Explaining The New CDFII W-Boson Mass In The Georgi-Machacek Extension Models

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Abstract: Georgi-Machacek model, which can preserve the custodial SU(2)_c symmetry at tree level, can hardly account for the new CDF II data on W-boson mass in its original form. As anticipated, unless tree level SU(2)_c custodial symmetry breaking effects are present, the new physics contributions Δm_W is always very small. Our numerical results show that ordinary GM model can contribute to Δm_W at most 0.0009 GeV, which can not explain the new CDF II data on W boson mass. We propose to introduce small misalignment among the triplet VEVs to increase Δm_W, which can reach 0.05 GeV for v_ξ - v_χ ~ 6 GeV, marginally explain the new data in 3σ range. We also propose to extend the GM model with low scale RH neutrino sector, which can adopt the leptogenesis mechanism and act as a SU(2)_c breaking source because of its correlation with h_{ij}. With low scale RH neutrino mass scale of order 100-1000 TeV, the new physics contribution Δm_W can reach 0.02 GeV. Combining both small SU(2)_c breaking effects, the 1σ range of CDF II data on W boson mass can be obtained even for tiny splitting among the triplet VEVs with v_ξ - v_χ ≈ 1 GeV.
1 Introduction

The standard model (SM) had already been corroborated by various contemporary collider experiments, including the 125 GeV Higgs boson by the Large Hadron Collider (LHC) [1, 2]. Despite its success, it still has many theoretical and aesthetical problems, such as the origin of tiny neutrino masses. Besides, new reported measurement of the W boson mass by the CDF II detector at the Fermilab Tevatron collider [3] gives

$$M_W = 80,433.5 \pm 6.4(stat) \pm 6.9(syst) = 80,433.5 \pm 9.4 MeV/c^2 ,$$ (1.1)

using data corresponding to $8.8 fb^{-1}$ of integrated luminosity collected in proton-antiproton collisions at a 1.96 TeV center-of-mass energy. This measurement is in significant tension with the standard model expectation [4]

$$M_W = 80,357 \pm 4(inputs) \pm 4(theory) MeV/c^2 .$$ (1.2)

If such value of the new W boson mass persist and get confirmed by other experiments, they will indicate the existence of new physics beyond SM [5]. Many discussions had been given on this new CDFII W-boson data [6].

Electroweak symmetry breaking (EWSB) mechanism with extended Higgs sectors is still allowed, given the uncertainties in Higgs boson coupling measurements. So the problems of SM can possibly be explained in the Higgs extended models, such as the extended Georgi-Machacek (GM) model [7–36]. On the other hand, any extended Higgs sector must be carefully constructed to satisfy the stringent constraints from electroweak precision data, the most important one of which is the $\rho$ parameter. We know that tree level relation $\rho_{tree} = 1$ is automatically satisfied by Higgs sector of SM, which can respect the custodial $SU(2)_c$ global symmetry. GM model, which augments the SM Higgs sector with a complex $SU(2)_L$ triplet of hypercharge $Y = 1$ and a real $SU(2)_L$ triplet of $Y = 0$, can protect at tree level the custodial $SU(2)_c$ symmetry. In the GM model, a larger triplet VEV $v_\Delta$ of order $\mathcal{O}(10)$ GeV is allowed due to the custodial symmetry, unlike the minimal Higgs triplet model (HTM), whose $v_\Delta$ is constrained to be much smaller because of its violation of $SU(2)_c$ at tree level. GW model can provide tiny mass to neutrinos via the Type-II neutrino seesaw mechanism.

However, we anticipate that the new physics contribution $\Delta m_W$ in GM model should be small because of the tree-level custodial $SU(2)_c$ symmetry, which protects the $\rho$ parameter from large deviations from unity. So, in order to increase $\Delta m_W$, explicitly $SU(2)_c$ symmetry breaking terms should be included. We propose to spoil the tree-level $SU(2)_c$ with slightly misalignment between the two triplet VEVs. We also propose an alternative GM extension model which augment the neutrino sector with low scale right-handed (RH) neutrino and adopt the type I+II seesaw mechanism similar to that in the Left-Right symmetric model. The presence of RH neutrino terms can not only be used to increase the value of $h_{ij}$ within the GM model, but also be used to understand the Baryon Asymmetry in the Universe (BAU). Satisfying Sakharov’s three conditions, leptogenesis mechanism can readily explain the BAU. Net baryon number is generated by out-of-equilibrium L-violating
decays of heavy Majorana neutrinos. Besides, low scale RH neutrino with large Yukawa couplings to leptons can have rich collider phenomenology.

This paper is organized as follows. In Sec 2, we review the GM model and discuss our low scale RH neutrino extension GM model. In Sec 3, we discuss the new physics contributions to CDFII W-boson mass in GM model and its various extensions. Sec 4 contains our conclusions.

2 GM model

In the GM model [7, 19], the Higgs sector contains the ordinary SM $SU(2)_L$ doublet Higgs field $\Phi$ with hypercharge $Y = 1/2$ and two $SU(2)_L$ triplet Higgs fields $\chi$ with $Y = 1$ and $\xi$ with $Y = 0$. These fields can be written in the form of $SO(4) \simeq SU(2)_L \times SU(2)_R$ symmetry

$$
\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} \chi^0 & \xi^+ & \chi^+ \\ \chi^- & \xi^0 & \chi^0 \\ \chi^- & \xi^- & \chi^0 \end{pmatrix},
$$

(2.1)

where $\sigma^a$ are the Pauli matrices, $T^a$ are the $3 \times 3$ matrix representation of the $SU(2)$ generators, and the similarity transformation relating the $SU(2)$ generators in the triplet and adjoint representations is given by

$$
P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i & 0 \\ i & 0 & \sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}.
$$

The scalar potential can trigger the EWSB by the VEVs

$$
\langle \phi \rangle = \frac{1}{\sqrt{2}} v_\phi, \quad \langle \chi \rangle = v_\chi, \quad \langle \xi \rangle = v_\xi.
$$

(2.3)

When the two triplet VEVs align, $v_\chi = v_\xi \equiv v_\Delta$, the global SO(4) symmetry is reduced to the custodial $SU(2)_C$ symmetry. The EWSB condition

$$
v^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{2G_F} \approx (246 \text{GeV})^2,
$$

(2.4)
can be recasted into $s_H$ variable with $s_H = \sin \theta_H$ for
\[ \tan \theta_H = 2\sqrt{2}v_\Delta / v_\phi. \] (2.5)

Majorana neutrino masses can be obtained from the term
\[ \mathcal{L} \supset h_{ij} \overline{L}_i^c \tau_2 \chi L_j + \text{h.c.}, \] (2.6)
via the type II seesaw mechanism to give
\[ m_\nu \approx h_{ij} v_\Delta, \] (2.7)
with the triplet VEV $v_\Delta$ triggered by the $\mu_1$ term once the doublet $\phi$ gets a VEV to break the electroweak symmetry. For $v_\Delta \sim \mathcal{O}(10)$ GeV, very tiny coupling $h_{ij} \sim 10^{-13}$ is needed to give tiny neutrino mass, which is rather unnatural.

### 2.1 RH-neutrino Extended GM Model

It will be clear soon, ordinary GM model can hardly explain the recent $\Delta m_W$ because of the tree-level $SU(2)_c$ symmetry. So, it is fairly interesting to seek extensions of GM model to account for the new CDFII result on $\Delta m_W$.

We propose to extend the GW model with RH neutrinos, which is given by
\[ \mathcal{L} \supset y_{ij} \overline{L}_i \phi N_R + \frac{1}{2} (M_R)_{ij} N_R^T C N_R, j + h_{ij} \overline{L}_i^c \tau_2 \chi L_j + \text{h.c.}, \] (2.8)

After EWSB, it will lead to the neutrino mixing mass matrix of the form
\[ M_\nu = \begin{pmatrix} h_{ij} v_\Delta & (y_{ij}^N)^T v_\phi \\ y_{ij}^N v_\phi & (M_R)_{ij} \end{pmatrix}. \] (2.9)

For simply, we choose $(M_R)_{ij} = M_{R,i} \delta_{ij}$ up to possible phases. So, the neutrino mass can be given as
\[ m_\nu \approx h_{ij} v_\Delta - v_\phi^2 (y_{ij}^N)^T M_R^{-1} (y_{jk}^N), \] (2.10)
for $M_R \gg v_\phi$, which is a mixed type I+II seesaw mechanism. Tiny neutrino mass of order $10^{-3}$ eV requires the cancelation
\[ h_{ij} v_\Delta \approx \frac{(y_{ij}^N v_\phi)^2}{M_R}. \] (2.11)

For $y_{ij}^N \sim \mathcal{O}(1)$, $M_R \sim \mathcal{O}(1)$ TeV and $h_{ij} \sim \mathcal{O}(1)$, tiny neutrino mass requires $v_\Delta \sim \mathcal{O}(10)$ GeV, which is naturally allowed by GM model. The neutrino mass matrix can be diagonalized by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $V_{PMNS}$, so the form of $h_{ij}$ should take the form
\[ h_{ij} = 2\sqrt{2} V_{PMNS}^T \frac{v(1 - s_H^2)}{s_H M_{R,i}} \delta_{ij} V_{PMNS}. \] (2.12)
The RH neutrino coupling form should take $y_{ij} = V_{PMNS}^{-1}$. However, large $|h_{ij}| \sim \mathcal{O}(1)$ is constrained stringently by lepton flavor violation (LFV) processes. In fact, the main LFV signatures of the seesaw mechanism stem from muon decay with the current bounds $BR(\mu \to e\gamma) < 4.2 \times 10^{-13}, BR(\mu \to 3e) < 10^{-12}$. Other LFV processes, such as $\mu - e$ conversion and LFV decays involving $\tau$ lepton, are subdominant. For scalar triplets of order 1 TeV, the typical magnitude of $|h_{ij}|$ is constrained to be less than the order $10^{-2}$ from $BR(\mu \to e\gamma)$ because

$$BR(\mu \to e\gamma) \sim \frac{\alpha_{EM}}{192\pi} |h_{ij}|^4 \left(\frac{m_W}{M_{H^{++}}}\right)^4.$$  \hspace{1cm} (2.13)

Due the correlation between $h_{ij}$ and $M_R$ in eq(2.12), the scale of $M_R$ should typically be heavier than 50 TeV with $s_H < 0.2$.

We should note that, in ordinary GM model, the coupling $h_{ij}$ should be very tiny of order $10^{-13}$. With the augmentation of the RH neutrino sector, the allowed value of $h_{ij}$ can be greatly increased. On the other hand, relatively large fine-tuning (FT) is still needed in the cancelation to get the observed tiny neutrino masses. The larger the $M_R$ value, consequently a smaller $h_{ij}$ coupling term ( indicating a smaller $SU(2)_C$ breaking source ), the smaller the FT is needed.

The introduction of RH majorana neutrino can be advantageous in cosmology. We would like the heavy Majorana neutrinos to be responsible for both the observed BAU and the neutrino oscillation data. In this context, it has been found [37, 38] that if the heavy singlet neutrinos have an hierarchical mass spectrum, a lower bound of about $10^8 \sim 10^9$ GeV on the leptogenesis scale can be derived. To obtain this lower bound, the size of the leptonic asymmetry between the heavy Majorana neutrino decay $N \to L\phi$ and its respective CP conjugate plays a key role. It has been shown in [37] that the leptonic CP asymmetry is not only analytically well-behaved, but also it can be of order unity if two of the heavy Majorana neutrinos have mass differences comparable to their decay widths. Due to this resonant effect, the bound on the RH neutrino mass scale from the requirement of generating sufficient lepton number asymmetry in resonant leptogenesis scenario can be significantly lower, even $M_1, M_2 \sim$ TeV are allowed to obtain sufficient baryogenesis. So, the $\mathcal{O}(100)$ TeV scale RH neutrino extension of GM model can still adopt the leptogenesis mechanism to provide enough baryon asymmetry.

3 New Contributions to $\Delta m_W$

The CDF II data on new W boson mass needs additional new contributions $\Delta m_W$ from new physics. Such new contributions to the W-boson masses can be calculated with the $S,T,U$ parameters [39–41]. Knowing the oblique parameters, one can obtain the shift of W-boson mass by new physics contributions in terms of the $S,T,U$ [39, 42] parameters

$$\Delta m_W = \frac{\alpha M_W}{2(c_W^2 - s_W^2)} \left(-\frac{1}{2} S + \frac{c_W^2}{2} T + \frac{c_W^2 - s_W^2}{4s_W^2} U \right),$$  \hspace{1cm} (3.1)
with
\[
\alpha S = 4 s_w^2 c_w^2 \left[ \Pi_{ZZ}^{'}(0) - \frac{c_w^2 - s_w^2}{s_w c_w} \Pi_{Z\gamma}^{'}(0) - \Pi_{\gamma\gamma}^{'}(0) \right],
\]
\[
\alpha T = \frac{\Pi_{WW}(0) - \Pi_{ZZ}(0)}{m_Z^2},
\]
\[
\alpha U = 4 s_w^2 \left[ \Pi_{WW}^{'}(0) - c_w^2 \Pi_{ZZ}^{'}(0) - 2 s_w c_w \Pi_{Z\gamma}^{'}(0) - s_w^2 \Pi_{\gamma\gamma}^{'}(0) \right],
\]
and \(\alpha^{-1}(0) = 137.035999084, s_w^2 = 0.23126.\)

We plot in fig.1 the possible values of \(S, T\) with \(U = 0\) that can give \(\Delta m_W\) up to \(1\sigma - 4\sigma\) range of new CDF II data. The green box is the \(1\sigma\) constraints on \(S\) and \(T\) from various inputs combined with \(M_Z\) [43]. It is obvious from the figure that unless the previous \(S, T, U\) bounds are relaxed, new contribution to \(\Delta m_W\) in the favored range of \(S, T, U\) can hardly reach \(4\sigma\).

![Figure 1](image_url)

**Figure 1.** Possible values of \(S, T\) with \(U = 0\) that can give \(\Delta m_W\) up to \(1\sigma - 4\sigma\) range of new CDF II data. The red box denotes the range \(S = 0.00 \pm 0.07\) and \(T = 0.05 \pm 0.06\) [43] while the green box denotes the range \(S = -0.01 \pm 0.10\) and \(T = 0.03 \pm 0.12\) [43].

Ordinary GM model is constrained from measurements of SM quantities in collider experiments, such as the electroweak precision tests, the determination of the \(Z\bar{b}b\) coupling, and the measurement of the Higgs boson signal strengths, the unitary bounds. The value of \(T\) in ordinary GM model is not large because of the tree-level custodial \(SU(2)\) symmetry.

\^1Fixing \(U = 0\) is motivated by the fact that \(U\) is suppressed by an additional factor \(M_{new}^2/M_Z^2\) compared to \(S\) and \(T\) [44]. Such a choice can greatly improves the precision on \(S\) and particularly \(T\).
Its concrete value depends on the logarithm of the triplet-like Higgs boson masses and therefore not important at the one-loop level \(^2\).

We scan the parameter space to see if the ordinary GM model can explain the CDF II data on W-boson mass. The code SARAH [45] is used to generate the input model file to link the Spheno [46] package, which has implemented various collider constraints. We require the predicted Higgs mass to lie between 124 GeV and 126 GeV. Bounds from stability of the electroweak vacuum, the unitarity of the perturbation theory are imposed and the value \(s_H\) is required to lie below 0.2. All the survived points need to pass the bounds in HiggsBounds [47], HiggsSignals [48] as well as GMCalc [49].

We show our numerical results in upper panels of fig.2. It can be seen that the survived points can not explain the CDF II data, which can contribute to \(\Delta m_W\) a maximal amount 0.0009 GeV at one loop level. The corresponding heavy Higgs mass \(m_{h_2}\) and the doubly charged Higgs \(m_{H^{++}}\) mass are also shown in the upper right panel. Such a heavy \(H^{++}\) can give very small contributions to \(\Delta a_\mu\) by the Barr-Zee diagrams, which can not account for the muon \(g - 2\) anomaly [50].

![Figure 2](image)

**Figure 2.** We show the new physics contribution \(\Delta m_W\) in ordinary GM model (the upper panels) and the GM model with a small misalignment among the triplet VEVs at the tree level (the lower panels). The dependences of \(\Delta m_W\) on the mass of doubly charged scalar are also shown in both models.

As noted previously, the \(T\) parameter can be increased in the GM model if the \(SU(2)_c\) custodial symmetry.

\(^2\)The \(T\) parameter can be increased in the GM model if we add terms that explicitly breaks the \(SU(2)_c\) custodial symmetry.
custodial symmetry breaking terms are added. The most economical recipe without putting in by hand new $SU(2)_c$ breaking terms is to split the triplet VEVs at the tree level, that is, requiring $v_\xi \neq v_\chi$ by a small amount. With a small misalignment among the triplet VEVs, the $T$ parameter can be increased accordingly because of the slightly breaking of $SU(2)_c$ custodial symmetry at the tree level. From our numerical results, we can see in the lower panels of fig.2 that the new physics contribution $\Delta m_W$ is indeed correlated to the $SU(2)_c$ breaking parameter $v_\xi - v_\chi$. With the misalignment parameter $v_\xi - v_\chi$ taking 6 GeV, new contribution $\Delta m_W$ can reach its maximum 0.048 GeV, which can explain the CDF II data to $3\sigma$ range. Larger $v_\xi - v_\chi$ up to 15 GeV can not increase further the value of $\Delta m_W$.

GM extension model with RH neutrinos and large Yukawa coupling strength $y_N \sim O(10^{-2})$ can lead to large $|h_{ij}| \sim O(10^{-2})$ couplings in eq.(2.12). As noted previously, FT in the cancelation of order $10^{-11}$ is needed to obtain tiny neutrino masses. Such cancelation is crucial to allow large $h_{ij}L^T_L^\chi L_L^\phi$ coupling. Although the introduction of the RH neutrino sector will not enter the W-boson loops at one-loop level, larger values of $h_{ij}$ term will lead to larger tree level $SU(2)_c$ symmetry breaking effects, which are welcome to increase the new $\Delta m_W$ contribution. We show the new physics contributions $\Delta m_W$ from such $h_{ij}$ term.  

We show our numerical results in the left panel of fig.3. We can see in the left panel of fig.3 that our RH neutrino extended GM scenario without misalignment among the triplet VEVs can not explain the new CDF II data on W-boson mass. The maximal contribution to $\Delta m_W$ is 0.016 GeV. Lower scale of $M_R$, which corresponds to larger value of $h_{ij}$, can possibly increase the new physics contributions to $\Delta m_W$, however, most of the points are ruled out by LFV constraints.

So, it is interesting to combine both small $SU(2)_c$ breaking effects to see the new physics contribution $\Delta m_W$, that is, the presence of both $h_{ij}$ coupling term and the small misalignment among the triplet VEVs. It can be seen in the right panel of fig.3 that large $\Delta m_W$ can indeed be obtained easily. The $1\sigma$ range of CDF II data on W boson mass can be obtained even with tiny splitting among the triplet VEVs for $v_\xi - v_\chi \approx 1$ GeV. So it is advantageous to introduce the RH neutrino sector, which can not only solve the CDF II data on W boson mass, but also accommodate the leptogenesis mechanism and neutrino masses. It can also be seen that the total $SU(2)_c$ breaking effects are still not very large. The maximum contribution $\Delta m_W$ is less than 0.25 GeV.

It is interesting to compare the GM model with the ordinary $SU(2)_L$ triplet extension model with $Y = 0$. In this case, the custodial $SU(2)_c$ symmetry is not kept. Consequently, we anticipate that this model can give large contribution to $\Delta m_W$. So, the new CDF II data on $m_W$ can at best be seen as a new constraint on such types of models. Numerical results show that the upper bound of doubly charged scalar is several TeV if the new CDF II data on $\Delta m_W$ is explained in this model.

4 Conclusions

Georgi-Machacek model, which can preserve the custodial $SU(2)_c$ symmetry at tree level, can hardly account for the new CDF II data on W-boson mass in its original form. As anticipated, unless tree level $SU(2)_c$ custodial symmetry breaking effects are present, the
Figure 3. We show the new physics contribution $\Delta m_W$ in the RH neutrino extended GM model without (left panel) and with (right panel) small misalignment among the triplet VEVs. The ranges of the corresponding values for $M_R$ are also shown.

new physics contributions $\Delta m_W$ is always very small. Our numerical results show that ordinary GM model can contribute to $\Delta m_W$ at most 0.0009 GeV, which can not explain the new CDF II data on W boson mass. We propose to introduce small misalignment among the triplet VEVs to increase $\Delta m_W$, which can reach 0.05 GeV for $v_\xi - v_\chi \sim 6$ GeV, marginally explain the new data in 3$\sigma$ range. We also propose to extend the GM model with low scale RH neutrino sector, which can adopt the leptogenesis mechanism and act as a $SU(2)_c$ breaking source because of its correlation with $h_{ij}$. With low scale RH neutrino mass scale of order 100-1000 TeV, the new physics contribution $\Delta m_W$ can reach 0.02 GeV. Combining both small $SU(2)_c$ breaking effects, the 1$\sigma$ range of CDF II data on W boson mass can be obtained even for tiny splitting among the triplet VEVs with $v_\xi - v_\chi \approx 1$ GeV.

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