A local realist theory of parametric down conversion

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Abstract

In a series of articles we have shown that all parametric-down-conversion processes, both of type-I and type-II, may be described by a positive Wigner density. These results, together with our description of how light detectors subtract the zeropoint radiation, indicated the possibility of a completely local realist description of all these processes. In the present article we show how the down-converted fields may be described as retarded fields generated by currents inside the nonlinear crystal, thereby achieving such a theory. Most of its predictions coincide with the standard nonlocal theory. However, the intensities of the down converted signals do not correspond exactly with the photon pairs of the nonlocal theory. For example, in a blue-red down conversion we would find about 1.03 red "photons" for every blue one. The theory also predicts a new phenomenon, namely parametric up conversion from the vacuum.

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1 Introduction

We have treated type-I parametric down conversion (PDC) processes, in the Wigner formalism, in a series of articles[1, 2, 3], and we extended this treatment to type-II PDC in [4]. In all of these processes, the resulting Wigner density is positive, as is, rather trivially, that of the vacuum. We have also proposed, in these articles, a theory of detection which is formally almost identical with the standard normal-ordering prescription of quantum optics. However, our description of the detection process recognizes that the vacuum fluctuations are real, so an important element of the theory is the manner in which detectors are able to extract signals from the rather large zeropoint noise background. This problem was discussed in [4], and we indicated the way towards its solution.
The approach of the above series of articles was a kind of compromise between the standard, nonlocal theory of Quantum Optics, where the interaction of the various field modes is represented by a Hamiltonian, and a fully Maxwellian theory, which would be both local and causal. In this latter case, the nonlinear crystal would be represented as a spatially localized current distribution, modified of course by the incoming electromagnetic field; the outgoing field would then be expressed as the retarded field radiated by this distribution. A preliminary attempt at such a theory was made\cite{5}, using first-order perturbation theory. However, we showed, in the above series of articles, that a calculation of the relevant counting rates, to lowest order, requires us to find the second-order perturbation corrections to the Wigner density, and the close formal parallel between these two theories means that the same considerations will apply to the Maxwellian theory.

If we were to take account of the tensor character of the polarizabilities, this would represent a rather formidable task. In this article we study a simplified model of the crystal, in which the electric field, and hence also the linear and nonlinear polarizabilities, are considered to be scalars. In such a model it is not possible to discuss the polarization correlation of the signal with the idler, so we are reverting to type-I PDC, which means we confine attention to the frequency and angular correlations between these two beams.

We shall also make the simplifying assumption that the crystal is infinitely large in the directions perpendicular to the pumping beam. This reduces the problem to a single spatial dimension, and, after making a certain linearization approximation, allows us to pass to a nonperturbative treatment of the process. We shall simplify the algebra by assuming a constant value for the nonlinear polarizability, but it will nevertheless be essential to retain an explicit frequency dependence for the linear part of the polarizability.

## 2 The linearization procedure

Provided the pumping laser is sufficiently intense and coherent, it is possible to neglect the depletion in its intensity which occurs when it interacts with other modes of the light field. This leads to a linearization of the field equation inside the nonlinear crystal. We remark that this procedure is essentially the same as is used in the standard photon analysis, where, by treating the laser amplitude as a $c$-number, a cubic interaction term in the Hamiltonian is reduced to a quadratic.

The scalar electric field $E(x, y, z, t)$ satisfies the wave equation

\[ \nabla^2 E - \ddot{E} = 0 \quad (1) \]

outside the crystal, and

\[ \nabla^2 E - \ddot{E} = -4\pi \bar{J} \quad (2) \]
inside the crystal, where \( J(x,y,z,t) \) is the current. The relation connecting \( J \) with \( E \) is

\[
J(r,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dt' i\omega f(\omega) e^{i\omega t-i\omega t'} E(r,t') + g'[E(r,t)]^2 ,
\]

where \( f(\omega) \) is analytic in the lower half plane, so that the integration on \( t' \) may be taken from minus infinity to \( t \) only. The refractive index is then given by

\[
\mu^2(\omega) = 1 + 4\pi f(\omega) .
\]

The Fourier transformed wave equation, inside the crystal, is

\[
\Delta \tilde{E}(\omega) - \omega^2 \mu^2(\omega) \tilde{E}(\omega) = -4\pi i\omega g' \int E^2(t)e^{i\omega t} dt .
\]

We shall suppose that the laser field inside the crystal is

\[
E_L(x,y,z,t) = V \cos[\omega_0 \mu(\omega_0) z - \omega_0 t] ,
\]

which represents a plane wave travelling from left to right. We should include a right-to-left wave resulting from internal reflection of the laser, but, in the linearization approximation we are about to describe, such a wave simply produces additional but independent pairs of down-converted signals. These are easily treated by our method, but for simplicity we shall not include them. Our linearization now consists of putting \( E = E_L + E' \), and discarding terms in \( E'^2 \).

Provided we do not have \( \omega \) close to a multiple of \( \omega_0 \), we may also discard the terms in \( E^2_L \), so our linearized wave equation, putting \( g = 4\pi g' V/\omega_0 \), is

\[
\Delta \tilde{E}'(\omega) - \omega^2 \mu^2(\omega) \tilde{E}'(\omega) = -i g\omega_0 [\tilde{E}'(\omega - \omega_0)e^{i\omega_0 \mu(\omega_0) z} + \tilde{E}'(\omega + \omega_0)e^{-i\omega_0 \mu(\omega_0) z}] .
\]

3 What is PDC?

It is necessary to pose this question, because, depending on the answer given, PDC may be described as either a local or a nonlocal phenomenon. This is because the view we are advocating requires us to recognize the reality of the zeropoint electromagnetic field (ZPF). The connection between a real ZPF and locality, in the context of PDC, was discussed in the last article of the Wigner series 4.

An example of the modern, nonlocal description is provided by Greenberger, Horne and Zeilinger 6. A nonlinear crystal (NLC), pumped by a laser at frequency \( \omega_0 \), produces conjugate pairs of signals, of frequency \( \omega \) and \( \omega_0 - \omega \) (see Fig.1). Since the modern wisdom is that light consists of photons, this means that an incoming laser photon “down converts” into a pair of lower-energy photons. Naturally, since we know that \( E = h\omega \), that means energy is conserved in the PDC process, which must be very comforting.
There is an older description, which I suggest is more correct than the modern one. It had only a short life. Nonlinear optics was born in the late 1950s, with the invention of the laser, and, up to about 1965, when Quantum Optics was born, the PDC process would have been depicted\cite{7} by Fig 2: an incoming wave of frequency $\omega$ is down converted, by the pumped crystal, into an outgoing signal of frequency $\omega_0 - \omega$. The explanation of the frequency relationships lies in the multiplication, by the nonlinear crystal, of the two input amplitudes; we have no need of $\hbar$!

This process persists when the intensity of the input is reduced to zero. This is because all modes of the light field are still present in the vacuum, and the nonlinear crystal modifies vacuum modes in exactly the same way as it modifies input modes supplied by an experimenter. What we see emerging from the crystal is the familiar PDC rainbow. This is because the angle of incidence $\theta$, at which PDC occurs, is different for different frequencies, on account of the variation of refractive index with frequency. We depict the process of PDC from the vacuum in Fig 3, but note that this figure shows only two conjugate modes of the light field; a complete picture would show all frequencies participating in conjugate pairs, with varying angles of incidence. In contrast with Fig 2, where we showed only the one relevant input, we must now take account also of the conjugate input mode of the zeropoint, since the first mode itself has only the zeropoint amplitude. The zeropoint inputs, denoted by interrupted lines in Fig 3, do not activate photodetectors, because the threshold of these devices is set precisely at the level of the zeropoint intensity, as discussed in ref\cite{4}. However, as we shall see in the next section, the two idlers have intensities above that of their corresponding inputs. Also there is no coherence between a signal and an idler of the same frequency, so their intensities are additive in both channels. Hence there are photoelectron counts in both of the outgoing channels of Fig 3.

The question we have posed in this section could be rephrased as “What is it that is down converted?” According to the thinking behind Fig 1, the laser photons are down converted, whereas according to Fig 3 it is the zeropoint modes; they undergo both down conversion, to give signals, and amplification, to give idlers.

4 The down-conversion process

We begin by reviewing the standard treatment of linear dielectric laminas, in the approximate form described in Freedman’s thesis\cite{8} and used by many since, including Aspect\cite{9} and ourselves\cite{10}. The approximation holds if the thickness $l$ of the lamina is large compared with the wavelength, and ignores interference effects between successive internal reflections. The lamina would need to be extremely accurately cut, in any case, for such effects to be observed. Let the
lamina occupy the region $0 < z < l$. A plane scalar wave is represented by

$$E(x, y, z, t) = e^{ip_xx + ip_yy + i\Omega_0(p)z - i\omega t},$$  \hspace{1cm} (8)

where

$$\Omega_0^2(p) = \omega^2 - p^2 \quad \text{and} \quad p^2 = p_x^2 + p_y^2.$$  \hspace{1cm} (9)

We shall put

$$E(x, y, z, t) = F(z, t)e^{ip_xx + ip_yy}.$$  \hspace{1cm} (10)

Then, for $g = 0$, a solution of eq.(7) may be found in the form

$$F(z, t) = e^{-i\omega t}(e^{i\Omega_0z} + Re^{-i\Omega_0z})(z < 0),$$

$$F(z, t) = e^{-i\omega t}(Ae^{i\Omega_0z} + Be^{-i\Omega_0z})(0 < z < l),$$

$$F(z, t) = e^{-i\omega t}Te^{i\Omega_0z}(z > l),$$  \hspace{1cm} (11)

where

$$\Omega^2(p) = \omega^2 \mu^2(\omega) - p^2.$$  \hspace{1cm} (12)

The four constants $(R, A, B, T)$ may be determined by imposing the conditions that $F$ and $\partial F/\partial z$ be continuous at $z = 0$ and $z = l$. A solution procedure consists of first putting $B = 0$ and imposing the boundary conditions at $z = 0$ only, thereby obtaining values for $R$ and $A$; then we impose the boundary conditions at $z = l$, using the value previously obtained for $A$, to obtain $B$ and $T$; then we go back to $z = 0$ with the new value of $B$ and calculate corrected values for $R$ and $A$; and so on. If we were to sum the infinite series, this would give us an exact solution. The first step gives

$$R_0 = \frac{\Omega_0 - \Omega}{\Omega_0 + \Omega},$$

$$A_0 = \frac{2\Omega_0}{\Omega_0 + \Omega}.$$  \hspace{1cm} (13)

The corresponding intensity coefficients are obtained by considering the ratios of the Poynting vector’s $z$-component, that is $E\partial E/\partial z$. They are

$$r_0 = R_0^2 \quad \text{and} \quad t_0 = A_0^2\Omega/\Omega_0 = 1 - r_0.$$  \hspace{1cm} (14)

Note that we are here neglecting the imaginary part of $\mu$, which gives rise to a small absorption rate. The second step gives, apart from a phase factor which we do not need,

$$B_0 = A_0R_0 \quad \text{and} \quad T_0 = A_0^2\Omega/\Omega_0,$$  \hspace{1cm} (15)

which again gives the intensity coefficients $r_0$ and $t_0$. Our approximation consists of simply multiplying by the appropriate intensity factors for all internal
reflections and transmissions, thereby arriving at the overall coefficients
\[ r = r_0 + r_0 t_0^2 + r_0^3 t_0^4 + \ldots = \frac{2r_0}{1 + r_0}, \]
\[ t = t_0^2 + t_0^2 r_0^2 + t_0^2 r_0^4 + \ldots = \frac{1 - r_0}{1 + r_0}. \]  (16)

These, naturally, are independent of the lamina thickness, and satisfy \( r + t = 1 \). Of course, in this case of \( g = 0 \), the above result is completely unaffected by the presence of a pumping laser.

Now we shall extend this theory to the nonlinear dielectric, described in the previous section. We put
\[ F(z, t) = \tilde{F}(z, \omega) e^{-i\omega t} + \tilde{F}(z, \omega - \omega_0) e^{i\omega_0 t - i\omega t}. \]  (17)

Then a solution of eq.(7) is
\[ \tilde{F}(z, \omega) = \sum_{r=1}^{4} \alpha_r e^{ik_r z}, \]
\[ \tilde{F}(z, \omega - \omega_0) = i \sum_{r=1}^{4} \beta_r e^{i[k_r - \omega_0\mu(\omega_0)]z}, \]  (18)

where
\[ [k_r^2 - \Omega_r^2(p)]\alpha_r = g\omega_0\omega_0\beta_r, \]
\[ ([k_r - \omega_0\mu(\omega_0)]^2 - \Omega_r^2(p))\beta_r = g\omega_0(\omega_0 - \omega)\alpha_r, \]  (19)

and
\[ \Omega_1(p) = +\sqrt{\omega^2 - p^2}, \]
\[ \Omega_2(p) = +\sqrt{(\omega_0 - \omega)^2 - p^2}. \]  (20)

We define also the corresponding free-space quantities
\[ \Omega_{10}(p) = +\sqrt{\omega^2 - p^2}, \]
\[ \Omega_{20}(p) = +\sqrt{(\omega_0 - \omega)^2 - p^2}. \]  (21)

We now put
\[ k_1 = \Omega_1 + \varepsilon_1, \]
\[ k_2 = \Omega_1 + \varepsilon_2, \]
\[ k_3 = -\Omega_1 + \varepsilon_3, \]
\[ k_4 = \Omega_2 + \omega_0\mu(\omega_0) + \varepsilon_4. \]  (22)

\(^1\)Here we make the approximation of discarding the coupling with waves of frequency \( \omega + \omega_0 \). This is justified because we are assuming that \( \omega \) and \( p \) are close to the PDC resonance condition given by eq.(23), and are far from the corresponding PUC resonance condition. In section \( \square \) we shall be assuming that the reverse is the case.
There is a particular value of $p$, which we shall designate $p_0$, satisfying the condition
\[ \Omega_1(p_0) + \Omega_2(p_0) = \omega_0 \mu(\omega_0) . \] (23)
This defines the direction $\theta(\omega)$ (see Fig.2) of the zeropoint field component, of frequency $\omega$, for which the PDC “resonance” has its maximum. We denote
\[ \omega_1 = \Omega_1(p_0) , \quad \omega_2 = \Omega_2(p_0) , \] (24)
and define $\omega_{10}$ and $\omega_{20}$ similarly. Then, for $g << 1$ and $|p-p_0| << \omega$, $\epsilon_r$ are given by
\[ \epsilon_1 + \epsilon_2 = (p - p_0) \frac{p_0(\omega_1 + \omega_2)}{\omega_1 \omega_2} , \] (25)
\[ \epsilon_1 \epsilon_2 = \frac{g^2 \omega(\omega_0 - \omega) \omega_1^2}{4 \omega_1 \omega_2} , \] (26)
\[ \epsilon_3 = - \frac{g^2 \omega(\omega_0 - \omega) \omega_1^2}{8(\omega_1 + \omega_2) \omega_1^2} , \] (27)
\[ \epsilon_4 = \frac{g^2 \omega(\omega_0 - \omega) \omega_2^2}{8(\omega_1 + \omega_2) \omega_2^2} . \] (28)
Because of the smallness of $\epsilon_2 - \epsilon_1$, the modes $k_1$ and $k_2$ are strongly coupled, and the phase relations between them, at both the frequencies $\omega$ and $\omega_0 - \omega$, are carried from one side of the crystal to the other. These modes all represent left-to-right waves. On the other hand, the modes associated with $k_3$ and $k_4$ represent right-to-left waves. Because of the smallness of $\epsilon_3$ and $\epsilon_4$, $k_3$ and $k_4$ are well separated, and these two waves are effectively unaltered by the interaction with the laser, which is explained by the fact that the laser wave travels in the opposite direction to them. We therefore put
\[ \alpha_1 = A_1 , \quad \alpha_2 = A_2 , \quad \alpha_3 = A_3 , \quad \beta_4 = A_4 , \] (29)
and then, neglecting $\epsilon_3$ and $\epsilon_4$, the internal field (18) becomes
\[ F(z, t) = A_1 \left[ e^{i(\Omega_1 + \epsilon_1)z - i\omega t} + \frac{2\epsilon_1 \epsilon_2}{g \omega \omega_1} e^{-i(\omega_0 \mu(\omega_0) - \Omega_1 - \epsilon_1)z + i(\omega_0 - \omega)t} \right] + A_2 \left[ e^{i(\Omega_1 + \epsilon_2)z - i\omega t} + \frac{2\epsilon_1 \epsilon_2}{g \omega \omega_1} e^{-i(\omega_0 \mu(\omega_0) - \Omega_1 - \epsilon_2)z + i(\omega_0 - \omega)t} \right] + A_3 e^{-\Omega_1 z - i\omega t} + A_4 e^{\Omega_1 z + i(\omega_0 - \omega)t} . \] (30)
This must be matched with
\[ F(z, t) = e^{i\Omega_{10} z - i\omega t} + R_1 e^{-i\Omega_{10} z - i\omega t} + R_2 e^{i\Omega_{20} z + i(\omega_0 - \omega)t} \] (31)
for $z < 0$, and
\[ F(z, t) = T_1 e^{i\Omega_{10} z - i\omega t} + T_2 e^{-i\Omega_{20} z + i(\omega_0 - \omega)t} \] (32)
for $z > l$. We must impose four boundary conditions (continuity of $F$ and of $\partial F/\partial z$ for both frequencies) at $z = 0$, and a similar four at $z = l$. This will determine the eight constants $(R_1, R_2, T_1, T_2, A_1, A_2, A_3, A_4)$. Our iteration procedure consists in putting $A_3 = A_4 = 0$, and then solving for $(R_1, R_2, A_1, A_2)$ using only the boundary conditions at $z = 0$; then, using these values for $A_1$ and $A_2$, we solve for $(A_3, A_4, T_1, T_2)$ using the boundary conditions at $z = l$; then, using these values of $A_3$ and $A_4$, we calculate the corrections to $(R_1, R_2, A_1, A_2)$ using the boundary conditions at $z = 0$; and so on. This series of iterations is greatly simplified by making the same approximation as we made above in the linear case, that is to say by neglecting interference effects between successive reflections inside the crystal, with the exception that in this case, for the reasons given above, we cannot neglect interference between $A_1$ and $A_2$.

The first step of the above procedure leads to the four equations

\[
\begin{align*}
1 + R_1 &= A_1 + A_2, \\
\omega_1\omega_1(1 - R_1) &= (\omega_1 + \epsilon_1)A_1 + (\omega_1 + \epsilon_2)A_2, \\
R_2 &= \frac{2\omega_1}{g\omega_0}(\epsilon_1 A_1 + \epsilon_2 A_2), \\
-\omega_2 R_2 &= \frac{2\omega_1\omega_2}{g\omega_0}(\epsilon_1 A_1 + \epsilon_2 A_2),
\end{align*}
\]  

(33)

to which the solution is

\[
\begin{align*}
R_1 &= \frac{\omega_1 - \omega_{10}}{\omega_{10} + \omega_1}, \\
R_2 &= 0, \\
A_1 &= \frac{2\epsilon_2\omega_{10}}{(\epsilon_2 - \epsilon_1)(\omega_{10} + \omega_1)}, \\
A_2 &= \frac{-2\epsilon_1\omega_{10}}{(\epsilon_2 - \epsilon_1)(\omega_{10} + \omega_1)}. \\
\end{align*}
\]

(34)

A similar matching at $z = l$ gives, for the next step in the procedure, the result below. As in the linear case some phase factors, and also some dissipation factors close to one, have been omitted.

\[
\begin{align*}
T_1 &= \frac{4\omega_1\omega_{10}}{(\omega_1 + \omega_{10})^2} (1 - i\epsilon_1 le^{-i\xi}\text{sinc}\xi), \\
T_2 &= \frac{2g\omega_0\omega_{10}(\omega_0 - \omega)}{(\omega_1 + \omega_{10})(\omega_2 + \omega_{20})}\text{sinc}\xi, \\
A_3 &= \frac{2(\omega_1 - \omega_{10})\omega_{10}}{(\omega_1 + \omega_{10})^2} (1 - i\epsilon_1 le^{-i\xi}\text{sinc}\xi), \\
A_4 &= \frac{g\omega_0\omega_{10}(\omega_2 - \omega_{20})(\omega_0 - \omega)}{\omega_2(\omega_1 + \omega_{10})(\omega_2 + \omega_{20})}\text{sinc}\xi, \\
\end{align*}
\]

(35)
where we have made use of eq.(26), and have put
\[ \xi = \frac{(\epsilon_1 - \epsilon_2)d}{2} \text{ and } \sin \xi = \frac{\sin \xi}{\xi}. \] (36)

In an obvious extension of the notation we used for the linear case, the Poynting vectors associated with these amplitudes may be written as
\[
\begin{align*}
\omega_{10}T_1T_1^* &= \omega_{10}T_1^2(1 + \gamma), \\
\omega_{20}T_2T_2^* &= \omega_{10}t_{10}t_{20}\gamma(\omega_0/\omega - 1), \\
\omega_1 A_3 A_3^* &= \omega_{10}t_{10}r_{10}(1 + \gamma), \\
\omega_2 A_4 A_4^* &= \omega_{10}t_{10}r_{20}\gamma(\omega_0/\omega - 1),
\end{align*}
\] (37)

where the new feature, associated with the down-conversion process, is the coefficient
\[ \gamma = \frac{g^2\omega_0^2(\omega_0 - \omega)}{4\omega_1\omega_2} \sin^2 \xi. \] (38)

As in the linear case, the overall reflection and transmission coefficients are obtained by simply adding the intensities associated with each set of internal reflections. Recalling that, because the \( k_3 \) and \( k_4 \) waves are nondegenerate, the factor \( \gamma \) is zero for right-to-left waves, this gives us, to first order in \( \gamma \),
\[
\begin{align*}
r_1 &= r_{10} + r_{10}T_{10}^2(1 + \gamma) + r_{10}^2T_{10}^2(1 + 2\gamma) + \ldots \\
&= \frac{2r_{10}}{1 + r_{10}} + \gamma r_{10} \frac{1 + r_{10}}{(1 + r_{10})^2}, \quad (39) \\
t_1 &= \frac{1 - r_{10}}{1 + r_{10}} + \gamma \frac{1 + r_{10}}{(1 + r_{10})^2}, \quad (40) \\
r_2 &= (\omega_0/\omega - 1)(t_{10}\gamma r_{20}t_{20} + t_{10}\gamma r_{20}^2t_{20} + t_{10}r_{10}^2\gamma r_{20}t_{20} + \ldots) \\
&= \frac{(\omega_0 - \omega)\gamma r_{20}}{\omega(1 + r_{10})(1 + r_{20})}, \quad (41) \\
t_2 &= \frac{(\omega_0 - \omega)\gamma}{\omega(1 + r_{10})(1 + r_{20})}, \quad (42)
\end{align*}
\]
where all of these coefficients have been defined, as in the linear case, by ratios between the \( z \)-components of the Poynting vectors. We deduce that
\[ t_1 + r_1 - 1 = \frac{\omega}{\omega_0 - \omega}(t_2 + r_2) = \frac{\gamma}{1 + r_{10}}. \] (43)

In the scalar version of stochastic electrodynamics\[12\] the modes of the zeropoint field all have the same amplitude, namely \( \sqrt{\hbar/2L^3} \); this differs from the
Maxwell version by a factor of $\sqrt{\omega}$, the difference being accounted for by the two expressions for the Poynting vector. Hence the number of “photons” in a given mode, including the undetected half photon of the zeropoint, is proportional to the modulus-square of that mode’s amplitude. So the above calculation gives, for the number of idler photons emerging from unit area of the face $z = l$, and arising from an incident wave of zeropoint amplitude and frequency $\omega$,

\[ n_i(\omega) = \frac{t_1 + r_1 - 1}{2}, \tag{44} \]

where we have subtracted the zeropoint intensity in accordance with the theory described in Ref.\[4\], while the number of signal photons is

\[ n_s(\omega - \omega) = \frac{(t_2 + r_2)\omega_{10}/(2\omega_{20})}{\cos[\theta(\omega)]} \tag{45} \]

Now we must refer back to Fig.3. It will be observed that the total number of “photons” in the $\omega$-channel is obtained by adding the idler photons from one input to the signal photons from the conjugate input, that is, using eq.(43),

\[ n_i(\omega) + n_s(\omega) = \frac{\gamma}{2} \left( \frac{1}{1 + r_{10}} + \frac{1}{1 + r_{20}} \frac{\cos[\theta(\omega_0 - \omega)]}{\cos[\theta(\omega)]} \right). \tag{46} \]

The corresponding output in the other channel is

\[ n_i(\omega_0 - \omega) + n_s(\omega_0 - \omega) = \frac{\gamma}{2} \left( \frac{1}{1 + r_{20}} + \frac{1}{1 + r_{10}} \frac{\cos[\theta(\omega)]}{\cos[\theta(\omega_0 - \omega)]} \right) . \tag{47} \]

Hence the ratio of the photon fluxes is

\[ \frac{n_i(\omega) + n_s(\omega)}{n_i(\omega_0 - \omega) + n_s(\omega_0 - \omega)} = \frac{\cos[\theta(\omega_0 - \omega)]}{\cos[\theta(\omega)]} . \tag{48} \]

So we conclude that the photon rate in a given channel is inversely proportional to the cosine of the rainbow angle.\[\footnote{Actually there are two rainbows — a forward one with intensities \((t_1, t_2)\) and a backward one with \((r_1, r_2)\). This simple relation is between the sums of the intensities in the two rainbows. It becomes rather more complicated if we confine attention to just the forward rainbow, but, since that contains about 96 percent of the total intensity, this simple relation is still almost exact.} \] In the standard nonlocal theory associated with Fig.1, by contrast, the above ratio is one. Indeed, it is an essential part of the energy-conservation argument that PDC “photons” must be created in pairs. In the local, consistently field-theoretic approach we are advocating here, energy is still conserved, but the units for energy transactions are no longer photons. Indeed, although the result we just obtained was stated in terms of photon fluxes, these are really just Poynting vectors with an appropriate zeropoint subtraction.

There seems little chance of finding out directly which of these theories is correct; the difference between the two ratios is small, since the rainbow...
angles are typically around 10 degrees, and it is not possible to measure at all accurately the efficiency of light detectors as a function of frequency. It is true that some of the experiments we have analysed, using the standard theory, in Refs. [1, 2, 3, 4], have slightly different results in the present theory, for example the fringe visibility in the experiment of Zou, Wang and Mandel [11]. Some details will be published shortly, but we can say that an experimental discrimination will be very difficult.

5 Parametric up conversion from the vacuum

There is, however, at least one prediction of the new theory which differs dramatically from the standard theory. An incident wave of frequency \( \omega \), as well as being down converted, by the pump, to give a PDC signal of frequency \( \omega_0 - \omega \), may also be up converted to give a PUC signal of frequency \( \omega_0 + \omega \). We depict this phenomenon, which is well known in classical nonlinear optics, in Fig.4. Note that the angle of incidence, \( \theta_u(\omega) \), at which PUC occurs is quite different from the PDC angle, which in Fig.2 was denoted simply \( \theta(\omega) \), but which we should now call \( \theta_d(\omega) \).

Now, following the same argument which led us from Fig.2 to Fig.3, we predict the phenomenon of PUC from the Vacuum, which we depict in Fig.5.

Let us calculate the intensity of this PUC rainbow. There is an important difference from the PDC situation, arising from a different relation between the frequency eigenvalues inside the crystal. Eqn.(26) must be replaced by

\[
e_1e_2 = \frac{g^2 \omega(\omega_0 + \omega)\omega_0^2}{4\omega_1\omega_2}.
\]

As a consequence we find that, in contrast with PDC, the intensity of the PUC idler is less than that of the input, and eq.(43) must be replaced by

\[
1 - t_1 - r_1 = \frac{\omega}{\omega_0 + \omega}(t_2 + r_2) = \frac{\gamma}{1 + r_{10}}.
\]

It now follows that the “photon” fluxes in the two outgoing channels are

\[
n_i(\omega) + n_s(\omega) = \frac{\gamma}{2} \left( \frac{1}{1 + r_{20}} \frac{\cos[\theta(\omega_0 + \omega)]}{\cos[\theta(\omega)]} - \frac{1}{1 + r_{10}} \right),
\]

\[
n_i(\omega_0 + \omega) + n_s(\omega_0 + \omega) = \frac{\gamma}{2} \left( \frac{1}{1 + r_{10}} \frac{\cos[\theta(\omega)]}{\cos[\theta(\omega_0 + \omega)]} - \frac{1}{1 + r_{20}} \right).
\]

The cosines occurring here all differ from one by a few percent. The reflection coefficients differ from zero by a few percent, and from each other by a few tenths
of a percent. Since (see Fig.5) the cosine of $\theta(\omega_0 + \omega)$ is greater than the cosine of $\theta(\omega)$, it follows that the photon flux in the $\omega$-channel is positive, while that in the $(\omega_0 + \omega)$-channel is negative, which means simply that the overall intensity there is below the zeropoint and that no detection events will occur. We expect to see detection events only in the $\omega$-channel. A comparison between eqs. (51) and (46) shows that, because of the sign difference, and the closeness of the cosines to one and of the reflection coefficients to zero, the intensity of the PUC rainbow is expected to be only a few percent of the PDC rainbow. This may explain why nobody has yet reported seeing it.

We may calculate quite easily the approximate relative positions of the PDC and PUC rainbows. In the PDC case the rainbow angle is determined by eq.(23). For simplicity we shall consider the case $\omega = \omega_0/2$. Then, defining

$$q_d = \sin^2[\theta_d(\omega_0/2)] \quad , \quad \mu_1 = \mu(\omega_0/2) \quad , \quad \mu_2 = \mu(\omega_0),$$

eq.(23) tells us that

$$q_d = \mu_2 - \mu_2^2.$$  \hspace{1cm} (54)

Now, similarly, let us define

$$q_u = \sin^2[\theta_u(\omega_0/2)] \quad , \quad \mu_3 = \mu(3\omega_0/2).$$

The equation defining the PUC rainbow angle at this frequency is

$$-\sqrt{\mu_3^2 - q_u} + \sqrt{9\mu_3^2 - q_u} = 2\mu_2,$$  \hspace{1cm} (56)

which gives

$$q_u = \frac{1}{16\mu_2^2} [36\mu_1^2\mu_3^2 - (9\mu_3^2 - 4\mu_2^2 + \mu_1^2)^2].$$

Substituting in eq.(57) this gives us that

$$q_u = 6q_d - \frac{25q_d^2}{4\mu_2^2}.$$  \hspace{1cm} (59)

So, if the degenerate PDC mode is at 10 degrees, then the PUC rainbow has its $\omega_0/2$ mode at about 25 degrees, which means that the two rainbows are well separated.

A detailed calculation of the intensity, using the above approximation, gives that the PUC intensity at this frequency is 3.3 percent of the PDC intensity. Naturally this should be taken as an order-of-magnitude prediction only, since the whole calculation was based on a scalar version of the Maxwell equations.

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Figure captions

1. PDC — photon-theoretic version. A laser photon down converts into a conjugate pair of PDC photons with conservation of energy.

2. Classical PDC. When a wave of frequency $\omega$ is incident, at a certain angle $\theta(\omega)$, on a nonlinear crystal pumped at frequency $\omega_0$, a signal of frequency $\omega_0 - \omega$ is emitted in a certain conjugate direction. The modified input wave is called the idler.

3. PDC from the vacuum — field-theoretic version. Both of the outgoing signals are above zeropoint intensity, and hence give photomultiplier counts.
4. PUC. In contrast with PDC the output signal has its transverse component in the same direction as that of the idler.

5. PUC from the vacuum. Only one of the outgoing signals is above the zeropoint intensity. The other one, depicted by an interrupted line, is below zeropoint intensity.
input(\omega) + idler(\omega_0 - \omega)

Figure 3:

zeropoint input(\omega_0 - \omega)

zeropoint input(\omega_0 - \omega)

signal(\omega_0 - \omega) + idler(\omega_0 - \omega)

idler(\omega) + signal(\omega)

Figure 4:

input(\omega)

\theta_\mu(\omega)

laser

NLC

signal(\omega_0 + \omega)

idler(\omega)

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Figure 5: