Combined Effect of Piezoviscous Dependency and Non-Newtonian Couple Stress on Squeeze-Film Porous Annular Plate

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Abstract: Squeeze film investigations focus upon film pressure, load bearing quantity and the minimum thickness of film. The combined effect of pressure viscous dependent and non-Newtonian couple stress in porous annular plate is studied. The modified equations of one dimensional pressure, load bearing quantity, non dimensional squeeze time are obtained. The conclusions obtained in the study are found to be in very good agreement compared to the previous results which are published. The load carrying capacity is increased due to the variation in the pressure dependent viscosity and also due to the couple stress effect. Finally this results in change in the squeeze film timings.

1. Introduction
In the field of hydrodynamic lubrication, the couple stress thin film characteristics of several bearings have studied for some decades [1-6] by considering the viscosity of the lubricant as constant, although it depends on both pressure and temperature. In recent years, the viscosity of variation of non-Newtonian couple stress with temperature and pressure has been given thoughtful study in many practical applications to engineering. Hanumagowda [7] investigated the characteristics of thin film for circular step bearing and considered the viscosity variation of couplestress fluid. Naduvinamani et al [8] discussed for parallel stepped plates.Lin et.al[9] investigated behaviour of Squeeze film for long partial journal bearings , Bartz and Ehler [10] discussed the influence of pressure viscosity oils on pressure, temperature and film thickness for elastohydrodynamic rolling contacts. In all these studies, the pressure dependent viscosity variation of couplestress lubricant on the squeeze film behaviour is discussed and found to be more important. All of the previous investigations were concerned with impermeable surfaces. Wu[12,13] studied theoretically, the squeeze-Film behaviour for annular Disks and rectangular plates by considering the porous facing. He found the importance of porous facing on the squeeze film behaviour. In this paper, therefore, the influence of piezo-viscous dependency on porous annular plates lubricating with non-Newtonian couplestress fluid is investigated.
2. Mathematical Formulation of the problem
The porous annular plates under investigation are shown in Figure 1. The conducting couples stress is considered in the squeeze film region. The lower plate of the bearing has porous facing and the upper plate approaches lower plate with squeezing velocity \( V (=h/t) \), where \( h \) being the thickness of the film between two annular plates. Let \( u \) and \( v \) are lubricant velocities in the \( r \) and \( y \) directions. The pressure, the viscosity, and a material constant which is associated for the couple stress fluids are respectively represented by the symbols \( P \), \( \mu \) and \( \eta \). The equations of motion for couple stress fluid in film region considered as:

\[
\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} = \frac{\partial p}{\partial r} \tag{2}
\]

\[
\frac{\partial p}{\partial y} = 0 \tag{3}
\]

If \( u^* \) and \( v^* \) represent the velocity components in the porous matrix then the equation of continuity in this region expressed as:

\[
\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} = 0 \tag{4}
\]

If the pressure \( P^* \) in the porous region, \( k \) is permeability parameter, \( \delta \) represents porous layer thickness and \( \beta \) \( (= (\eta / \mu) / k) \) represents ratio of microstructure size to pore size. On the account of modified Darcy’s law, the velocity components \( u^* \) and \( v^* \) are expressed as:

\[
u^* = -\frac{k}{\mu(1-\beta)} \frac{\partial p^*}{\partial x} \tag{5} \\
v^* = -\frac{k}{\mu(1-\beta)} \frac{\partial p^*}{\partial y} \tag{6}
\]

The valid boundary condition to the pressure \( P^* \) in porous matrix are expressed by

\[
\frac{\partial p^*}{\partial y} \bigg|_{y=0} = 0 \tag{7}
\]

\[
p^*(x,0) = p(x,0) \tag{8}
\]

Equations (12) and (13) indicate the pressure at solid boundary and its continuity at film-plate interface respectively.

Substituting expressions for \( u^* \) and \( v^* \) (from equations(5) and (6)) into equation (4) and Integrating with respect to \( y \) over porous layer and making use of the boundary conditions are given by (7) and (8), we obtain

\[
v^* \bigg|_{y=0} = \frac{k \delta \delta^2 p^*}{\mu \delta x^2} \tag{9}
\]

The appropriate boundary conditions for velocities at upper and lower are expressed as:
\[ u = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{and} \quad v = -\frac{\partial h}{\partial t} \]  
(10)

\[ u = 0, \quad \text{and} \quad v = 0 \]  
(11)

Solution of equation (2) using (1) the expression for \( u \) and further by applying the boundary conditions given by (10) and (11) gives

\[ u = -\frac{1}{2\mu} \frac{\partial p}{\partial r} \left[ y^2 - hy + 2l^2 \left( 1 - \frac{\cosh \left( \frac{2y - h}{2l} \right)}{\cosh \frac{h}{2l}} \right) \right] \]  
(12)

where \( l = \left( \frac{\eta}{\mu_0} \right)^{1/2} \) being the couple stress parameter.

Substituting for \( u \) from (12) in the continuity equation (1) and integrating with the boundary conditions (10) and (11) gives

\[ \frac{\partial}{\partial r} \left[ A(h, m, \alpha, p, \psi) \frac{\partial p}{\partial r} \right] = 12r\mu_0 \frac{dh}{dt} \]  
(13)

Where,

\[ A(h, m, \alpha, p, \psi) = h^3 e^{-2\alpha p} - 12m^2 h e^{-2\alpha p} + 24m^3 e^{-2.5\alpha p} \tanh \left( \frac{he^{\alpha p/2}}{2m} \right) + \frac{12\delta k e^{-\alpha p}}{1 - \beta} \]

The non-dimensional physical quantities are introduces as:

\[ r^* = \frac{r}{b}, \quad a^* = \frac{a}{b}, \quad l^* = \frac{m}{h_0}, \quad h^* = \frac{h}{h_0}, \quad G = \frac{\alpha \mu_0 b^2 (-dh/dt)}{h_0^3}, \quad \text{and} \quad \psi = \frac{k\delta}{h_0} \]

Substituting non dimensional quantities in (13) the equation of Reynolds type is obtained as follows

\[ \frac{\partial}{\partial r^*} \left[ f^*(h^*, l^*, G, p^*, \psi) r^* \frac{\partial p^*}{\partial r^*} \right] = -12r^* \]  
(14)

Where,

\[ f^*(h^*, l^*, G, p^*, \psi) = e^{-Gp^*} h^* - 12e^{Gp^*} h^* - 24l^3 e^{-2Gp^*} \tanh(e^{0.5Gp^*} h^*/2l^*) + \frac{12\delta k e^{Gp^*}}{1 - \beta} \]  
(15)

The non-dimensional Reynold’s equation (14) are found to be highly nonlinear. So in order to obtain the first order solution analytically for small values of the viscosity parameter \( 0 \leq G = 1 \), a small perturbation method is adopted for the film pressure by putting

\[ p^* = p^*_0 + Gp^*_1 \]  
(16)

into the Reynold’s type equation and neglecting second and higher order of \( G \), we get the following two equations responsible for pressures \( p^*_0 \) and \( p^*_1 \):

\[ \frac{\partial}{\partial r^*} \left[ r^* \frac{dp^*_0}{dr^*} \right] = \frac{-12r^*}{f^*_0(h^*, l^*)} \]  
(17)

\[ \frac{d}{dr^*} \left[ r^* \frac{dp^*_1}{dr^*} \right] = -f^*_1(h^*, l^*, \psi) \frac{d}{dr^*} \left[ p^*_0 \right] \]  
(18)
Where,

\[ f'_0(h^*, I^*, \psi) = h^{13} - 12l^{12}h^* + 24l^{13} \tanh(h^*/2l^*) + \frac{12\psi}{(1 - \beta)} \]

\[ f'_1(h^*, I^*, \psi) = -h^{13} + 6l^{12}h^* \left( 4 + \sech^2(h^*/2l^*) \right) - 60l^{13} \tanh(h^*/2l^*) - \frac{12\psi}{(1 - \beta)^2} \]

Solving the equations (17) and (18) under the pressure boundary conditions

\[ \frac{dp_0^*}{dr^*} = \frac{dp_1^*}{dr^*} = 0 \text{ at } r^* = a^* \]

\[ p_0^* = p_1^* = 0 \text{ at } r^* = 1 \]

The dimensionless pressure obtained in the film region is given by

\[ p^* = \frac{3(a^{*2} - 1)}{f'_0(h^*, I^*, \psi)} \left[ \frac{\log a^* - (r^{*2} - 1)}{\log a^* - (a^{*2} - 1)} \right] - \frac{9(a^{*2} - 1)^2 f'_1(h^*, I^*, \psi)}{2f'_0(h^*, I^*, \psi)} \left[ \frac{(\log a^*)^2}{(a^{*2} - 1)} \right] \]

The load carrying capacity \( W \) on the annular bearing is defined as:

\[ W = 2\pi \int_a^1 prdr \]

The non-dimensional load-carrying capacity \( W^* \) is obtained in the form

\[ W^* = \frac{3\pi(a^{*2} - 1)^2}{2f'_0(h^*, I^*, \psi)} \left[ \frac{1}{\log a^* - (a^{*2} - 1)} \right] - \frac{9\pi(a^{*2} - 1)^2 f'_1(h^*, I^*, \psi)}{4f'_0(h^*, I^*, \psi)} \left[ \frac{3(a^{*2} - 1)}{2\log a^* - (a^{*2} - 1)} - \frac{2(a^4 + a^{*2} + 1)}{3(a^{*2} - 1)} \right] \]

The squeezing time can be calculated by integrating (24) with respect to \( h^* \) under the condition \( h^* = 1 \) at \( t^* = 0 \) as follows

\[ T^* = \int_0^1 \left[ \frac{3\pi(a^{*2} - 1)^2}{2f'_0(h^*, I^*, \psi)} \left[ \frac{1}{\log a^* - (a^{*2} - 1)} \right] - \frac{9\pi(a^{*2} - 1)^2 f'_1(h^*, I^*, \psi)}{4f'_0(h^*, I^*, \psi)} \left[ \frac{3(a^{*2} - 1)}{2\log a^* - (a^{*2} - 1)} - \frac{2(a^4 + a^{*2} + 1)}{3(a^{*2} - 1)} \right] \right] dh^* \]

3. Results and discussions

Combined effect of pressure dependent viscosity and non-Newtonian couple stresses in Porous Annular plate has been analysed in this paper. The dimensionless pressure, load carrying capacity and squeeze film time are the function of \( I^*, \) step length \( h \) and \( \psi. \) The parametric range used for the discussion of squeeze film characteristics are:

\[ \psi = 0.0001, 0.001, 0.01, 0.1 \quad G = 0, 0.02, 0.04, 0.06 \text{ and } l^* = 0, 0.1, 0.2, 0.3 \]
3.1. The thin film Pressure
Variation of dimensionless pressure $P^*$ with $r^*$ for several values of $G$ with $h^* = 0.001$, $a^* = 0.3$, $\beta = 0.2$ is depicted in Figure 2. It is observed that the pressure increases with increasing values of $G$. Also non-dimensional pressure $P^*$ is varied with respect to $r^*$ for different values of $l^*$ with $h^* = 0.7$, $G = 0.04$, $a^* = 0.3$, $\psi = 0.001$, $\beta = 0.2$ the same is depicted in Figure 3 and is observed that $P^*$ increases with increasing values of $l^*$. The variation of non-dimensional pressure $P^*$ with $r^*$ for different values of $\psi$ with $h^* = 0.7$, $l^* = 0.3$, $a^* = 0.3$, $G = 0.04$, $\beta = 0.2$ presented in figure 3. This gives a clear observation that as $\psi$ value increases the pressure decreases.

3.2. The thin film load Carrying capacity
From Figure 5, we can clearly understand that the variation of load carrying capacity $W^*$ with $h^*$ and found that $W^*$ increases with increasing values of $G$. We can see the significant increase in the dimensionless load carrying capacity $W^*$ when $l^*$ is increased from the figure 6. Figure 7 shows the variation of load $W^*$ verses $h^*$ for different values of $\psi$, with the fixed values $l^* = 0.3$, $G = 0.04$, $a^* = 0.3$, $\beta = 0.2$. It has been observed that load decreases with increasing values of $\psi$.

3.3. Squeeze film time
In Figures 8, 9 and 10 the dimensionless squeeze time $T^*$ is plotted against $h_{1}^*$ for various different values of $G$, couplestress parameter $l^*$ and permeability parameter $\psi$ respectively keeping the other parameter are fixed. In all these, as decreasing values of $h_{1}^*$, dimensionless squeeze time $T^*$ increases with increasing values of viscosity($G$), couplestress($l^*$) where as $T^*$ decreases with increasing value of permeability($\psi$).

4. Conclusions
The squeeze film characteristics for pressure dependent viscosity and non-Newtonian couple stresses in porous annular plates is studied.

The pressure, load, squeezing time decreases with increasing values of $\psi$.

The pressure, load, squeezing time increases with increasing values of $G$ and $l^*$.

![Figure 1: Physical geometry of porous annular plates](image-url)
Figure 2: Variation of non-dimensional pressure $P^*$ with $r^*$ for different values of $G$ with $h^* = 0.7$, $l^* = 0.3$, $a^* = 0.3$, $\varphi = 0.001$, $\beta = 0.2$

Figure 3: Variation of non-dimensional pressure $P^*$ with $r^*$ for different values of $l^*$ with $h^* = 0.7$, $G = 0.04$, $a^* = 0.3$, $\varphi = 0.001$, $\beta = 0.2$

Figure 4: Variation of non-dimensional pressure $P^*$ with $r^*$ for different values of $\varphi$ with $h^* = 0.7$, $l^* = 0.3$, $a^* = 0.3$, $G = 0.04$, $\beta = 0.2$.

Figure 5: Variation of non-dimensional Load carrying capacity $W^*$ with $h^*$ for different values of $G$, with $l^* = 0.3$, $a^* = 0.3$.

Figure 6: Variation of non-dimensional Load carrying capacity $W^*$ with $h^*$ for different values of $l^*$, with $G = 0.04$, $a^* = 0.3$, $\varphi = 0.001$, $\beta = 0.2$.

Figure 7: Variation of non-dimensional Load carrying capacity $W^*$ with $h^*$ for different values of $\varphi$, with $l^* = 0.3$, $G = 0.04$, $a^* = 0.3$, $\beta = 0.2$. 
Figure 8: Variation of squeeze film time $T^*$ with $h_1^*$ for different values of $G$, with $l^* = 0.3$, $a^* = 0.3$, $\varphi = 0.001$, $\beta = 0.2$.

Figure 9: Variation of squeeze film time $T^*$ with $h_1^*$ for different values of $l^*$, with $G = 0.04$, $a^* = 0.3$, $\varphi = 0.001$, $\beta = 0.2$.

Figure 10: Variation of squeeze film time $T^*$ with $h_1^*$ for different values of $\varphi$, with $G = 0.04$, $l^* = 0.3$, $a^* = 0.3$, $\beta = 0.2$.

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