1. Introduction

One of the most important characteristics of spiral galaxies, rotation curves (hereafter RCs), do not show Keplerian fall-off thus indicating the presence of invisible mass component (Ashman, 1992). The question of the nature of the dark matter (DM) which dominates the outer parts of spirals, but can also be found in the regions where the ordinary matter is present (Persic, Salucci, Stel, 1995, hereafter PSS), may lead towards the answer to the important questions concerning galaxy formation, cosmology and particle physics. In this paper we made the attempt to show whether massive neutrinos could play the important role in the above mentioned “conspiracy” concerning RCs (Peebles, 1993).

2. Neutrino Mass

The question whether neutrinos have masses is still unresolved and we present here the laboratory limits on the neutrino mass that follow purely from kinematics: $m_{\nu_e} \lesssim 5 \text{ eV}$, $m_{\nu_\mu} \lesssim 250 \text{ KeV}$, and $m_{\nu_\tau} \lesssim 23 \text{ MeV}$ (Montanet et al., 1994). Recent measurements that have been done using the LSND (Liquid Scintillator Neutrino Detector) (Athanassopoulos et al., 1995, Hill, 1995) strongly suggest that neutrinos do have masses thus giving us a possibility to determine their place in the universe. The contribution of neutrinos to the cosmological density today (Primack, 1996) is:

$$\Omega_\nu = \frac{\sum_i m_{\nu_i}}{94 h^2 \text{eV}},$$

where the density parameter $\Omega_\nu = \frac{\rho_\nu}{\rho_{\text{crit}}}$, $\rho_{\text{crit}} = \frac{3H_0^2}{8\pi G} \simeq 1.88 \times 10^{-27} h^2 \frac{\text{g}}{\text{cm}^3}$. $h$ is dimensionless parameter used in the parametrization of the Hubble constant $H_0$, $H_0 = 100 \ h \frac{\text{km/s}}{\text{Mpc}}$ and $0.4 < h < 1$. According to the well-known cosmological upper limit (e.g., Sarkar, 1996) we have the following constraint:

$$\sum_i m_{\nu_i} \left(\frac{g_{\nu_i}}{2}\right) \leq 94 \text{eV},$$

where the sum goes over all species that were relativistic at decoupling i.e. $m_{\nu_i} \lesssim 1 \text{ MeV}$. 
Perhaps the most noticeable evidence of the existence of the DM came from the measurements of luminosity profiles and RCs of spiral galaxies (Rubin et al., 1985). The case of the spiral galaxy NGC3198 (“everyone’s favourite”) is very important and could lead us to some important results concerning the problem of the nature of the DM. Van Albada et al., (1985) found that the amount of the DM inside the last point of the rotational curve is at least four times larger than the amount of visible matter. Mass-to-light ratio adjusted to fit the inner part of the rotational curve of this galaxy is:

\[
\frac{M}{L} = 5.3 \, h \, \frac{M_\odot}{L_\odot}
\]

(Peebles, 1993), where \( M \) stands for the mass, \( L \) for the luminosity, symbol \( \odot \) denotes the Sun. While in the central parts of this galaxy there exists good agreement with Newtonian model, in the outskirts, observed velocities do not show Keplerian fall-off. It is assumed that the mass in these outer parts is dominated by low-luminosity material — a dark halo.

This material could be in two forms: baryonic and nonbaryonic. According to Persic and Salucci (1996), (hereafter PS96):

\[
\Omega_{\text{gal}}^\text{bar} = \Omega_{\text{E}}^\text{bar} + \Omega_{\text{S}}^\text{bar} = 2 \times 10^{-3},
\]

where \( \Omega_{\text{bar}} = \frac{\rho_{\text{bar}}}{\rho_{\text{crit}}} \).

PS96 found that spiral (S) and elliptical (E) galaxies contribute the same cosmological stellar mass density. However, recent measurements imply that the density parameter \( \Omega_0 = 1 \) (with estimated standard error ±0.2) thus indicating the existence of non-baryonic DM (Kolb and Turner, 1993).

If we want to make the model of a spiral galaxy (for example, NGC3198) we could use the following Richstone and Tremaine (1986) approach for the mass density:

\[
\rho(r) = \frac{\rho_c}{(1 + \frac{r^2}{r_c^2})^{\frac{3}{2}}},
\]

where \( r_c \) is the core radius. For small values of \( r \), i.e. \( r \ll r_c \):

\[
\rho = \rho_c(1 - \frac{3}{2} \frac{r^2}{r_c^2} + \cdots).
\]

The velocity distribution is isotropic and independent of position, the mass-to-light ratio is independent of radius. In this model there exists a well defined, flat central core. The velocity dispersion is \( \sigma = \langle v^2 \rangle^{\frac{1}{2}} \) in one dimension, the pressure is \( p = \rho \sigma^2 \) and the pressure per unit volume is \( -\frac{\partial p}{\partial r} \). The condition for gravitational equilibrium is:

\[
\frac{\partial p}{\partial r} = \frac{3 \rho_c \sigma^2 r}{r_c^2} = \frac{GM(r)}{r^2} \rho = \frac{4\pi}{3} G \rho_c^2 r.
\]

Thus, one gets the central mass density:
\[ \rho_c = \frac{9\sigma^2}{4\pi G r_c^2} \]

(Peebles, 1993, Padmanabhan, 1993).

Schramm and Steigman (1981) considered relic neutrinos with mass \( m_\nu \gtrsim \frac{1}{2} \) eV and obtained the same result. According to this paper neutrinos could have collapsed gravitationally during the formation of astrophysical systems whose potential wells are sufficiently deep:

\[ \frac{GM(\leq r_c)}{r_c} \approx 3\sigma^2 \gtrsim v_\nu^2. \]

Following Peebles’ (1993) reasoning, one can establish a possible model for the present mean distribution of neutrinos in a dark halo via spherically symmetric isothermal form:

\[ N_f = N_0 e^{-\frac{1}{\sigma^2} \left( \frac{r^2}{v_\nu^2} + \varphi(r) \right)} \quad (2) \]

where \( N_f \) is defined as:

\[ N_f = \left\langle e^{\frac{1}{v_\nu^2}} \right\rangle, \quad T_\nu = \left( \frac{4}{11} \right)^{\frac{1}{2}} T_0 (1 + z) \]

where \( T_0 \) is the present cosmic microwave background temperature, \( T_0 = 2.73 \pm 0.01 \) K (Smooth and Other, 1995). In the equation (2) the gravitational potential per unit mass at radius \( r \) is \( \varphi(r) \), with \( \varphi(0) = 0, N_0 \leq 0.5 \).

This distribution can be used in order to find the mean density:

\[ \rho_\nu(r) = \frac{2m_\nu}{(2\pi \hbar)^2} \int N_f d^3p = \frac{N_0 m_\nu^4 \sigma^3}{2^{\frac{1}{2}} \pi^{\frac{3}{2}} h^3} e^{-\frac{\varphi(r)}{\sigma^2}} \quad (2a) \]

(Peebles, 1993).

After solving the Poisson equation for the gravitational potential:

\[ \nabla^2 \varphi = \frac{d^2\varphi}{dr^2} + \frac{2}{r} \frac{d\varphi}{dr} = 4\pi G \rho_\nu(r) \]

one obtains, for \( r \gg \alpha, \alpha^2 = \frac{h^3}{2^{\frac{1}{2}} G N_0 m_\nu^4 \sigma} \):

\[ \rho_\nu(r) = \frac{\sigma^2}{2\pi G r_c^2} = \frac{v^2}{6\pi G r_c^2}. \quad (3) \]

We put in the value for \( v^2 \), i.e. the mean square velocity \( < v^2 >, < v^2 > = 3\sigma^2 \) (Padmanabhan, 1993).

It follows from equations (2a) and (3) that the mass is equal to:

\[ m_\nu^4 = \frac{1}{6\pi} \left( \frac{3}{2\pi} \right)^{\frac{1}{2}} \frac{h^3}{G r_c^2}, \quad (4) \]
where we have assumed that \( \frac{\sigma(v)}{\sigma_{0}} \to 0 \) and \( N_{0} = 0.5 \).

If one inserts the values characteristic for the Milky Way \( (v \sim 230 \text{ km s}^{-1} \text{ and } r_{c} \sim 8 \text{ kpc}) \), one obtains the following result for the neutrino mass:

\[
m_{\nu} \approx 27 \text{ eV}.
\]

This value is the lower limit for \( m_{\nu} \), while the upper limit is given in the equation (1). The obtained value plays the crucial role in the decaying dark matter (DDM) theory, firstly proposed by Melott (1984) and later developed by Sciuma (1990a, 1990b, 1993). According to this theory the mass of the tau neutrino is:

\[
m_{\nu\tau} = 29.21 \pm 0.15 \text{ eV}.
\]

Melott et al. (1994) while considering decaying neutrinos in galaxy clusters obtained the lower limit for the neutrino lifetime \( \tau_{23} \) in the units \( 10^{23} \text{ s} \): \( \tau_{23} > (3 \pm 1)(\frac{29 \text{ eV}}{m_{\nu}}) \). Such a decay could be observed (e.g., Samurović and Čelebonović, 1995). Experiment that has been proposed, EURD, will have to prove the existence of a decay line derived from the photons with energy \( \sim 15 \text{ eV} \).

3. Rotational Curves of Spiral Galaxies

PSS presented the profiles of 134 RCs (references on the measurements can be found therein). We used the equation (4) which after inserting the appropriate values becomes:

\[
m_{\nu}[\text{eV}] = \frac{299.0362}{v^{1.4}r_{c}^{0.2}}.
\]

For the estimation of the core radius, \( r_{c} \), we used Kormendy’s relation (Ashman, 1992) according to which:

\[
r_{c} \approx 5.9 \left( \frac{L_{B}}{10^{9}L_{\odot}} \right)^{0.34} \text{[kpc]}.
\]

In the Table (1) we present the values from PSS together with estimated values for the neutrino mass (most probably of the tau neutrino). Although the most important galactic parameters are not known accurately (Spergel, 1996), one can see that the values of the neutrino mass are gathered within the range \( 20 \lesssim m_{\nu} \lesssim 30 \text{ eV} \); the mean value is \( 22\pm7 \text{ eV} \). The obtained result could lead to the conclusion that massive neutrinos could dominate in the spiral galaxies, but additional measurements of galaxies’ parameters and laboratory measurements of neutrino mass are indispensible.

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1 URL: [http://www.laeff.esa.es/eng/laeff/activity/eurd.html](http://www.laeff.esa.es/eng/laeff/activity/eurd.html)
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| Name   | $M_B$ | $v$ [km s$^{-1}$] | $r_c$ [kpc] | $m_\nu$ [eV] | Name   | $M_B$ | $v$ [km s$^{-1}$] | $r_c$ [kpc] | $m_\nu$ [eV] |
|--------|-------|------------------|-------------|-------------|--------|-------|------------------|-------------|-------------|
| U 3269 | -20.01 | 185               | 20.58       | 17.87       | U 3282 | -21.57 | 220               | 24.53       | 15.68       |
| U 4375 | -19.59 | 190               | 13.19       | 22.17       | U 11810 | -20.39 | 185               | 16.95       | 16.96       |
| U 12417 | -19.45 | 126               | 12.63       | 25.12       | U 3282 | -21.57 | 220               | 23.62       | 15.98       |
| U 4375 | -19.45 | 126               | 12.63       | 25.12       | U 12417 | -19.45 | 126               | 12.63       | 25.12       |
| U 12533 | -19.93 | 210               | 14.68       | 20.51       | U 12533 | -19.93 | 210               | 14.68       | 20.51       |
| N2629 | -20.12 | 200               | 15.58       | 20.15       | U 12810 | -21.45 | 220               | 23.62       | 15.98       |
| N2629 | -20.12 | 200               | 15.58       | 20.15       | U 12810 | -21.45 | 220               | 23.62       | 15.98       |
| N2724 | -21.26 | 220               | 16.46       | 40-012      | N2724 | -21.26 | 220               | 16.46       | 40-012      |
| N2724 | -21.26 | 220               | 16.46       | 40-012      | N2724 | -21.26 | 220               | 16.46       | 40-012      |
| N2724 | -21.26 | 220               | 16.46       | 40-012      | N2724 | -21.26 | 220               | 16.46       | 40-012      |
| N2724 | -21.26 | 220               | 16.46       | 40-012      | N2724 | -21.26 | 220               | 16.46       | 40-012      |
| N2724 | -21.26 | 220               | 16.46       | 40-012      | N2724 | -21.26 | 220               | 16.46       | 40-012      |
| N2724 | -21.26 | 220               | 16.46       | 40-012      | N2724 | -21.26 | 220               | 16.46       | 40-012      |
| N2724 | -21.26 | 220               | 16.46       | 40-012      | N2724 | -21.26 | 220               | 16.46       | 40-012      |
| N2724 | -21.26 | 220               | 16.46       | 40-012      | N2724 | -21.26 | 220               | 16.46       | 40-012      |
Table 1. Values of radius $r$ in kpc, velocity $v$ in km/s and mass of neutrino $m_\nu$ for 134 spiral galaxies.

| Name  | $M_B$ | v [km/s] | $r_c$ [kpc] | $m_\nu$ [eV] | Name  | $M_B$ | v [km/s] | $r_c$ [kpc] | $m_\nu$ [eV] |
|-------|-------|----------|------------|-------------|-------|-------|----------|------------|-------------|
| N4682 | -19.58 | 175      | 13.15      | 22.67       | 383-088 | -20.04 | 180      | 15.19      | 20.95       |
| N4800 | -18.76 | 172      | 10.17      | 25.89       | 437-030 | -20.46 | 205      | 17.33      | 18.99       |
| N5033 | -20.67 | 218      | 18.50      | 18.09       | 439-018 | -21.71 | 245      | 25.63      | 14.93       |
| N5055 | -20.65 | 215      | 18.39      | 18.21       | 439-020 | -19.91 | 215      | 14.58      | 20.45       |
| N5371 | -21.72 | 240      | 25.71      | 14.98       | 444-047 | -19.44 | 145      | 12.59      | 24.29       |
| N5585 | -18.47 | 80       | 9.29       | 32.80       | 444-086 | -19.95 | 210      | 14.77      | 20.44       |
| N5673 | -20.30 | 138      | 16.48      | 21.49       | 445-058 | -20.72 | 205      | 18.80      | 18.23       |
| N5905 | -20.87 | 235      | 19.70      | 17.21       | 446-044 | -19.22 | 148      | 11.75      | 25.01       |
| N5907 | -20.69 | 225      | 18.62      | 17.89       | 481-002 | -18.22 | 119      | 8.59       | 30.89       |
| N6503 | -18.62 | 122      | 9.74       | 28.83       | 499-005 | -20.84 | 170      | 19.52      | 18.75       |
| N6674 | -21.33 | 255      | 22.75      | 15.69       | 502-002 | -20.25 | 210      | 16.22      | 19.50       |
| N7083 | -21.27 | 220      | 22.33      | 16.43       | 507-007 | -21.10 | 263      | 21.17      | 16.14       |
| N7331 | -20.92 | 243      | 20.01      | 16.93       | 509-091 | -20.03 | 150      | 15.14      | 21.96       |
| N7339 | -19.21 | 170      | 11.73      | 24.20       | 533-004 | -19.23 | 150      | 11.79      | 24.89       |
| N7536 | -20.18 | 183      | 15.87      | 20.41       | 543-012 | ... | 174      | ...        | ...        |
| N7591 | -20.34 | 200      | 16.69      | 19.47       | 548-032 | -17.58 | 66       | 7.03       | 39.57       |
| N7593 | -19.61 | 150      | 13.28      | 23.45       | 555-016 | -20.19 | 222      | 15.92      | 19.42       |
| N7606 | -21.15 | 265      | 21.50      | 15.98       | 563-014 | ... | 148      | ...        | ...        |
| N7631 | -20.18 | 198      | 15.87      | 20.01       | 564-020 | -19.00 | 92       | 10.97      | 29.16       |
| N7793 | -17.76 | 111      | 7.51       | 33.62       | 566-022 | -18.91 | 138      | 10.66      | 26.72       |
| I 467 | -19.77 | 150      | 13.96      | 22.87       | 601-009 | -21.55 | 258      | 24.37      | 15.11       |
| I 2974 | ...   | 230      | ...        | ...         | M-3-1042 | ... | 148      | ...        | ...        |
| U 2259 | -16.54 | 81       | 5.08       | 44.24       | ...   | ...   | ...      | ...        | ...        |