Comparing bio-inspired heuristic algorithm for the mean-CVaR portfolio optimization

E P Setiawan
Bachelor of Science in Statistics Programme, Department of Mathematics Education, Faculty of Mathematics and Natural Science, Universitas Negeri Yogyakarta, Yogyakarta, Indonesia
E-mail: ezra.ps@uny.ac.id

Abstract. Risk aversion parameter is a coefficient that denotes the trade-off between the risk and the return in an optimal investment. This coefficient had widely used to modify the mean-variance portfolio optimization procedure. In this study, we develop become a mean-CVaR optimization problem with risk aversion. We investigate the usage of several biological-based heuristic algorithms such as genetic algorithm, grasshopper optimization, firefly optimization, moth flame optimization, particle swarm optimization, grey-wolf optimization, and dragonfly optimization to solve this portfolio optimization procedure. Empirical study with Indonesia Stock data show that the Grey-Wolf Optimization yields better performance than the others.

1. Introduction
Since Markowitz’s seminal work [1], portfolio optimization becomes an enthralling subject in financial mathematics. In his approach, the variance was used as a measure for the investment’s risk. The optimal portfolio defined as the minimum-variance portfolio given the level of expected return. As a consequence, Markowitz’s approach widely known as mean-variance portfolio optimization.

Further analysis shows that Markowitz’s mean-variance model assumed that the investor is risk-averse since they focused on the smallest risk portfolio. In contrast, a risk-seeker investor might want to maximize the return without pay attention to the investment’s risk. Another type of investor examines both the return and the risk of the investment. Consequently, a model using utility theory was developed to implement the investor’s level of risk-averse. The utility theory was implemented as a utility function, defined as a function in real numbers and giving real value. Each (random) portfolio was evaluated using the utility function, and the higher one will be preferred [2]. There are several types of the utility function, however, they should be a concave function in order to represent the risk-aversion level. Several studies integrating mean-variance portfolio optimization with utility functions were available, such as [3] and [4].

In the mean-variance portfolio optimization model, the variance became a risk measure. On the other hand, the usage of variance as a risk measure yields several problems. First, the number of estimated parameters from the data would be large, especially when there are many assets included in the portfolio. Second, it relies on the assumption that the assets return are normally distributed. More than a half-century ago, Mandelbrot [5] show that assets return data are heavy-tailed and contain outliers.

To improve the portfolio optimization procedure, Uryasev [6] uses a new risk measure called Conditional Value-at-Risk (CVaR), which developed by Artzner [7]. In several kinds of literature such
as [8], CVaR is also known as Expected Tail Loss (ETL), Expected Shortfall (ES), mean excess loss, or tail Value-at-Risk. Pflug [9] show that the CVaR is a coherent risk measure, and it also proved to be convex. Based on these properties, the CVaR was used in portfolio optimization, as did by [6] and [10]. The CVaR also used in developed portfolio optimization procedures, such as with additional transaction lots and cardinality constraints [11,12].

In this paper, we extend the usage of CVaR into portfolio optimization with a utility function, which represented by the risk-averse coefficient. Following the usage of heuristic method in portfolio optimization such as in [13], [14], and [15], we investigate several new bio-inspired heuristic methods such as the grasshopper optimization algorithm [16], dragonfly algorithm [17], cuckoo search [18], moth flame optimization [19], grey-wolf optimization [20], and firefly algorithm [21].

The rest of this paper is as follows. Section 2 discusses the definition of CVaR, its properties, and its usage in portfolio optimization. Section 3 presents several biological-based heuristic optimization algorithms used in this study. Empirical or numerical study with real data from a stock exchange market is presented in section 4. The last section presents the conclusion and some suggestions for future research.

2. Conditional value-at-risk and its optimization

Following [10], consider a portfolio with vector weights \( w \) and random returns denoted by \( r \) with probability density function \( p(r) \). Let \( f(w,r) \) denote the loss function when one chose a portfolio \( W \) among the set of feasible portfolio. Hence the cumulative distribution function of the loss related to the weight \( w \) could be written as

\[
\Psi(w,x) = \int_{f(w,r) \leq x} p(r) \, dr
\]  

(1)

For a confidence level \( \alpha \), the value-at-risk (VaR) of the portfolio defined as the \((1-\alpha)\) quantile of loss function, or

\[
\text{VaR}_\alpha (w) = \min \left\{ x \in \mathcal{R} : \Psi(w,x) \geq \alpha \right\}
\]  

(2)

Based on these result, we define the conditional value-at-risk (CVaR), Expected Tail Loss (ETL), Expected Shortfall (ES), related to the portfolio as

\[
\text{CVaR}_\alpha (w) = \frac{1}{1-\alpha} \int_{f(w,r) \leq \text{VaR}_\alpha (w)} f(w,r) \, p(r) \, dr.
\]  

(3)

A concise explanation related to the properties of CVaR were presented in [9] and [10]. However, a simpler auxiliary function can be used to simplify the calculation of CVaR [6], so that the minimum CVaR portfolio could be defined as

\[
\text{minimize} \quad \alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^{t} u_k
\]  

subject to linear constraint

\[
u_k \geq 0
\]  

(5)

\[
w^T y_k \geq -\alpha - u_k
\]  

(6)

\[
w^T 1 = \sum_{j=1}^{n} w_j = 1
\]  

(7)

With the additional utility function, let \( \lambda \) be the risk aversion coefficient, as defined in [12]. Using this notation, the optimization above become

\[
\text{maximize} \quad \left( 1-\lambda \right) \sum_{k=1}^{t} r_k - \lambda \left[ \alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^{t} u_k \right]
\]  

(8)

while the constraint (5-7) still hold. This means that the objective is maximize the difference between the total return and the risk (CVaR).
3. Heuristic optimization procedure

In this section, we describe several bio-inspired heuristic optimization procedures that used in this study. In general, these procedures were used with almost similar objective function, same number of population, and same number of maximum iteration.

3.1. Calculation using Genetic Algorithm (GA)

Genetic Algorithm (GA) was proposed by Holland [22] based on the concept of evolution. In this method, the solution candidates are represented by the chromosomes, each consists of several genes that stand for optimized variable. Two kind of processes, namely the crossover and mutation, are used to change the value of the gene in the chromosome. Fitness function and elitism principle were used to evaluate each solution candidate as well as keep the best solution during the iterative process. The asset’s weights were represented by the genes in the chromosomes.

Since 1990s, the GA were used in several portfolio optimization procedures, especially those involve additional constraint such as the transaction lots and cardinality constraints [12, 23] show that the genetic algorithm can perform better than several meta-heuristic optimization problem such as the Tabu Search and the Simulated Annealing.

3.2. Calculation using Grasshopper Optimization (GO)

The idea of Grasshopper Optimization (GO) were prompted by the navigation systems of grasshopper swarms. Introduced by Merjalili et al. [16], this model use the grasshopper’s position in the swarm as a representation of the solution candidate. Mathematically, the position of the $i$-th grasshopper ($X_i$) in the swarm is defined to be a sum of its social interaction ($s$), its gravity force ($g$ and $e_g$), and the wind affection ($u$ and $e_w$). Moreover, these swarms were equipped with several parameters that represent the exploration and exploitation process in several optimization stages. As a result, the mathematical formula of each grasshopper’s position is

$$X_i = \sum_{j=1, j\neq i}^{N} s\left[|x_j - x_i| \right] \frac{x_j - x_i}{d_{ij}} - g e_g + u e_w \quad (8)$$

This formula can be modified to add the lower and upper bound on the grasshopper’s position, as well as represent the best obtained solution in the iteration. Our literature survey show that the grasshopper optimization still not widely used in portfolio optimization procedure.

3.3. Calculation using Firefly Optimization (FO)

Firefly Optimization Algorithm, or simply known as Firefly Algorithm (FA), is a meta-heuristic optimization method developed by Dr. Xin-Shi Yang [21]. This algorithm were inspired from the behavior of fireflies or lighting bugs, a nocturnal luminous insect that could be found in tropical and temperate regions. The light produced by the fireflies help them to communicate each other as well as to attack the prey or other fireflies.

The optimization mechanism of firefly algorithm is as follows. The candidate solution were represented as the fireflies. Next, it is assumed that the brightness of a firefly is represent the objective function, and that a firefly will be attracted to other fireflies proportional to the brightness regardless of their sex. Based on these assumption, the variation of the attractiveness $\beta$ with the distance $r$ is

$$\beta (r) = \beta_0 e^{-mr}, m \geq 1 \quad (9)$$

and the movement of a firefly $i$ which attracted to the other firefly $j$ (which is brighter) could be determined by

$$X_i = X_i + \beta_0 \left( X_j - X_i \right) \exp \left( -\gamma \sum_{k=1}^{d} \left( X_{j,k} - X_{i,k} \right)^2 \right) + \alpha \varepsilon_i \quad (10)$$

This optimization algorithm were used to solve several portfolio optimization problems by Bacanin and Tuba [24, 25, 26].
3.4. Calculation using Dragonfly Algorithm (DA)
Dragonfly algorithm (DA), proposed by Merjalili [17], is a swarm-intelligence optimization method that adopt the behavior of dragonflies. To represent the optimization procedure, it is assumed that all dragonflies (which represent the solution candidate) always follow five different patterns of movement: separation from the swarm, alignment, cohesion, attracted to the foods, and distraction from their enemies. Each pattern above has its own mathematical function to represent the result of their movement, so that the movement can be formulated as

\[
\Delta X_{jt+1} = \left[ s \left( \sum_{j=1}^{N} (X - X_j) \right) + a \left( \frac{1}{N} \sum_{j=1}^{N} V_j \right) + c \left( \frac{1}{N} \sum_{j=1}^{N} X_j - X \right) \right] + w \Delta X_j
\]

where \(s\), \(a\), \(c\), \(f\), and \(e\), represent the weight of swarm, alignment, cohesion, attraction to the food source, and distraction by the enemies, respectively. Usage of dragonfly algorithm in portfolio optimization has been studied by Zhang and Li [27].

3.5. Calculation using Grey Wolf Optimization (GWO)
Grey Wolf Optimization (GWO) is one of the recent bio-inspired optimization algorithm which were introduced by Mirjalili et al. [20]. This algorithm is inspired by the behavior of grey wolves when hunting for prey in a certain social hierarchy level, namely alpha (the most powerful wolf), beta, delta, and omega. In addition, this algorithm also mimic the grey wolf’s hunting behavior, which consists of chasing, encircling, harassing, and attacking process. As in [28], the grey-wolf optimization were used in many optimization procedure. The usage of grey-wolf optimization in portfolio optimization were studied by Ren et al. [29].

3.6. Calculation using Cuckoo Search Optimization (CSO)
Cuckoo Search Algorithm is a kind of meta-heuristic optimization algorithm developed by Gandomi, Yang, and Alavi [18]. This algorithm were inspired by the behavior of Cuckoo, a kind of bird. In this method, the solution candidates are represented by the nest. The ‘cuckoo’ will generate new solution candidate using, which will be evaluated. If the new solution is better than a randomly-chosen nest, the nest will be replaced by the new solution. Worse nests would be abandoned, since the number of nest is fixed. The nests with higher quality solution would be kept and compared to find the best solution. Our literature survey show that the cuckoo search algorithm were used for portfolio optimization with cardinality constraint [30].

3.7. Calculation using Moth Flame Optimization (MFO)
Moth Flame Optimization were proposed in 2015 by Merjalili [19]. The inspiration for this method were taken from moths, a kind of insects that undergo metamorphosis from larvae to adult via cocoons. In the night, moths can navigate using the moon light. However, the moth’s navigation can be disturbed by the presence of artificial light such as flame. The moth will change the fly pattern, from the straight line into a spiral path encircling the flame. In the optimization method, the moths are the search agents that moving around the search area, while the flames represent the best position of the moths obtained during the iteration. By moving around the flame, the moths would obtain better solution and then the best solution.

4. Numerical result
To implement these heuristic algorithm, we select three sets consist of 5 stocks, 10 stocks, and 15 stocks that randomly chosen from LQ-45 index in Indonesia Stock Exchange. Each stock return were calculated based on the daily price as shown at the Yahoo Finance site. The performance of each stock can be seen in figure 1 as follows.
As seen in figure 1, all of these asset’s data are contain several outliers, which represented by the small bullet below and above the horizontal whisker line. Although these assets return data exhibit similar median, the variability were quite different. The return of BBCA exhibit smallest variability, whereas PTBA exhibit largest variability. Statistic normality test using Shapiro-Wilk method for each return data shows that at 5% significance level only BBRI and GGRM that follow Gaussian or normal distribution. Therefore, the usage of CVaR instead of variance as a risk measure is reasonable.

The portfolio with 15 assets contains all assets presented in figure 1, while the portfolio with 10 assets only contains the first 10 assets. In addition, the portfolio with 5 assets consists of the first 5 assets in figure 1 above.

For each sets of data, we calculate the optimal portfolio weight using each optimization method, with different level of risk aversion coefficient, that is, $\lambda = 1$ and $\lambda = 5$. All of the heuristic optimization procedure were done using R, with library `metaheuristicOpt` version 2.0 [31], while the CVaR calculated using library `PerformanceAnalytics` version 1.5.3 [32]. The optimization parameter in this study could be seen in table 1.

| Optimization method                  | N of population | Max iteration | Other parameter(s)                                      |
|--------------------------------------|-----------------|---------------|--------------------------------------------------------|
| Genetic Algorithm (GA)               | 50              | 400           | mutation probability = 0.1, crossover probability = 0.6. |
| Cuckoo Search Optimization (CSO)     | 50              | 400           | abandoned fraction = 0.5                               |
| Moth Flame Optimization (MFO)        | 50              | 400           | -                                                       |
| Grasshopper Optimization (GO)        | 50              | 400           | -                                                       |
| Dragonfly Algorithm Optimization (DO)| 50              | 400           | -                                                       |
| Grey Wolf Optimization (GWO)         | 50              | 400           | -                                                       |
| Firefly Algorithm Optimization (FO)  | 50              | 400           | attractiveness = 1, light absorption coefficient = 1, randomization parameter = 0.2 |

Since a heuristic method could being fall into a local optimum, each calculation for each combination of risk aversion and number of stocks were repeated five times with various parameters. The summary of the result were reported in table 2.
The performance of these meta-heuristic algorithms could be represented by the average and best (maximum) optimal value. Since the objective of this optimization problem is to maximize a function, higher optimal value means better performance. Furthermore, the portfolio’s performance measured by the Sharpe Ratio.

Table 2. Result of optimal portfolio with several bio-inspired heuristic optimization methods

| N of stocks | Risk aversion $\lambda$ | Optimization method | Average time (sec) | Best objective function Average | Best objective function Maximum | Sharpe ratio Average | Sharpe ratio Maximum |
|-------------|-------------------------|---------------------|-------------------|-------------------------------|-------------------------------|---------------------|---------------------|
| 5           | 0.50                    | GA                  | 210.01            | 0.16383                       | 0.16390                       | 0.23802             | 0.24918             |
|             |                         | CSO                 | 404.45            | 0.16288                       | 0.16316                       | 0.10879             | 0.27871             |
|             |                         | MFO                 | 208.38            | 0.16392                       | 0.16392                       | 0.24551             | 0.24551             |
|             |                         | GO                  | 211.08            | 0.16339                       | 0.16392                       | -0.33402            | 0.24551             |
|             |                         | DO                  | 209.91            | 0.16339                       | 0.16392                       | -0.34098            | 0.24631             |
|             |                         | GWO                 | 205.55            | 0.16392                       | 0.16392                       | 0.24551             | 0.24555             |
|             |                         | FO                  | 210.72            | 0.15366                       | 0.15690                       | 0.38884             | 0.53390             |
| 5           | 0.85                    | GA                  | 312.97            | 0.03640                       | 0.03640                       | 1.28209             | 1.32963             |
|             |                         | CSO                 | 400.53            | 0.03547                       | 0.03609                       | 1.26845             | 2.03694             |
|             |                         | MFO                 | 205.47            | 0.03640                       | 0.03640                       | 1.27701             | 1.27701             |
|             |                         | GO                  | 207.23            | 0.03640                       | 0.03640                       | 1.27701             | 1.27702             |
|             |                         | DO                  | 209.77            | 0.03640                       | 0.03640                       | 1.27573             | 1.27841             |
|             |                         | GWO                 | 205.14            | 0.03640                       | 0.03640                       | 1.26781             | 1.27752             |
|             |                         | FO                  | 212.55            | 0.03141                       | 0.03315                       | 1.01266             | 2.70903             |
| 10          | 0.50                    | GA                  | 322.30            | 0.50061                       | 0.50469                       | -1.08802            | -1.06689            |
|             |                         | CSO                 | 413.30            | 0.30317                       | 0.31128                       | -1.72711            | 0.06844             |
|             |                         | MFO                 | 214.09            | 0.51442                       | 0.51442                       | -1.09429            | -1.09429            |
|             |                         | GO                  | 236.81            | 0.46501                       | 0.51442                       | -2.39006            | -1.09429            |
|             |                         | DO                  | 226.57            | 0.30844                       | 0.51442                       | -2.36465            | -1.09429            |
|             |                         | GWO                 | 221.44            | 0.51442                       | 0.51442                       | -1.09429            | -1.09429            |
|             |                         | FO                  | 314.73            | 0.29953                       | 0.33395                       | 0.28693             | 0.54918             |
| 10          | 0.85                    | GA                  | 322.63            | 0.33034                       | 0.33291                       | -1.09537            | -1.08024            |
|             |                         | CSO                 | 413.39            | 0.14443                       | 0.15949                       | -1.72711            | 0.06844             |
|             |                         | MFO                 | 212.31            | 0.34582                       | 0.34582                       | -1.09429            | -1.09429            |
|             |                         | GO                  | 236.25            | 0.34578                       | 0.34582                       | -1.10680            | -1.09429            |
|             |                         | DO                  | 223.80            | 0.11707                       | 0.29932                       | -1.67139            | -0.63374            |
|             |                         | GWO                 | 230.92            | 0.34582                       | 0.34582                       | -1.09429            | -1.09429            |
|             |                         | FO                  | 314.26            | 0.17227                       | 0.21340                       | -0.84229            | -0.10588            |
| 15          | 0.50                    | GA                  | 348.27            | 0.44330                       | 0.45810                       | 0.01215             | 0.25122             |
|             |                         | CSO                 | 447.98            | 0.33111                       | 0.33111                       | 1.37669             | 1.78875             |
|             |                         | MFO                 | 229.97            | 0.51442                       | 0.51442                       | -1.25647            | -1.25647            |
|             |                         | GO                  | 270.17            | 0.43076                       | 0.48665                       | 0.20357             | 1.22327             |
|             |                         | DO                  | 246.04            | 0.30114                       | 0.34806                       | 0.97490             | 2.54796             |
|             |                         | GWO                 | 245.44            | 0.51442                       | 0.51442                       | -1.25647            | -1.25647            |
|             |                         | FO                  | 341.64            | 0.36140                       | 0.36140                       | 0.46619             | 1.14394             |
| 15          | 0.85                    | GA                  | 347.92            | 0.30385                       | 0.30713                       | 0.66124             | 1.26648             |
|             |                         | CSO                 | 448.43            | 0.17975                       | 0.20146                       | 1.35161             | 2.02343             |
|             |                         | MFO                 | 231.52            | 0.34842                       | 0.34842                       | -1.14804            | -1.14804            |
|             |                         | GO                  | 270.30            | 0.28792                       | 0.30664                       | 1.12479             | 1.72862             |
|             |                         | DO                  | 244.08            | 0.21823                       | 0.21823                       | 1.39939             | 1.88540             |
|             |                         | GWO                 | 240.57            | 0.34842                       | 0.34842                       | -1.14804            | -1.14803            |
|             |                         | FO                  | 341.92            | 0.23541                       | 0.25341                       | 0.82095             | 1.72199             |
Table 2 above shows that among seven meta-heuristic algorithm used in this study, the grey wolf optimization (GWO) always reach higher value of the objective function, followed by the moth flame optimization (MFO) algorithm, then by genetic algorithm (GA). In contrast, the Cuckoo Search Optimization (CSO) algorithm frequently yield lower average optimal value. Among five replications used in each study, the highest value of the objective function obtained using GWO and MFO algorithm were higher than the highest value obtained using these other methods. All of these meta-heuristic algorithms could yield the optimal result in reasonable time, yet this result is depends on the performance of the computer used in this study.

By examining the objective function value for the portfolio with 5 and 10 assets, there were several heuristic optimization algorithm that yield same average and/or same best result. All average and/or best value that are same were higher than the other result. Hence, it can be inferred that the same value of average and/or best value were the highest average and/or highest best value for this combination of assets and risk aversion level $\lambda$. On the other hand, the same value between the average and the best value of the objective function means that most of the simulation yield same result. Since there are no restriction on the number of assets, it is possible that only one asset is used in the portfolio. If the investor wants to prevent this result, they can put a constraint on the number of assets used or put a constraint on the asset weight.

Further analysis were done to check the performance of each calculated portfolio. Using the out-of-sample data, we calculate the total return and standard deviation, then use them to compute the Excess return-to-standard deviation ratio, which known as Sharpe Ratio. This ratio were depends on the performance of each assets as well as the risk aversion $\lambda$. For the portfolio with 5 and 15 assets, the positive value of Sharpe Ratio indicates that most of these methods can yield higher total return than the risk-free asset return. In contrast, almost all portfolio consists of ten assets exhibit negative Sharpe Ratio, which means that the total return were lower than the risk-free asset return. Also there are no correlation between the average optimum value and the average Sharpe Ratio, which means that comparison of the Sharpe Ratio between optimization methods are meaningless. Related to the risk aversion, it could be seen that higher $\lambda$ means higher weight for the investment risk. Therefore, several portfolios with $\lambda = 0.85$ have higher Sharpe Ratio than the portfolios with $\lambda = 0.5$. By choose a suitable value of $\lambda$, the investor may obtain a portfolio with good performance from these available assets.

5. Conclusion and Recommendation
The Grey Wolf Optimization (GWO) algorithm could yield better optimal value among several meta-heuristic optimization algorithms when used to solve the mean-CVaR portfolio optimization with risk aversion coefficient. The objective function, the assets, and the risk aversion coefficient should be chosen carefully to ensure that the portfolio weight with higher objective value function is a higher-performance portfolio. Further study can be done to grasp the performance of GWO in portfolio optimization with more practical constraint.

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