Mobility analysis of hybrid-driven ten-bar mechanism

Limin Wang¹ and Ying Wang²,³

¹ Office of Science and Technology Research, North University of China, Taiyuan, China
² School of Energy and Power Engineering, North University of China, Taiyuan, China
³ E-mail: 20040092@nuc.edu.cn

Abstract. For a new mechanism, it should be able to move continuously according to the predetermined motion law. Each bar of the mechanism should meet certain size conditions. In this paper, a new type of three degree of freedom ten-bar mechanism with symmetrical structure is proposed. According to the motion requirement, the condition of crank existence is analyzed. The size of the mechanism is determined. The possible limit position space of this mechanism is analyzed. It lays a theoretical foundation for the application of the mechanism.

1. Introduction

Hybrid-driven mechanism is a multi-degree of freedom mechanism, which uses real-time uncontrollable motor (conventional motor) and real-time controllable motor (servo motor) as its power source. The uncontrollable motor provides the main power for the system. The controllable motor plays the role of motion regulation, which can accurately meet the requirements of the given motion. When the motion flexibility is obtained, the servo motor power can be effectively reduced, and the needs of low-cost, high-power and programmable machines can be realized, which can meet the needs of diversified functions of modern machines. This system not only has the advantages of traditional mechanical system, but also has certain flexibility. It can provide a variety of output motion laws through the controllability of servo motor [1].

The hybrid-driven mechanism can meet the requirements of large load and continuous motion of the machine. For the new mechanism, we know that when the active bars of the mechanism can move according to their own motion law, each component should meet certain size conditions. This is the condition of mobility. Zhang Chun et al. [2] analyzed the mobility of planar composite linkage mechanism. Taking RRRPP configuration as the research object, Song Shengtao et al. [3] analyzed the reachable workspace and singular configuration of the mechanism and obtained the movable boundary conditions of the mechanism. Wu Xiaowei et al. [4] analyzed the necessary conditions for the existence of double crank of hybrid-driven five-bar mechanism. He Shuiqin et al. [5] puts forward a kind of planar 3-DOF hybrid mechanism and studies the mobility of the mechanism through the crank condition of the bar.

2. Mobility analysis of hybrid-driven ten-bar mechanism

In order to make the mechanism have better adaptability and controllability, a kind of motion chain with symmetrical structure is selected. Here, 3-DOF ten-bar kinematic chain is selected as the research
object, as shown in Figure 1. The mechanism structure formed by this kinematic chain is symmetrical to the left and right, which is easy to generate symmetrical structure and increase its bearing capacity.

For the hybrid-driven mechanism in order to improve the stability of the mechanism, four pairs of bar is selected as the frame. The mechanism diagram is shown in Figure 2. From the analysis of controllability and easy driving, the connecting bar is generally selected as the driving bar. The mechanism has three degrees of freedom, and four pairs of bar is selected as the frame, which meets the requirements that the connecting bars are all driving bars. In order to increase the working space of the mechanism, it is necessary to have at least one crank for the whole rotation. Therefore, this paper analyzes the crank existence condition of the connecting bars.

![Figure 1. 3-DOF 10-bar kinematic chain.](image1)

![Figure 2. The diagram of hybrid-driven mechanism.](image2)

All the hinges of this mechanism are rotating pairs. It contains four connecting bars. Three of the four connecting bars can be used as driving bars. According to the mechanism diagram, the mechanism consists of three closed chains $ABCD'E, A'B'C'D'E', EDFF'D'E'$. The bar $FF'$ is the output bar. It can be seen from the diagram that the mechanism is composed of two five-bars mechanisms $ABCD'E, A'B'C'D'E'$ in parallel. In order to make it have better adaptability and controllability, the structure is symmetrical. The length of each bar meets the following conditions:

\[
AB = A'B' = l_1, \quad BC = B'C' = l_2, \quad CD = C'D' = l_3, \quad DE = D'E' = l_4
\]

\[
CF = C'F' = l_5, \quad FD = F'D' = l_6, \quad FF' = l_7
\]

The frame

\[
AE = A'E' = l_5, \quad EE' = l_9
\]

The multi degree of freedom hybrid-driven mechanism is composed of several closed kinematic chains. When analyzing the crank condition of the mechanism, it is necessary to analyze the crank condition of the closed loop where the driving bar is located. There are three situations in the closed loop of kinematic chain: four link loop, five link loop and more than five link loop. The existence condition of the crank of the driving bar is analyzed according to the number of components in the circuit.

The planar parallel mechanism is composed of two five-bar mechanisms in parallel. Therefore, the existence condition of the crank of the mechanism depends on the existence condition of the crank of the hinged five-bar mechanism and the six-bar mechanism $EDFF'D'E'$. composed of two five bar mechanisms. The mechanism selects component $AB, DE, D'E'$ as the driving bars, and component $DE$ is the crank.

The necessary and sufficient conditions for assembling a single ring n-bar mechanism composed of rotating pairs are as follows:

\[
l_{\text{max}} \leq \sum_{k=1}^{n} l_k - l_{\text{max}} \tag{1}
\]

If any member $l_i$ is a rotatable member, then:

\[
l_{\text{max}} + l_i \leq \sum_{k=1, k \neq i}^{n} l_k - l_{\text{max}} \tag{2}
\]
2.1. Existence condition of crank in five-bar mechanism

According to the above analysis, the crank existence condition of the hinged five-bar mechanism satisfies the Grashof extension criterion

\[ l_{\text{max}} + l_{\text{min1}} + l_{\text{min2}} < l_a + l_b \]  

(3)

Where \( l_{\text{min1}} \) and \( l_{\text{min2}} \) represent the shortest two components, \( l_{\text{max}} \) represent the longest component, \( l_a \) and \( l_b \) represent the other two components respectively.

According to Formula (3), the existence condition of crank of five-bar mechanism can be obtained:

(1) When the connecting bar and floating bar are the shortest two components, the mechanism is a crank rocker mechanism;

(2) When two connecting bars are the shortest two members, the mechanism is a double crank mechanism;

(3) When the connecting bar and frame are the shortest two components, the mechanism is a three-crank mechanism.

Therefore, in the five-bar mechanism \( ABCDE \), it is required that the connecting \( DE \) is one of the shortest bars, the other connecting \( AB \) and the frame \( AE \) are not one of the shortest bars, so the condition that the \( DE \) can be crank is satisfied:

\[ l_{\text{min4}} = l, \quad l_{\text{min1}} \neq l_{\text{min2}}, \quad l_{\text{min5}} \neq l_{\text{min2}} \]  

(4)

2.2. Existence condition of crank in six-bar mechanism

In the six-bar mechanism \( EDFF'D'E' \), it is necessary to use \( DE \) and \( D'E' \) as crank. When \( EE' \) is the longest bar, the six-bar mechanism can reach the position of drawing collinear and overlapping collinear, as shown in Figure 3.

\[ l_{\text{min4}} + l_{\text{min5}} + l_{\text{max}} < l_a + l_b \]  

(5)

When the overlapping is collinear, the necessary and sufficient conditions for the assembly of the five-bar linkage in Formula (2) should also be satisfied.

When \( l_{\text{min4}} - l_{\text{min5}} \) is the longest bar:

\[ l_{\text{min4}} - l_{\text{min5}} > \max(l_{\text{min4}}, l_{\text{min5}}, l_{\text{max}}) \]

So:

\[ l_{\text{min4}} > \max(2l_{\text{min4}}, l_{\text{min5}} + l_{\text{max}}) \]  

(6)

The necessary and sufficient conditions for assembly are as follows:

\[ l_{\text{min4}} - l_{\text{min5}} < l_{\text{min4}} + l_{\text{min5}} + l_{\text{max}}, \quad \text{so} \quad l_{\text{min4}} < 2(l_{\text{min4}} + l_{\text{min5}}) \]  

(7)

According to the Formulas (5) ~ (7), the following results can be obtained:

\[ 2l_{\text{min4}} + l_{\text{min5}} > l_{\text{min4}} > \max(2l_{\text{min4}}, l_{\text{min5}} + l_{\text{max}}) \]  

(8)
(2) When $l_7$ is the longest bar:

$$l_7 > \max(l_4, l_7, l_8, l_9 - l_4)$$  \hspace{1cm} (9)

The necessary and sufficient conditions for assembly are as follows:

$$l_7 < l_9 - l_4 + l_8 + l_7 + l_4$$
$$l_7 < l_7 + l_8 + l_9$$  \hspace{1cm} (10)

According to the Formulas (5), (9), (10), the following results can be obtained:

$$l_7 > \max(l_4, l_7, l_9 - l_4) \text{ and } l_9 < 2l_7 + l_8$$  \hspace{1cm} (11)

(3) When $l_8$ is the longest bar:

$$l_8 > \max(l_5, l_7, l_9 - l_4)$$  \hspace{1cm} (12)

The necessary and sufficient conditions for assembly are as follows:

$$l_8 < l_9 - l_4 + 2l_7 + l_4$$
$$l_8 < 2l_7 + l_9$$  \hspace{1cm} (13)

Equations (5) and (13) contradict each other, so the mechanism cannot be assembled under this condition.

(4) When $l_4$ is the longest bar:

$$l_4 > \max(l_5, l_7, l_9 - l_4)$$  \hspace{1cm} (14)

The necessary and sufficient conditions for assembly are as follows:

$$l_4 < l_9 - l_4 + 2l_7 + l_8$$  \hspace{1cm} (15)

According to the Formulas (5), (14), (15), the following results can be obtained:

$$l_9 < 2l_7 + l_8$$  \hspace{1cm} (16)

According to the above analysis, the mechanism does not exist when $EE'$ is the longest bar and $l_8 > \max(l_5, l_7, l_9 - l_4)$. When $l_9 < 2l_7 + l_8$, the crank $DE$ exists. According to this condition, the length ratio of each bar is determined as follows:

$$l_1/l_2/l_4/l_5/l_7/l_8/l_9 = 11/12/5/10/7/6/10$$

$DE$ is a constant motor for crank rotation. $AB$ and $D'E'$ apply servo motor to make rocker movement. $FF'$ is an output bar. This mechanism can be used as a rocking chair mechanism to realize the function of swinging up and down, left and right. Its structure is shown in Figure 4.

![Figure 4](image)

**Figure 4.** A rocking chair mechanism.

### 3. Motion analysis of hybrid-driven ten-bar mechanism

By analyzing the motion of the mechanism, the motion track of the mechanism can be obtained according to the crank existence condition and the ratio of the length of each bar. The three limit positions of the mechanism are shown in Figure 5(a), 5(b) and 5(c). According to the diagram of each position of the mechanism in Figure 5, it can be seen that the continuity of motion can be realized by using constant motor drive when the crank $DE$ is in the whole rotation. $AB$ and $D'E'$ servo motor drive
can increase the variability of motion. The output bar FF’ can swing up and down, left and right. The functional requirements of rocking chair are realized.

![Diagram of limit positions](image)

(a) the First limit position  
(b) the Second limit position  
(c) the Third limit position

**Figure 5.** The limit position diagram.

4. Conclusions

Based on the analysis of the mobility and motion of the hybrid-driven ten-bar mechanism, the existence conditions of the crank and its motion trajectory are studied. It provides a reliable theoretical basis for the further research of hybrid-driven mechanism.

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