^2 \), i.e., in the limit of \( R_c \gg r \), and neglecting the Poisson noise, the variance in a fractal is

\[
\sigma_g^2 \approx N_c^{-1} \left[ Q \left( 1 + \frac{2D\gamma(\gamma)}{3} \right) - 1 \right] g^2,
\] (9)

where \( \gamma = 3 - D \) and \( J_2(\gamma) = 72/[3(3 - \gamma)(4 - \gamma)(6 - \gamma)2^\gamma] \). Notice that the relative error on \( \hat{g} \) is independent on the scale, which, as a matter of fact, is found numerically (Amici & Montuori 1998). For instance, for \( Q = 1 \) and \( D = 2 \), as some observations suggest, \( \sigma_g^2 \approx N_c^{-1}(8/5)g^2 \).

An important consequence of equation (9) is that the relative error of the conditional density measured in a single cell can be very large for a fractal (more than 100%). Then the average density in a sample of galaxies around us, a conditional density, has such a large variance, in a fractal, that in practice it gives no information. A further consequence is that the variance of the amplitude of the correlation function, and of related quantities like \( \xi \) and \( \sigma_g \), makes the use of the correlation amplitude, as opposed to its slope, useless in the case of fractals. This problem applies also to the case in which the density of a sample \( n(r) \) is measured as a function of the distance from the observer, without averaging over several cells.

If we define the scale of homogeneity as the scale at which \( \hat{g} \) flattens so that \( D \geq 2.9 \), then we can quantify it in any given CDM model. Clearly, the higher the normalization \( \sigma_g \), the larger this scale will be. It turns out that if \( \sigma_g \approx 1.5 \), as observed for bright galaxies in SSRS2 (Benoist et al. 1996), the CDM homogeneity scale can be as large as 50 Mpc \( h^{-1} \) and reach \( \approx 70 \) Mpc \( h^{-1} \) for clusters (\( \sigma_g \approx 2 \)); therefore, the gap between the pure fractal model and the standard scenarios begins to be significant only at scales larger than 50 Mpc \( h^{-1} \).

The \( \hat{g} \) statistic in spherical cells has been applied to several galaxy surveys (Pietronero et al. 1997; Sylos Labini & Montuori 1998; Sylos Labini et al. 1998; Cappi et al. 1998). Here we summarize the results from only the deepest of such surveys, SSRS2 (da Costa et al. 1994). SSRS2 includes 3600 galaxies in 1.13 sr of the southern sky, down to an apparent magnitude of 15.5. The results of the analysis in Cappi et al. (1998) indicate that the conditional density decreases as \( r^{-1} \) from 1 to 40 Mpc \( h^{-1} \) in all the volume-limited samples con-
Fig. 1.—Two of the VL subsamples of LCRS that we analyze in this work.

Table 1: VL Subsamples

| Sample      | \(\langle M \rangle\) | \(N_{\text{tot}}\) | \(\Gamma\) | \(\sigma_{g|a}\) |
|-------------|----------------------|-------------------|-----------|----------------|
| 147-297     | -20.43               | 878               | 0.10      | 1.10           |
| 190-330     | -20.75               | 1081              | 0.55      | 1.55           |
| 224-437     | -21.34               | 810               | 0.30      | 1.50           |
| 280-410     | -21.26               | 793               | 0.40      | 1.65           |

We applied our method of the radial cells to the LCRS, the deepest redshift survey so far studied. LCRS contains fields that include galaxies with magnitudes between 16.0 and 17.3 and fields with limits between 15.0 and 17.7. Every field \(i\) has an associated sampling factor of \(0 < f_i < 1\), which is the fraction of the galaxies randomly chosen out of the total number in the field within the magnitude limits; the weights \(1/f_i\) must be taken into account in the statistics. We considered the only slice with all fields of large magnitude range (slice at \(-12^\circ\)). We evaluated the conditional density integrated in radial cells with a shape and orientation such as not to intersect the survey boundaries (Palladino 1997). We cut the sample into four VL subsamples, denoted as VL 147-297, VL 190-330, VL 280-410, and VL 224-437 (see Fig. 1 and Table 1), where the numbers give the lower and upper cutoff distance (the two cuts are necessary because LCRS has two limiting magnitudes). The use of volume-limited samples avoids the uncertainties connected to the radial selection functions. The correlation function in the full magnitude-limited sample is evaluated in Tucker et al. (1997). The results for \(g(r)\) are shown in Figure 2. The errors are calculated from equation (6) using two models: the standard CDM, with galaxy normalization \(\sigma_{g|a} = 1.5\) and \(\Gamma = 0.3\), and the pure fractal, with \(D = 2\) and \(Q = 1\). For comparison, we evaluated the errors from a simulation of a standard CDM model and found that our theoretical estimate of the errors is approximate to better than 30%. It can be seen that there is an approximate \(D = 2\) fractality on small scales, up to 20 or 30 Mpc \(h^{-1}\), just as in the SSR S2 case, followed by a flattening of the slope. The results from SSR S2 of Cappi et al. (1998; sample with magnitude cut at \(-20\)), obtained with full spheres, are reported in Figure 2 along with VL 147-297, the LCRS sample with an average magnitude closer to SSR S2. As we can see, the scales we reach here are the largest scales ever reached for the \(g(r)\) statistic. In the case of VL 280-410, the

Fig. 2.—The function \(g(r)\) in various VL samples of LCRS with the errors expected in a CDM model and in a fractal \(D = 2\) model (dotted lines). The thin line is the tentative CDM fit (see text). In the upper right panel, we also report \(g(r)\) from a sample of SSR S2 for comparison, from Cappi et al. (1998).
trend is decreasing, albeit with a change in slope, down to more than 100 Mpc $h^{-1}$, while for VL 224-437 (the most sparse sample), a very noisy flattening is reached already at 40 Mpc $h^{-1}$. In Figure 3, we plot the fractal dimension $D$ as a function of $r$, evaluated as the least square slope of sets of five contiguous points of $\hat{g}(r)$. The tendency to $D = 3$ is very clear. It is also seen that the dimension is not really constant in any range, although the scatter is too large to infer a clear trend.

From these results, we can conclude that there is a tendency to homogenization at around 50–100 Mpc $h^{-1}$, as expected from a CDM model. However, we remark that we did not detect clear homogeneous behavior, i.e., $\hat{g}(r) = 1$, not even at more than 100 Mpc $h^{-1}$, except for the most sparse sample. This again leaves space for a fractal behavior with larger scales, especially in view of the large errors associated with a fractal. The dimension, however, would be closer to 3 at large scales.

We compared our results with the CDM-like power spectrum

$$P(k) = A k^2 T^2(\Gamma, k) G(\Omega_m, \Omega_\Lambda, \sigma_8, \sigma_m, \sigma_g) ,$$

where $T^2(\Gamma, k)$ is the transfer function of Bond & Efstathiou (1984), $G$ includes the redshift correction of Peacock & Dodds (1994) and the nonlinear correction of Peacock & Dodds (1996), $\sigma_g$ is the line-of-sight velocity dispersion, and the subscripts $\Lambda$, $g$, and $m$ refer to the cosmological constant, the galaxies, and the total matter, respectively. In Figure 2, we compare LCRS with a tentative model with $\Gamma = 0.24$ and $\sigma_8 = 1.4$, assuming for the other parameters, the values $\sigma_m = 1$, $\Omega_{\text{tot}} = 1$, $\Omega_m = 0.4$, $\Omega_\Lambda = 0.6$, $h = 0.6$, and $\sigma_g = 300$ km s$^{-1}$. Finally, we find the best fit to $\hat{g}$ by varying $\Gamma$ and $\sigma_8$ for scales larger than 30 Mpc $h^{-1}$, in order to avoid the nonlinear corrections at smaller scales. The results are listed in Table 1. The average for all four samples is $\sigma_8 = 1.5$ and $\Gamma = 0.3$.

To summarize, in this Letter, we have applied the technique of radial cells to LCRS, pushing the analysis of fractality to 200 Mpc $h^{-1}$. We have shown that a proper treatment of the errors is important, in order to compare alternative models, and is particularly crucial for a fractal, in which the variance tends to be very large. We have also shown that the use of radial cells allows us to probe at very large scales. The conclusion is that LCRS is well described by a CDM model with a high normalization, up to $\sigma_g \approx 1.6$ for the deepest and thus brightest sample. The trend can also be approximated as a $D \approx 2$ fractal up to 20–30 Mpc $h^{-1}$ but shows a clear flattening afterward.

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Fig. 3.—The function $D(r)$ for the four samples, obtained as the least square slope of sets of five contiguous values of $\hat{g}(r)$. 

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