Peculiarities of the reflective echelon in applications to the kinetic spectroscopy and information optics

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Abstract. In this paper we study specifics of an ultrashort pulse transformation with a device similar to the Michelson reflective echelon. The initial impulse transformation process is treated using the conception of pulse response of the optical system, in particular, of the dynamics of scattering of a $\delta(t-z/c)$ wave by the echelon. The advantage of this method is the possibility to calculate the response of any optical system to a signal of an arbitrary amplitude dependence on time by calculation of convolution of the function describing the signal with the pulse response, whether the pulse response exists. The urgency of the problem is due to the prospects of the echelon application to the solution of numerous modern problems of optics and optical informatics, such as modulation of the THz radiation spectrum, interferometry of ultrashort laser pulses and problems of absorption spectroscopy with high resolution in time.

1. Introduction

The reflective Michelson echelon is one of linear systems designed to analyze and transform the spectrum of a single ultrashort light pulse. Reflection of such a pulse from the set of elements of the echelon converts it into a sequence of copies of the pulse. The spectrum of the obtained series of pulses differs from that of the incident pulse. Our study is aimed onto the development of technique of analysis of the reflective echelon response on a delta-shaped in time plane wave. The urgency of the study is due to the possibility of the echelon application to the solution of a number of modern problems of optics and its technical applications, such as multiple-beam interferometry of ultrashort laser pulses, modulation of radiation, including that of THz band, as well as formation of an equidistant frequency grid in the visible and near-IR bands [1, 2, 3]. The ultrashort light pulse illuminating the echelon is transformed into a sequence of reflected and diffracted impulses, propagating with some time delay relative to each other. For the aims of single-shot spectroscopy, the delay between reflected pulses in time determines the temporal resolution of the system, and even the simplest echelon allows one to achieve the time delay of a few picoseconds. Thus, the use of an echelon makes it possible to obtain a sequence of equidistant ultrashort light pulses and permits to study the dynamics of irreversible superfast processes in solid materials or other patterns [3]. The echelon can also be used in the information optics, as shown in [4], its components can be provided with LCD cells, which can control not only the amplitude but also the phase and polarization of the reflected waves. One may assign logical “1” to the impulse in some “position”, or “0” when being absent, thus forming the coded
massage. As a result, the Michelson echelon has not lost its relevance and can be used in a number of modern problems of optics and its applications.

2. Michelson reflective echelon pulse response
Consider the echelon as an optical system consisted of a set of $N$ parallel plane infinitely long mirrors of equal widths $b$, shifted with respect to one another by $b$ in the direction tangent to the mirror surface and by $d$ in the direction normal to their surfaces. The normal projection of the edges of adjacent mirrors onto a plane parallel to the mirror surfaces are in contact to each other (later on, one can easily transform our formulas to analyze the case when there is some gap between mirror edges). To derive the basic relations describing the operation of echelon over the $\delta$-wave, examine the Fig. 1. For this figure simplification, we assume that the initial wave incidents onto the set of mirrors in a direction normal to their surfaces, i.e. the angle of wave incidence is $\alpha = 0$. The diffracted wave is observed at an angle $\beta \neq 0$ [5].

![Figure 1](image)

**Figure 1.** Reflective Michelson echelon and diffracted waves. The positive $\delta$-wave falls normally onto the mirrors. Positive $\delta$-waves are shown with solid lines, touches are used for negative waves. With double dashed lines the reflected waves are shown, double fat line indicate the diffracted waves observation plane.

Diffraction of waves from one perfect plane mirror of the constant width $b$ is equivalent to the diffraction from the slit of width $b$. As it follows from [5, 6], the pulse response consists of:
- the reflected wave of permanent width $b$ propagating at the velocity of light $c$ normally to the mirror surface (in the case under consideration), and
- two boundary cylinder $\delta$-waves which radius increases at the velocity $c$, in full correspondence with Young’s conceptions;
- at the boundaries between the space illuminated with the reflected wave and the shadow, the edge $\delta$-waves change their sign, so that in the shadow it coincides with that of reflected wave, and in the illuminated zone it is opposite (these peculiarities are shown in Fig. 1 with dashed and solid circles that represent the projection of cylinders onto the plane of figure).

To facilitate the observation of the pulse response at $\beta$ angle, one can use the perfect lens of the focal length $f$ which optical axis is oriented at $\beta$, and detect the wave amplitude (not the intensity!) in its focal point.
On transformation of formula obtained in [5] for the $\delta$-wave scattered from the slit and applying it to one mirror in the case of oblique incidence ($\alpha \neq 0$), for small $\alpha$ and $\beta$ one gets the pulse response of one elementary mirror in the form:

$$h_1(t, f) \approx \frac{1}{2\pi f(\alpha + \beta)}[-\delta(t - \frac{\Delta t_1}{2}) + \delta(t + \frac{\Delta t_1}{2})],$$

(1)

where $\delta(t)$ is a Dirac delta function. The time lag between two edge waves $\delta$-impulses from one mirror is

$$\Delta t_1 = \frac{b}{c}(\sin \alpha + \sin \beta)$$

On neglecting the scattering of light by "vertical" elements $d$ of the echelon, it is quite easy to show that for the initial wave falling on the set of mirrors at the angle $\alpha \neq 0$ and observation of the scattered waves at the angle $\beta \neq 0$, the pulse response of the echelon consisted of $N$ mirrors can be represented as:

$$h(t) = \left[\text{comb}\left(\frac{t}{\Delta t_2}\right) \otimes h_1(t, f)\right] \text{rect}\left(\frac{t}{N\Delta t_2}\right).$$

(2)

In this formula, the convolution (denoted $\otimes$) of the functions in square brackets is nothing but a train consisting of an infinite number of one mirror responses (1). The external factor $\text{rect}(\ldots)$ is the box-function of $N\Delta t_2$ length that takes into account the finite number of $N$ sequence of elements. The symbol comb denotes the "Dirac comb" function with a time step $\Delta t$. The value $\Delta t$ is determined by the difference between the intervals $\Delta t_1$ and $\Delta t_2$,

$$\Delta t = \Delta t_1 - \Delta t_2 = \frac{1}{c}[b(\sin \alpha + \sin \beta) - d(\cos \alpha + \cos \beta)].$$

(3)

Here $\Delta t_2 = \frac{2}{c}(\cos \alpha + \cos \beta)$ is the period of repetition of the responses $h_1$. The $\Delta t_2$ value has the form typical for the multibeam interference devices and determine the time interval for the initial wave to pass from some step of the echelon to the next one and backwards. The form of pulse response for the echelon consisting of four steps is shown in Fig. 2.

![Figure 2. Pulse response of the Michelson echelon when observed in the direction $\beta \neq 0$ under the conditions of illumination by normal incident $\delta$-wave.](image)

At a small angles of incidence and observation, the formula (3) for time interval $\Delta t$ turns to very simple one:

$$\Delta t \approx \frac{b}{c}(\alpha + \beta) - \frac{2d}{c}$$
By calculation the Fourier transform of (2) one can find the diffraction field of the scattered monochrome wave, the result is visually similar to that one for the reflective diffraction grating [7].

There is a profound difference between grating and echelon: the value $\Delta t$ is determined not only by $b$ that is identical to the "width of grooves" of the diffraction grating, but also by the depth of steps $d$. In particular, $\Delta t_2$ can be greater than $\Delta t_1$, their difference can turn to zero, which corresponds to the specular reflection of waves not at $\alpha \neq -\beta$ but at $(\alpha + \beta) = 2 \arctan(\frac{d}{c})$.

Let us consider the change of spectrum of a pulse scattered by a real echelon, taking into account its finite dimensions. Let us take for an example the parameters of echelon used in [1]. The dimensions of echelon are $50 \times 50 \times 10 \text{ mm}$, the width of the step is $150.0 \pm 0.2 \mu \text{m}$, the step height is $69.0 \pm 0.02 \mu \text{m}$. The echelon was turned in a way that the direction along the $150.0 \mu \text{m}$ - widths of steps was normal to the incident beam. Thus, the number of steps in the echelon was about $N \approx 303$. The signal reflected from the grating was also observed in a direction perpendicular to the tiers of echelon, so $\alpha = \beta = 0$. In this case, the pulse response of a single mirror has the form of time first derivative of the delta function [6]:

$$h_1(t, f) = \delta'(t).$$

As a result, the pulse response of Michelson echelon consisting of a finite number of mirrors is to be rewritten as following:

$$h(t) = \left[ \text{comb}\left(\frac{t}{\Delta t_2}\right) \otimes \delta'(t) \right] \text{rect}\left(\frac{t}{N \Delta t_2}\right). \tag{4}$$

Note that the derivative of delta function has the property

$$\int_{-\infty}^{\infty} g(x) \delta'(x - y) dx = -g'(y).$$

Applying the Fourier transform to the impulse response (4), we find the amplitude spectrum of the signal reflected from the echelon under the described conditions:

$$h(\nu) = \left[ \text{comb}(\nu \cdot \Delta t_2) \left(\frac{-i \nu}{2 \pi}\right) \right] \otimes \text{sinc}(\nu \cdot N \Delta t_2). \tag{5}$$

Figure 3. The reflected $\delta$ - pulse spectrum from the echelon consisting of $N = 303$ mirrors
The spectrum consists of a sequence of \( \text{sinc} \) functions, the distance to the first zero of each function is \( N \) times smaller than the period of repetition of the functions (Fig. 3). The transition to the wavelength scale can be carried out using the condition \( f(\lambda)d\lambda = f(\nu)d\nu \).

3. Results
We will consider a quasimonochrom pulse of Gaussian form with a central wavelength \( \lambda = 800\text{nm} \) and a width \( \Delta \lambda = 12\text{nm} \) illuminates the Michelson echelon, as in [1]. The power spectrum of the signal reflected from the Michelson echelon can be obtained from (5):

\[
H(\nu) = \left\{ \text{comb}(\nu \cdot \Delta t_2) \left( \frac{-i\nu}{2\pi} \right) \otimes \text{sinc}(\nu \cdot N \Delta t_2) \right\}^2 \cdot g(\nu),
\]

where \( g(\nu) \) is Gaussian spectral line contour.

The spectrum of radiation reflected from the echelon with the indicated characteristics is shown in Fig. 4a.

Figure 4. The power spectrum of the radiation reflected from the Michelson echelon with a) \( N = 303 \) mirrors, b) \( N = 6 \) mirrors. Dashed line represents the spectrum of incident pulse.

This result contradicts the data presented in [1]: the recorded intensity spectrum consisted of overlapping maxima, and there were significantly non-zero minima between them. The registered shape of the reflected pulse spectrum was not discussed in the paper. The width of \( \text{sinc} \)-functions depends on the number of illuminated steps of echelon. Supposing that only 6 mirrors are illuminated, the resulting spectrum is given in Fig. 4b. It practically coincides with that one recorded in [1]. The incident beam width is not specified in [1], but it is assumed that its surface is illuminated uniformly. It is unlikely that when measuring the spectrum, the authors specially narrowed the beam so that only a zone of the order of 0.1mm in diameter was illuminated.

At the same time, the imperfection of the echelon structure itself strongly affects the form of the pulse response and the corresponding spectrum. Technology of the echelon manufacturing used in [1] was not described, but it can be assumed that in the process of polishing mirrors the
thickness of one step differed from the other within some error (to avoid such errors, Michelson used overlapping pieces of one plane-parallel plate, carefully removing the air gap between them). Let the step thickness vary in a certain range \( b \pm \Delta b \) and \( \Delta b \) take a random value distributed uniformly in the interval \([b - \Delta b, b + \Delta b]\). Figure 5 shows how the spectrum of reflected pulse varies with the value of \( \Delta b \).

It can be seen that beginning from \( \Delta b \approx 60 \text{ nm} \) the modulation of spectrum decreases, and disappears with further increase of the manufacturing error.

To summarize, it should be noted that the considered transformation of spectrum of the ultrashort pulse reflected from the echelon is determined not only by the width and height of the step, but also by the effective number of echelon steps involved in the formation of the resulting signal, and hence by the way it is illuminated. Our study demonstrates also the strong dependence of the obtained spectrum on the position of the echelon steps in space. Given analytical expressions for the reflected signal as a function of time make it possible to calculate the expected spectrum in any specific experiment conditions. The obtained relations are in agreement with the classical theory of the reflective Michelson echelon at the usual ways of its illumination.

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