TURBULENT MIXING IN CLUSTERS OF GALAXIES

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ABSTRACT

We present a spherically symmetric, steady state model of galaxy clusters in which radiative cooling from the hot gas is balanced by heat transport through turbulent mixing. We assume that the gas is in hydrostatic equilibrium and describe the turbulent heat diffusion by means of a mixing length prescription with a dimensionless parameter $\alpha_{\text{mix}}$. Models with $\alpha_{\text{mix}} \sim 0.01$–0.03 yield reasonably good fits to the observed density and temperature profiles of cooling core clusters. Making the strong simplification that $\alpha_{\text{mix}}$ is time independent and that it is roughly the same in all clusters, the model reproduces remarkably well the observed scalings of X-ray luminosity, gas mass fraction, and entropy with temperature. The break in the scaling relations at $kT \sim 1$–2 keV is explained by the break in the cooling function at around this temperature, and the entropy floor observed in galaxy groups is reproduced naturally.

Subject headings: cooling flows — galaxies: clusters: general — hydrodynamics — instabilities — turbulence — X-rays: galaxies

1. INTRODUCTION

Recent X-ray observations (Peterson et al. 2001, 2003, and references therein) show that there must be some heat source balancing the radiative cooling of hot gas in galaxy clusters. Current ideas include (1) heat transport from the outer regions of the cluster to the center by thermal conduction (e.g., Bertschinger & Meiksin 1986; Bregman & David 1988; Rosner & Tucker 1989; Narayan & Medvedev 2001; Voigt et al. 2002; Zakamska & Narayan 2003, hereafter ZN03), (2) energy injection by jets or radiation from a central active galactic nucleus (AGN; Churazov et al. 2000, 2002; Ciotti & Ostriker 2001; Brüggen & Kaiser 2002; Kaiser & Binney 2003), or (3) a combination of both (e.g., ZN03; Ruszkowski & Begelman 2002; Brighenti & Mathews 2002, 2003; Brüggen 2003).

Models based on electron conduction are able to reproduce the observed profiles of gas density and temperature in several clusters (ZN03). The models are also much less unstable than models without conduction (Kim & Narayan 2003, hereafter KN03). These features make the conduction model rather attractive. In addition to conduction, another diffusive process that may be potentially important in galaxy clusters is turbulent mixing. Clusters are highly dynamic entities that are constantly being stirred by the infall of groups and subclusters, the motions of galaxies, and outflows from AGNs. The dynamical motions supply a large amount of turbulent kinetic energy, perhaps comparable to the thermal energy of the cluster (e.g., Deiss & Just 1996; Roettiger, Stone, & Burns 1999; Ricker & Sarazin 2001). The turbulence also causes diffusive mixing, which tends to erase gradients in the specific entropy and chemical composition (Sarazin 1988) and transports heat efficiently (Cho et al. 2003). The latter feature of turbulence is the focus of this Letter. We describe a model of galaxy clusters in which turbulent heat transport balances radiative cooling of the hot gas, and we show that this model is consistent with a wide range of observations.

2. MODEL

In the presence of turbulence, gas elements move around randomly and transport specific entropy from one point to another. When the elements mix, there is a net heat flux from regions of higher entropy to regions of lower entropy. The heat flux due to turbulence is thus proportional to the entropy gradient and may be written as

$$F = -\kappa_{\text{mix}} \rho T \nabla s, \quad \kappa_{\text{mix}} = \alpha_{\text{mix}} c_s H_p,$$

$$H_p \approx (r^2 + r_c^2)^{1/2},$$

where $\rho$ is the gas density, $T$ is the temperature, and $s$ is the entropy per unit mass. The diffusion constant $\kappa_{\text{mix}}$ is expected to be of the order of $v_{\text{turb}} l_{\text{coh}}^2/3$, where $v_{\text{turb}}$ is the rms turbulent velocity and $l_{\text{coh}}$ is the coherence length of the velocity field (e.g., Deiss & Just 1996; Cho et al. 2003). We expect $v_{\text{turb}}$ to be a fraction of the sound speed $c_s$ (the only relevant velocity in the problem) and $l_{\text{coh}}$ to be a fraction of the local pressure scale height $H_p$ (the obvious scale, as in the mixing length theory of convection). Over most of the cluster, the scale $H_p$ is comparable to the radius, but it is of the order of the core radius $r_c$ inside the core (see ZN03 for the definition of $r_c$).

These arguments lead to the scalings for $\kappa_{\text{mix}}$ and $H_p$, shown in equation (1). The specific entropy for a classical perfect gas is given by $s = c_s \ln (P \rho^{-\gamma})$, where $P = \rho c_s^2$ is the pressure, $c_s$ is the specific heat at constant volume, and $\gamma = 5/3$ is the adiabatic index of the gas.

We consider a spherically symmetric cluster in hydrostatic equilibrium, $(\rho P dr) = -d\Phi/dr$, where $\Phi$ is the gravitational potential. We assume steady state, so that the radiative cooling of each gas element is balanced by the divergence of $F$,

$$\nabla \cdot F = -j,$$

where $j$ is the radiative energy-loss rate per unit volume. For $T \gtrsim 2$ keV, $j$ is dominated by thermal bremsstrahlung from free electrons and ions, while for lower temperatures it is mostly due to atomic transitions. We should note that the assumption of steady state is a strong simplification. Since turbulent motions in a cluster are likely to be episodic, e.g., as a result of AGN activity (Kaiser & Binney 2003) or subcluster infall and merger (Deiss & Just 1996; Ricker & Sarazin 2001), equations (1) and (2) should be viewed as time-averaged relations.
model, however, we find that we obtain fairly good fits for all 10 clusters, with reasonably small (and physically justifiable) values of $\alpha_{\text{mix}}$. The five clusters (A1795, A1835, A2199, A2390, and RX J1347.5−1145) that ZN03 found to be consistent with conduction have a median $\alpha_{\text{mix}}$ of 0.013, while the other five clusters (A2052, A2597, Hydra A, Sersic 159-03, and 3C 295) have a median $\alpha_{\text{mix}}$ of 0.026. The larger $\alpha_{\text{mix}}$ in the latter group may reflect the fact that these clusters have powerful AGNs that cause extra turbulence. Figure 1 shows the results of the model fitting for two clusters, A1795 (one of the better-fitting clusters) and Hydra A (the worst example). We see that the mixing model is generally consistent with the observations, although the $\chi^2$ is not always small.

Apart from providing a qualitative explanation for the observed density and temperature profiles of cluster gas, another major attraction of the thermal conduction model of ZN03 is the fact that conduction helps to control the thermal instability of the gas (KN03). We have now repeated the global stability analysis of KN03 for the turbulent mixing model. For A1795, with $\alpha_{\text{mix}} = 0.011$, we find that mixing suppresses the instability in all radial modes except the fundamental (nodeless) mode. The growth time of the lone unstable mode is very much longer than the Hubble time. In the case of Hydra A, with $\alpha_{\text{mix}} = 0.021$ (and thus a larger $\kappa_{\text{mix}}$), we find that all modes, including the nodeless fundamental mode, are stable. Thus, the equilibrium models described here are for all practical purposes stable.

4. SCALING RELATIONS

Groups and clusters of galaxies form when primordial density perturbations in the universe grow, gravitationally collapse, and merge together according to the standard hierarchical clustering scenario. The statistical properties of these collapsed systems contain many clues to the process of cosmic structure formation. Numerous X-ray studies have been published on the

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**TABLE 1**

**BEST-FIT PARAMETERS OF THE TURBULENT MIXING MODEL**

| Name          | $T(0)$ (keV) | $n_e(0)$ (cm$^{-3}$) | $\alpha_{\text{mix}}$ | $\chi^2$/dof |
|---------------|--------------|----------------------|------------------------|--------------|
| A1795         | 2.1          | 0.052                | 0.011                  | 8.6/15       |
| A1835         | 3.6          | 0.070                | 0.014                  | 6.2/15       |
| A2199         | 1.3          | 0.10                 | 0.013                  | 6.8/13       |
| A2390         | 2.9          | 0.070                | 0.012                  | 8.5/13       |
| RX J1347.5−1145 | 4.0       | 0.034                | 0.025                  | 7.5/13       |
| A2052         | 1.0          | 0.043                | 0.014                  | 62/29        |
| A2597         | 1.4          | 0.070                | 0.026                  | 6.6/19       |
| Hydra A       | 2.6          | 0.057                | 0.021                  | 35/13        |
| Sersic 159-03 | 2.0          | 0.035                | 0.026                  | 3.8/11       |
| 3C 295        | 1.7          | 0.22                 | 0.030                  | 4.1/11       |

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**Fig. 1.**—Observed and calculated density and temperature profiles for (a) A1795 and (b) Hydra A. Results for the turbulent mixing model are shown by solid lines and those for the conduction model are shown by dotted lines. Plus signs correspond to Chandra data (Ettori et al. 2002 for A1795 and David et al. 2001 for Hydra A). Here $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_{\Lambda} = 0.3$, and $\Omega_m = 0.7$ are adopted. The parameters for the mixing model are given in Table 1. The parameters for the conduction model are $f = 3.5$, $n_e(0) = 0.06$ cm$^{-3}$, and $T(0) = 2.0$ keV for A1795, and $f = 5.5$, $n_e(0) = 0.06$ cm$^{-3}$, and $T(0) = 3.0$ keV for Hydra A. Because $f > 1$ for Hydra A, the conduction model is not viable for this cluster (ZN03).
power-law scalings of the size, temperature, X-ray luminosity, mass, entropy, and gas mass fraction of galaxy clusters. The best-fit values of the power-law indices depend on the particular sample of clusters used and on the specific methods employed to estimate the mass and temperature. Nevertheless, there is a broad consensus on the observed scalings as functions of temperature $T$, as summarized in the first three columns of Table 2. A clear break in cluster properties is seen at a characteristic temperature $\sim 1–2$ keV.

Small clusters and galaxy groups are found to have a relatively constant entropy. The prevailing explanations for this surprising “entropy floor” include (1) preheating of intracluster gas (Kaiser 1991; Evrard & Henry 1991) via galactic winds (Loewenstein 2000), AGNs (Valageas & Silk 1999; Wu & Xue 2002; Nate & Roychowdhury 2002), and accretion shocks (David et al. 2001; Tozzi & Norman 2001; Dos Santos & Doré 2002), (2) removal of cold low-entropy gas via galaxy formation in clusters (Bryan 2000; Muanwong et al. 2001; Wu & Xue 2002; Davé, Katz, & Weinberg 2002), and (3) both radiative cooling and supernova feedback (Voit & Bryan 2001; Voit et al. 2002). None of these models includes the effects of thermal conduction or turbulent mixing.

When thermal conduction by electrons is the dominant heating mechanism, the local heat flux is given by $F = -k_{\text{cond}}\nabla T/v$, with $k_{\text{cond}} \propto T^{0.2}$ (Spitzer 1962). For $kT > 2$ keV, we can assume $j \propto n_e^2 T^{1/2}$, corresponding to thermal bremsstrahlung. Then, using equation (2) and substituting $\nabla \sim 1/r$, we find that the gas density in equilibrium scales as $n_e \propto T^0$, which gives a gas mass $M_e \propto n_e r_e \propto T^{5/2}$, gas fraction $f_g \equiv M_g/(M + T^{0.3})$, X-ray luminosity $L_X \propto r_e^3 \propto T^3$, and entropy $S \propto \exp (s/c_s) \sim T n_e^{-1/2} \propto T^{0.5}$. For the mixing model, on the other hand, equation (1) with $c_s \propto T^{1/2}$ and $H \propto r \propto T^{1/2}$, combined with equation (2), yields $n_e \propto T^{1/2}$, $M_e \propto T^2$, $f_g \propto T^{0.2}$, $L_X \propto T^4$, and $S \propto T^{3/2}$. These scaling relations are shown in the last two columns of Table 2. Remarkably, the turbulent mixing model yields scaling relations for $kT > 2$ keV that are in very good agreement with observations. The scalings obtained with the conduction model agree less well.

At lower temperatures, the resonance radiation of highly ionized metals such as O, Si, and Fe is responsible for most of the cooling. From the equilibrium cooling functions provided by Sutherland & Dopita (1993), we obtain approximately $j \propto n_e^2 T^{0.7} \propto T^{0.7}$ for $kT \sim 0.05–1$ keV (for a plasma with solar to half-solar metal abundance). Following the same steps as above, this gives a different set of scaling relations, as shown in Table 2. Both the conduction and the mixing model predict a dramatic change in the scaling below $\sim 2$ keV, in agreement with the observations. The entropy floor that is observed in small clusters and groups is also reproduced naturally in the models.

5. DISCUSSION

Diffusive processes are quite attractive as a heat source in galaxy clusters, since they transport energy to the cluster centers from the outside and also help to control the thermal instability. In this Letter, we have presented equilibrium models of galaxy clusters in which the hot gas maintains energy balance between radiative cooling and heating by turbulent diffusion. To quantify the amount of heat transported by mixing, we adopt a mixing length prescription in which the heat flux is proportional to the local gradient of specific entropy, with a diffusion coefficient parameterized by a dimensionless constant $\alpha_{\text{mix}}$ (eq. [1]). The mixing model fits the observed density and temperature profiles of hot gas in clusters fairly well (§ 3; Fig. 1). The resulting gas configurations are also very stable. In both respects, the mixing model performs better than the conduction model. This is presumably because turbulent mixing transports energy more efficiently in the low-temperature high-density cores of clusters (Cho et al. 2003).

What is the origin of the turbulence invoked in the mixing model? It could be from the infall of subclusters, the motion of the dark matter potential, the orbital motions of galaxies, or energy input from an AGN. Deiss & Just (1996) studied the turbulence generated by motions of individual galaxies and found that the diffusivity constant is $\kappa_{\text{mix}} \sim 1–10$ kpc$^2$ Myr$^{-1}$ in the Coma and Perseus Clusters. Ricker & Sarazin (2001) found that the turbulent velocities driven by cluster mergers are generally subsonic, with $v_{\text{turb}} \sim 0.1–0.3 c$, $\sim 100–300$ km s$^{-1}$. It is difficult to measure the coherence length of the velocity field, but Faraday depolarization measurements suggest that $l_b \sim 5–20$ kpc for magnetic field tangling near the centers of typical clusters (e.g., Carilli & Taylor 2002 and references therein). The corresponding diffusion coefficient is $\kappa_{\text{mix}} \sim 0.5–6$ kpc$^2$ Myr$^{-1}$. In comparison, the values of $\alpha_{\text{mix}} \sim 0.01–0.03$ required in the mixing models presented here correspond to $\kappa_{\text{mix}} \sim 1–6$ kpc$^2$ Myr$^{-1}$ for $r \sim 50–300$ kpc. These values are in good agreement with the other independent estimates.

As shown in § 3, some clusters that contain relatively strong AGNs have somewhat larger values of $\alpha_{\text{mix}}$ compared to clusters that do not have obvious AGN activity. This is natural if jets from the AGN cause some of the turbulence in the centers.

### Table 2

| Physical Quantity | Observation | References | Conduction Model | Mixing Model |
|-------------------|-------------|------------|------------------|--------------|
| Size scale $r_e$  | $T^{0.3}$   | 1, 2, 3    |                  |              |
| Total mass $M_e$  | $T^{1.7-1.9}$ | 3, 4       |                  |              |
| Electron density $n_e$ | $T^{0.3}$   | 1, 2, 3    |                  |              |
| X-ray luminosity $L_X$ | $T^{0.3}$   | 1, 2, 3    |                  |              |
| Entropy $S$       | $T^{0.3}$   | 1, 2, 3    |                  |              |
| Gas mass fraction $f_g$ | $T^{0.3}$   | 1, 2, 3    |                  |              |

REFERENCES.—(1) Vikhlinin, Forman, & Jones 1999; (2) Mohr, Mathiesen, & Evrard 1999; (3) Sanderson et al. 2003; (4) Finoguenov, Reiprich, & Böhringer 2001; (5) White, Jones, & Forman 1997; (6) Markevitch 1997; (7) Allen & Fabian 1998; (8) Wu, Fabian, & Nulsen 2000; (10) Helsdon & Ponman 2000; (11) Ponman, Cannon, & Navarro 1999; (12) Lloyd-Davies, Ponman, & Cannon 2000; (13) Pratt & Arnaud 2003; (14) Ponman, Sanderson, & Finoguenov 2003; (15) Finoguenov et al. 2002; (16) Reiprich 2001.
of these clusters. The influence of AGN jets has been studied by Churazov et al. (2002), Brüggen & Kaiser (2002), and Kaiser & Binney (2003), who showed that powerful jets mix gas with different entropy and thereby greatly reduce the mass deposition rate in cluster cores. Brüggen & Kaiser (2002), in particular, showed that the entropy increase at the cluster center is quite insensitive to the amount of energy injection by AGNs, which is consistent with our result that the values of $\alpha_{\text{mix}}$ in AGN-dominated clusters are within a factor of 2 of those in clusters without strong AGNs.

The model considered in this Letter is highly idealized. In reality, we expect both turbulent mixing and electron conduction to influence heat transport in clusters. Mixing may be inhomogeneous; e.g., a central AGN may induce more efficient mixing near the cluster center compared to the outer regions. Mixing is also likely to be variable in time. Furthermore, turbulent dissipation is an additional source of heat that we have not included in equation (2). These effects need to be investigated in more detail. Another problem is that we have not considered how a cluster got to the state that it is observed in. Bregman & David (1988) showed that cluster models with thermal conduction have a balance between cooling and heating via turbulent heat transport, and if we further assume that $\alpha_{\text{mix}}$ is roughly constant across the entire population, then the model makes clear predictions for how various cluster properties should change as a function of temperature (§ 4; Table 2). Remarkably, the predicted scalings obtained with the mixing model are in excellent agreement with observations. The model also explains the observed discontinuity in cluster properties at $kT \sim 1-2$ keV as a consequence of the change in the cooling function at this temperature and provides a natural explanation for the entropy floor seen in galaxy groups. These results, although not inconsistent with some of the previously proposed explanations (see references in § 4), suggest that those models should be generalized to include the effect of turbulent heat transport.

Note added in manuscript.—After this Letter was submitted to the journal, Voigt & Fabian (2003) posted a paper in which they have suggested independently that turbulent heat transport may be important in galaxy clusters.

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