Synchronizing the EMRIs and IMRIs in AGN Accretion Disks

Peng Peng and Xian Chen

1 Astronomy Department, School of Physics, Peking University, 100871 Beijing, People’s Republic of China; xian.chen@pku.edu.cn
2 Kavli Institute for Astronomy and Astrophysics at Peking University, 100871 Beijing, People’s Republic of China

Received 2022 November 18; revised 2023 April 4; accepted 2023 April 17; published 2023 June 6

Abstract

Extreme-mass-ratio inspirals (EMRIs) and intermediate-mass-ratio inspirals (IMRIs) are important gravitational-wave (GW) sources for the Laser Interferometer Space Antenna (LISA). So far, their formation and evolution have been considered to be independent. However, recent theories suggest that stellar-mass black holes (sBHs) and intermediate-mass black hole (IMBHs) can coexist in the accretion disk of an active galactic nucleus (AGN), which indicates that EMRIs and IMRIs may form in the same place. Motivated by the fact that a gas giant migrating in a protoplanetary disk could trap planetesimals close to its orbit, in this paper we study a similar interaction between a gap-opening IMBH in an AGN disk and the sBHs surrounding it. We analyze the torques imposed on the sBHs by the disk and also by the IMBH, and show that the sBHs can be trapped by the IMBH if they are inside the orbit of the IMBH. We then implement the torques in our numerical simulations to study the migration of an outer IMBH and an inner sBH, which are both embedded in an AGN disk. We find that their migration is synchronized until they reach a distance of about 10 Schwarzschild radii from the central supermassive black hole, where the pair break up due to strong GW radiation. This result indicates that LISA may detect an EMRI and an IMRI within several years from the same AGN. This GW source will bring rich information about the formation and evolution of sBHs and IMBHs in AGNs.

Unified Astronomy Thesaurus concepts: Active galactic nuclei (16); Intermediate-mass black holes (816); Stellar mass black holes (1611); Gravitational waves (678)

1. Introduction

Extreme-mass-ratio inspirals (EMRIs) and intermediate-mass-ratio inspirals (IMRIs) are important gravitational-wave (GW) sources for milli-Hertz (mHz) GW detectors, such as the Laser Interferometer Space Antenna (LISA, Amaro-Seoane 2018). An EMRI normally consists of a supermassive black hole (SMBH) of $10^6$–$10^7 M_\odot$ and a stellar-mass black hole (sBH) of tens of solar masses, so that the mass ratio $q$ is much smaller than $10^{-4}$. An IMRI has a typical mass ratio of $10^{-4} \lesssim q \lesssim 10^{-3}$ (Amaro-Seoane et al. 2007). Therefore, it can be formed either by an intermediate-mass black hole (IMBH) of $10^2$–$10^3 M_\odot$ and a stellar-mass compact object (i.e., a white dwarf, neutron star, or sBH), or by an SMBH and an IMBH. Since the GW-radiation timescale is proportional to $q^{-1}$, EMRIs and IMRIs could well be present in the LISA band for many years. During this period, they accumulate $10^4$–$10^5$ GW cycles in the data, which provides rich information about the spacetime geometry close to the horizon of a massive black hole (Amaro-Seoane et al. 2007).

The accretion disk of an active galactic nucleus (AGN) is an important place for EMRI and IMRI formation, and it is known to be the breeding ground of sBHs (e.g., Syer et al. 1991; Artymowicz et al. 1993; Šubr & Karas 1999; Karas & Šubr 2001; Levin 2003; Goodman & Tan 2004). The sBHs could form from the massive stars in the outer part of the disk (Goodman & Tan 2004) or be captured from the nuclear star cluster by the disk (Syer et al. 1991). Once inside the disk, the sBHs could migrate toward the central SMBH due to various hydrodynamical effects, including hydrodynamical drag (Sigl et al. 2007), the interaction with density waves (Levin 2003; Kocsis et al. 2011, 2012; Sánchez-Salcedo 2020), and head or tail winds if the disk is geometrically thick (Chakrabarti 1993; Molteni et al. 1994; Basu et al. 2008; Kocsis et al. 2011). Those sBHs that eventually reach a few Schwarzschild radii from the central SMBH will produce EMRIs. Such “wet” EMRIs may predominate the event rate, as recent calculations suggest (Gröbner et al. 2020; Pan & Yang 2021; Pan et al. 2021; Derdzinski & Mayer 2022).

The accretion disk of an AGN may also contain IMBHs, and hence could also produce IMRIs. On one hand, IMBHs can be brought into the galactic nucleus by galaxy mergers or inspiraling globular clusters (Volonteri et al. 2003; Mastrobuono-Battisti et al. 2014). These IMBHs could later be captured by the accretion disk in a way similar to the capture of sBHs (Ivanov et al. 1999). On the other hand, IMBHs could also grow from the aforementioned sBHs that have accumulated in the AGN disk. The growth is due to either the accretion of the surrounding gas (McKernan et al. 2011, 2012a; Tagawa et al. 2021) or the merger with the other sBHs in the disk (McKernan et al. 2012b; Bartos et al. 2017; Stone et al. 2017).

Earlier studies have shown that the probability of IMBH formation could be particularly high at a series of radii in the AGN disk, which are known as the “migration traps” (Bellovary et al. 2016; Secunda et al. 2019, 2020; Peng & Chen 2021). These traps typically reside at tens to hundreds of Schwarzschild radii ($R_S$) from the central SMBH. Inside the migration trap, the hydrodynamical torque that drives the migration of sBHs vanishes. Consequently, sBHs will accumulate in the trap, forming binaries. The binaries will continue to harden due to the interaction with either the surrounding gas (Baruteau et al. 2011; Antoni et al. 2019) or the other sBHs (Leigh et al. 2018; Yang et al. 2019b). A likely outcome is that the binaries will merge within the lifetime of...
the AGN. Moreover, the merger remnants will remain in the accretion disk because the recoil velocity (due to anisotropic GW radiation) is typically much smaller than the escape velocity from the central SMBH (Miller & Lauburg 2009; Yang et al. 2019b; Tagawa et al. 2021). Therefore, the post-merger sBHs could participate in the next generation of mergers. Through such “hierarchical mergers” (Yang et al. 2019b; Gerosa & Fishbach 2021), sBHs could gradually grow into the intermediate-mass range. Interestingly, this channel of heavy black hole (BH) formation in AGN disk can well explain the merger rate (McKernan et al. 2018; Gröbner et al. 2020; Secunda et al. 2020) and the mass function (Ghayathri et al. 2020) of the binary BHs that have been detected by the Laser Interferometer Gravitational-wave Observatory (LIGO) and the Virgo detectors.

As the IMBH in an AGN disk grows, the tidal torque that is induced by the IMBH on the surrounding gas would eventually be strong enough to open a gap in the disk (Ivanov et al. 1999; Gould & Rix 2000). This gap-opening process is similar to what is happening around the gas giants in protoplanetary disks (Lin & Papaloizou 1979a, 1979b, 1986; Artymowicz & Lubow 1994). After opening a gap, the IMBH could continue exchanging energy and angular momentum with the accretion disk by its tidal force, and hence keep migrating toward the central SMBH (Gould & Rix 2000; Armitage & Natarajan 2002).

According to this picture of the formation and evolution of sBHs and IMBHs in an AGN accretion disk, it is likely that an IMBH could encounter other sBHs during its migration. For example, the IMBH could catch up with an sBH if the migration of the latter is slower, or the IMBH may at some point pass by a migration trap of sBHs. The subsequent interaction seems analogous to the interaction between a gap-opening gas giant and a sub-gas-opening terrestrial planet in a protoplanetary disk. Previous studies of this interaction suggest that the smaller terrestrial planet could be trapped in one of the resonance belts around the larger gas giant (Hahn & Ward 1996; Pierens & Nelson 2008; Podlewska & Suszczewicz 2008).

For the same reason, we also expect a gap-opening IMBH in an AGN disk to trap an sBH along its way toward the central SMBH. If such a pair could be maintained until the IMBH reaches the last few Schwarzschild radii from the central SMBH, then we would detect an interesting event—an EMRI happening at the same time as an IMRI. This GW source could reveal rich triple dynamics in the regime of strong gravity (see, e.g., Gupta et al. 2021, for the interaction between two EMRIs). In fact, Yang et al. (2019a) have shown that two sub-gap-opening sBHs in the accretion disk of an AGN could indeed be trapped at the mean-motion resonance and migrate, as a pair, toward smaller radius. However, they also showed that the pair will decouple when the inner sBH reaches tens of $R_S$, where GW radiation of the inner EMRI becomes important.

The situation may be different if an IMBH traps an sBH within its orbit. First, the coupling between the sBH and the IMBH is expected to be tighter than an sBH-sBH pair because the IMBH exerts a larger tidal force and excites stronger density waves in the disk. Second, the inspiral timescale of the IMBH due to GW radiation could be shorter than that of the sBH, so that the IMBH could keep up with the inward migration of the sBH even when the latter has entered the GW-radiation regime. To test these postulations, in this paper we conduct numerical simulations of the migration and interaction of the IMBHs and sBHs in an AGN accretion disk.

This paper is organized as follows. In Section 2, we review the physical processes that will lead to the formation and migration of sBHs and IMBHs in the accretion disk of an AGN. We show that a gap-opening IMBH can catch up with the migration of an inner sBH. In Section 3, we calculate the hydrodynamical torque of the gas and the tidal torque of the IMBH exerted on the sBH. Based on the understanding of the torques, we conduct 1D and 2D simulations in Section 4 to show how the IMBH and the sBH are synchronized in their migration toward the central SMBH. Finally, we discuss the possibility of detecting an EMRI-IMRI pair by the LISA mission and the relevant parameter space that can produce such a special GW source in Section 5.

2. Formation and Migration of sBHs and IMBHs in an AGN Disk

Previous studies suggest that sBHs can be produced in the accretion disks of AGNs in two ways. First, sBHs can originate from the nuclear star cluster surrounding an AGN. If their orbits intersect the accretion disk, then the repeated collision with the disk could cause energy and angular-momentum loss, and some sBHs could eventually be captured by the disk (Syrer et al. 1991). In particular, Fabj et al. (2020) and Nasim et al. (2023) showed that about 10% of the sBHs with a semimajor axis of $a_s \sim 10^3$–$10^6 R_S$ relative to the SMBH can be captured from the nuclear star cluster into the accretion disk during the lifetime of the AGN. In the case of an SMBH with a mass of $M_{SMBH} = 10^6 M_\odot$, about 100 sBHs can be captured because there are about 1000 sBHs in the nuclear star cluster whose orbits intersect the accretion disk (see, e.g., Section 5.5 in Tagawa et al. 2020). Second, sBHs could be produced by the massive stars in the accretion disk. These stars are either born in the disk (Shlosman & Begelman 1987; Goodman & Tan 2004) or captured from the nuclear star cluster (Artymowicz et al. 1993). However, recent calculations suggest that this channel does not dominate the formation of sBHs in AGN disks because only a small fraction of the stars (~1%) would evolve into sBHs (Tagawa et al. 2020). Therefore, in the following analysis, we focus on the sBHs captured from the nuclear star cluster.

A captured sBH will excite density waves in the accretion disk (Goldreich & Tremaine 1978, 1979). The back reaction of the density waves effectively exerts a torque on the sBH, causing the sBH to migrate in the disk, which is known as a Type-I migration (Goldreich & Tremaine 1980). The corresponding migration timescale can be calculated with

$$T_I = \frac{f_I h^2 M_{SMBH}^2}{m_{sBH} \Sigma a_s^2 \Omega_s}$$

$$= 10^4 \left( \frac{m_{sBH}}{10 M_\odot} \right)^{-1} \left( \frac{M_{SMBH}}{10^6 M_\odot} \right)^{-2} \times \left( \frac{\Sigma}{10^6 \text{g cm}^{-2}} \right) \left( \frac{a_s}{1000 R_S} \right)^{-1/2} \text{yr},$$

where $m_{sBH}$ is the mass of the sBH; $\Omega_s$ is its angular velocity; $h$ and $\Sigma$ are, respectively, the aspect ratio (between the scale height and the radius) and the surface density of the accretion disk; and $f_I$ is a function of the temperature and density.
gradients of the disk near the sBH (e.g., Paardekooper et al. 2011). Given the parameters of our interest, we find that the Type-I migration timescale is much shorter than the lifetime of the AGN (∼10 Myr; Gonçalves et al. 2008; Hopkins & Hernquist 2009; Gabor & Bournaud 2013).

The direction of the Type-I torque depends on the relative strength of the density waves leading and trailing the sBH, which is characterized by the parameter $f_i$ in Equation (1). In a large range of radius, the trailing wave is stronger, and so the torque on the sBH is negative. Therefore, the sBH loses angular momentum and migrates inward toward the central SMBH. However, there are regions in the disk where the temperature and density profiles are discontinuous due to the change of opacity. In this region, the leading wave could become stronger, and the sBH will feel a positive torque and migrate outward (Bellovary et al. 2016). The transition from a negative to a positive torque will result in a zero-torque region. Here, the inwardly migrating sBHs will meet the outwardly migrating sBHs, and the migration will stall once the sBHs enter the region. For this reason, the region with zero torque is known as the migration trap. There are a series of migration traps at a radial range between several tens and several thousands of $R_S$ (Bellovary et al. 2016; Secunda et al. 2019).

Inside a migration trap, the sBHs, assisted by the surrounding gas, could experience multiple mergers. The number of mergers depends on the number of sBHs in the trap. Secunda et al. (2020) recently estimated that for a SMBH of $M_{\text{SMBH}} = 10^6 M_\odot$, every $\sim 10^7$ yr an sBH will be transported to the Type-I migration trap. Consequently, a total number of $\sim 100$ sBHs will end up in the trap during the AGN’s lifetime. This result suggests that an IMBH with a final mass of $\sim 1000 M_\odot$ could form (see Figure 4 in Secunda et al. 2020). For a smaller SMBH, although there are less sBHs in the nuclear star cluster (see, e.g., Section 5.5 in Tagawa et al. 2020), the capture of sBHs by the disk could be more efficient because the dynamic timescale is shorter. In addition, the Type-I migration timescale also decreases with decreasing $M_{\text{SMBH}}$. Therefore, the number of sBHs transported to the migration trap, as well as the final mass of the IMBH, could be even larger.

Besides forming in the migration trap, the IMBHs in AGN disks could come from other channels. For example, McKernan et al. (2012b) and Tagawa et al. (2020) showed that an IMBH of $100 M_\odot$ could form outside the migration trap, partly due to the mutual capture of sBHs and partly to gas accretion. Moreover, galaxy mergers and inspiraling globular clusters could also bring in IMBHs, as has been mentioned in Section 1.

Therefore, IMBHs and sBHs are likely to coexist in an AGN accretion disk. To understand the subsequent evolution, it is important to realize that their migration could be different. While an sBH undergoes Type-I migration, an IMBH could open a gap in the accretion disk. This characteristically changes the subsequent evolution. The criterion of opening a gap is determined by two factors. First, the tidal force of the IMBH should be stronger than the viscous torque of the disk, which gives

$$\frac{(m_{\text{IMBH}}/M_{\text{SMBH}})^{1/3}}{h} \left( \frac{h}{1600 \alpha} \right)^{1/3} > 1 \quad (2)$$

(Lin & Papaloizou 1986), where $m_{\text{IMBH}}$ is the mass of the IMBH and $\alpha$ is the viscosity parameter of the disk. Second, the tidal force of the IMBH should be stronger than the pressure at the edge of the gap, which gives an additional condition

$$\frac{(m_{\text{IMBH}}/M_{\text{SMBH}})^{1/3}}{h} > 1$$

(Ward 1997). So for typical parameters, $M_{\text{SMBH}} = 10^6 M_\odot$, $\alpha = 0.01$, and $h \lesssim 0.05$, the IMBH needs to be more massive than $\sim 100 M_\odot$ to open a gap. This mass corresponds to $\sim 10$ mergers of sBHs. This number is smaller than the typical number of sBHs in an migration trap.

A gap-opening IMBH is no longer subject to the Type-I torque because the density profile of the disk is drastically altered. Instead, the IMBH is coupled to the viscous evolution of the disk. It exchanges energy and angular momentum with the disk by tidally interacting with the gas at the edges of the gap. The corresponding migration is known as a Type-II migration (Lin & Papaloizou 1986). The timescale can be calculated with

$$T_H = \frac{1}{\alpha h^2 \Omega_I} = 10^6 \left( \frac{M_{\text{SMBH}}}{10^6 M_\odot} \right) \left( \frac{a_I}{1000 R_S} \right)^{1/2} \times \left( \frac{h}{10^{-2}} \right)^{-2} \left( \frac{\alpha}{10^{-2}} \right)^{-1} \text{yr}, \quad (4)$$

where $a_I$ and $\Omega_I$ are the semimajor axis and the orbital angular velocity of the IMBH. The direction of the migration may depend on the mass ratio between the IMBH and the central SMBH (Miranda et al. 2017; Muñoz et al. 2020). However, for the IMBHs of our interest ($m_{\text{IMBH}} \lesssim 10^3 M_\odot$), the mass ratio is small and the migration is normally toward the SMBH.

Equation (1) indicates that the migration of sBHs will slow down as they approach the SMBH, since $T_I \propto a_I^{-1/2}$. The migration may even stall if the sBHs enter one of the Type-I migration traps. However, for IMBHs, Equation (4) suggests that their Type-II migration will accelerate because $T_I \propto a_I^{-1/2}$. This differential migration will result in an interesting and important consequence—an outer IMBH can catch up with an inner sBH.

In this work, we will neglect the increase of the mass due to gas accretion onto the sBHs or the IMBH because the mass increases by one e-folding on the Salpeter timescale of $T_{\text{sal}} \approx 3 \times 10^7$ yr (Salpeter 1964), if we assume that the BHs are accreting at the Eddington limit $M_{\text{Edd}} = L_{\text{Edd}}/(0.1 c^2)$. This timescale is much longer than the migration timescale of the BHs in the disk (see Equations (1) and 4). Therefore, the masses of the BHs should not significantly change during their migration in the disk.

3. Interaction Between sBHs and IMBHs

When an outer IMBH catches an inner sBH, two factors will affect their subsequent evolution. First, the sBH will start interacting with the inner edge of the gap opened by the IMBH. Since the edge of the gap has a sharp density and temperature profile, it will modify the Type-I torque on the sBH. Second, the IMBH could exchange energy and angular momentum with the sBH if the two BHs are caught in a resonance. We will analysis these two effects in this section and show that the migration of the two BHs can be synchronized if the sBH is inside the orbit of the IMBH.
perturbation theory (Ford et al. 2000), which was developed for planetary systems. This theory is applicable in our case because both the IMBH and the sBH are much lighter than the central SMBH, and their orbital eccentricities and inclinations remain small due to a hydrodynamical damping effect (Tanaka & Ward 2004). According to this theory, the interaction is the strongest at the mean-motion resonances, where the orbital frequencies of the IMBH and the sBH take integer ratios, such as \( \Omega_s/\Omega_t = 1:2, 2:1, 3:4, 4:5, \) etc (Murray & Dermott 1999). The loci of the most important resonances are shown in the Figure 1 as dotted-vertical lines.

The resonant interaction exchanges energy and angular momentum between the IMBH and the sBH, so the semimajor axis \( a_s \) and the orbital eccentricity \( e_s \) of the sBH will secularly change. We will ignore the back reaction on the evolution of the IMBH because the mass of the sBH is small. Take the 2:1 resonance for example, i.e., \( \Omega_s/\Omega_t = 2 \) and the sBH is inside the orbit of the IMBH. The evolution is governed by the equations

\[
\frac{\dot{a}_s}{a_s} = \text{sgn}(\Omega_s - \Omega_t) f_d e_s \Omega_s \sin(\varphi) \left( \frac{4 m_{\text{IMBH}}}{M_{\text{SMBH}}} \right),
\]

\[
\dot{\varphi} = -f_d \Omega_s \sin(\varphi) \left( \frac{m_{\text{IMBH}}}{M_{\text{SMBH}}} \right),
\]

\[
\varphi = -\Omega_s + 2\Omega_t - f_d e_s^{-1} \Omega_s \cos(\varphi) \left( \frac{m_{\text{IMBH}}}{M_{\text{SMBH}}} \right)
\]
(Murray & Dermott 1999), where a dot denotes the time derivative, \( f_d \) is a factor of order unity which takes different values for different resonances, and \( \varphi \) is the resonant argument. In Equation (7), we have omitted the higher-order terms of \( e_s \) and \( m_{\text{IMBH}}/M_{\text{SMBH}} \). We note that \( f_d < 1 \), and it is negative when \( \Omega_s > \Omega_t \). Besides the 2:1 resonance, we have also computed the 3:4, 4:5, and 5:6 resonances to the first order of \( e_s \) and included them in our model. The other resonances of higher order of \( e_s \) are neglected in this work because \( e_s \) remains small (see below). In the case where the sBH is outside the orbit of the IMBH, we have \( \Omega_s < \Omega_t \) and the factor \( f_d \) in Equation (5) will be positive.

The last three equations indicate that the sBH could be trapped by the IMBH in one of the mean-motion resonances when \( \Omega_s > \Omega_t \). For example, Equation (7) suggests that \( \varphi \) evolves on a timescale that is much longer than the orbital period when the sBH resides at the exact location of the resonance \( \Omega_s/\Omega_t = 2 \). This location is where \( \dot{\varphi} = 0 \). If the sBH deviates from this location and moves, say, further inward (outward), then \( -\Omega_s + 2\Omega_t \) will become negative (positive), also making \( \varphi \) negative (positive). Consequently, in Equation (5), \( \dot{a}_s \) will be positive (negative) because \( f_d < 0 \) when \( \Omega_s > \Omega_t \). Therefore, the sBH feels a positive (negative) torque and will move back into the resonance. For this reason, the sBH will be trapped at a location where \( \varphi \approx 0 \) and \( \dot{a}_s \approx 0 \). In fact, when \( \Omega_s < \Omega_t \), i.e., the sBH is outside the orbit of the IMBH, the equations for resonant interaction (not shown here) will lead to the same result.

This physical picture of a resonance trap needs to be modified in our problem because (i) the IMBH migrates and (ii) gas is present around the sBH. The consequences are twofold. On one hand, to keep up with the inward Type-II migration of the IMBH, the sBH should satisfy the condition \( \dot{a}_s < 0 \). This

![Figure 1](image-url)

**Figure 1.** Upper panel: Surface density of the disk before (blue-dashed curve) and after (orange-solid curve) the gap is opened. The black dot marks the location of the IMBH. The vertical dotted lines, from left-hand to right-hand, show the loci of the 2:1, 3:2, 4:3, 5:4, 6:5, and 1:2 resonances. Lower panel: Strength of the Type-I torque (in arbitrary unit) near the gap. The green-dashed line shows the negative torque, which drives a sBH migrating inward, and the blue-solid line shows the positive torque, driving the sBH outwards.

### 3.1. Modified Type-I Torque Due to the Gap

To evaluate the first effect, we use the 1D hydrodynamical code presented in Fontecilla et al. (2019) to calculate the surface-density profile of the gas near the gap opened by the IMBH. The result is shown in the upper panel of Figure 1. The parameters are \( m_{\text{IMBH}} = 100 M_\odot \), \( M_{\text{SMBH}} = 10^6 M_\odot \), and the accretion rate of the disk \( M_{\text{Edd}} \) is set to the Eddington rate \( M_{\text{Edd}} = L_{\text{Edd}}/(0.1 c^2) \). We can see that near the gap, the surface density decreases by several orders of magnitude within a radial range of only 0.1 dex.

The sharp change of the density, as well as the temperature, will significantly affect the value and the sign of \( f_1 \) in Equation (1). Using the formulae given in Paardekooper et al. (2010) to calculate \( f_1 \), we can calculate the strength of the Type-I torque at the edges of the gap. The result is shown in the lower panel of Figure 1. We find that the Type-I torque at the inner (outer) edge of the gap is negative (positive). This result indicates that the torque tends to push sBHs away from the gap. Moreover, the magnitude of the torque increases sharply toward the gap, which indicates that it is difficult for sBHs to enter the gap.

### 3.2. Mean-motion Resonance

The second effect is the gravitational interaction between the IMBH and the sBH. This interaction can be modeled by the
The aspect ratio does not significantly change with time because the tidal heating by the IMBH is not strong when compared to the intrinsic viscous heating of the disk.

The initial location of the IMBH is $a_2 = 3000R_S$, which coincides with a Type-I migration trap in the unperturbed disk. This choice is motivated by the prediction that an IMBH could be produced inside a Type-I migration trap (see Section 2).

However, if the IMBH comes from a capture event (as mentioned in Section 1), then it could also migrate to the first Type-I migration trap and interact with the sBHs there. In this latter case, our choice of $a_2$ is also reasonable. We notice that this location is different from the Type-I migration trap shown in Secunda et al. (2019). The cause for this is our choice of a much smaller mass for the central SMBH and a higher accretion rate. We find in our simulations that the radius of the Type-I migration trap in general increases with a decreasing mass of the SMBH or an increasing accretion rate.

For sBHs, we initially place one at $a_s = 0.8a_2$ and another at $-1.2a_2$ to mimic the result found by earlier hydrodynamical simulations of the Type-I migration trap (Secunda et al. 2019, 2020). The masses of the sBHs are set to $m_{sBH} = 10M_\odot$. To compute the speed of migration, we consider both the Type-I torque and the torque due to mean-motion resonance. (i) The Type-I torque is calculated according to the equations in Paardekooper et al. (2011). In the calculation, the disk profile (in particular the profiles at the edges of the gap) is given by our 1D simulation. (ii) For the resonance torque, we notice that in our simulations the sBHs do not enter the annulus between the 6:7 and 7:6 resonances. Therefore, we only consider the 2:1, 3:4, 4:5, 5:6, 1:2, 4:3, 5:4, and 6:5 resonances. The corresponding equations governing the evolution of $a_s$ and $e_s$ are adopted from Murray & Dermott (1999).

For the two sBHs, we also consider the hydrodynamical damping of their orbital eccentricities. We model the effect by a characteristic damping timescale, $T_d \equiv -e_s/\dot{e}_s$. When the eccentricity is smaller than the aspect ratio of the disk, i.e., when $e_s < h$, we use $T_d = h^2T_I$ to calculate the damping timescale (Tanaka & Ward 2004). Correspondingly, the damping rate, $-\dot{e}_s/T_d$, is proportional to $e_s$. However, when $e_s > h$, the damping rate will decrease as $e_s$ increases (e.g., Papaloizou & Larwood 2000). In this case, the evolution of the sBHs is more complex. We will study this in more detail in the next subsection.

In our simulation, the orbit of the IMBH is kept circular. We have used the criterion derived by Duffell & Chiang (2015) to check whether or not the eccentricity of the IMBH will be exited by the Lindblad resonances. This criterion is based on a comparison of the strength of the external Lindblad resonance, which excites the eccentricity, and the strength of the other resonances, which damps the eccentricity. Accordingly, the eccentricity of our IMBH would not be excited, mainly because the mass of the IMBH is relatively small and the viscosity of the disk is relatively strong. We notice that the criterion given by Duffell & Chiang (2015) is derived from the simulations of fixed semimajor axes for the planets. According to more recent simulations, this criterion is only accurate for small planets, e.g., with a mass ratio smaller than $q = 10^{-3}$ with respect to the central star (Scardoni et al. 2022). Since we have $q = 10^{-4}$ in our fiducial model, we are allowed to adopt the same criterion.

In addition, Sari & Goldreich (2004) showed that the orbital eccentricity of a gap-opening object cannot exceed the ratio between the gap width and the orbital radius. Otherwise, the
the Type-I torque less effective of the disk is larger than that of the outer part, due to the higher eccentricity of the IMBH would remain small. This result also indicates that the eccentricity of the IMBH would remain small.

Figure 3 shows the early evolution of the IMBH and the two sBHs when the condition \( e_s < h \) can be satisfied. We can see that the sBH inside the orbit of the IMBH remains trapped during the Type-II migration of the IMBH. The migrations of the two BHs are synchronized. However, the outer sBH outside the IMBH is initially pushed away from the IMBH. This behavior can be explained by the positive Type-I torque at the outer edge of the gap. Moreover, this outer sBH cannot keep up with the Type-II migration of the IMBH. This result is consistent with the prediction made in the previous section based on a qualitative analysis of the resonances. As the IMBH leaves its original location, we observe that the outer sBH stays more or less at the same initial radius. This is caused by a restore of the disk to the unperturbed state and a recovery of the Type-I migration trap at the location of the outer sBH.

We find that the coupling between the IMBH and the inner sBH is facilitated by different mechanisms at different evolutionary stages. (i) At the beginning, the IMBH migrates relatively slowly. The negative Type-I torque at the inner edge of the gap (see Figure 1) is sufficient to drive the sBH inward, at the same pace as the IMBH. The torque due to mean-motion resonance is subdominant at this stage because the orbital eccentricity of the sBH remains small (see the inset of Figure 3). (ii) When the IMBH reaches a semimajor axis of \( a_I \sim 10^2 R_s \), the migration becomes relatively fast because \( T_{\text{IH}} \propto a_I^{3/2} \). At the same time, the aspect ratio \( h \) of the inner part of the disk is larger than that of the outer part, due to the higher temperature in the inner disk. The larger aspect ratio also makes the Type-I torque less effective \( (T_I \propto h^{2}) \). As a result, the torque due to mean-motion resonance starts to dominate. Since the resonance torque is proportional to \( e_s \), as Equation (5) suggests, the sBH needs to maintain a higher eccentricity with respect to the eccentricity in the earlier evolutionary stage to keep pace with the IMBH. For this reason, in the inset of Figure 3, we see a fast rise of \( e_s \) toward the end of the simulation. In fact, the sBH is trapped in the 3:2 resonance at this stage because we find that the ratio \( a_s/a_I \) converges to \((2/3)^{2/3} \approx 0.76\).

### 4.2. Late Evolution in the GW Regime

When the IMBH migrates to a distance of \( a_I \sim 30R_s \) from the central SMBH, GW radiation becomes important and hence the IMBH migrates even faster. A direct consequence of this is that \( e_s \) is excited to a higher value than the aspect ratio \( h \) of the disk. At this stage, we can no longer calculate the hydrodynamical damping timescale of the eccentricity by \( h^2T_I \).

To properly calculate the damping timescale for \( e_s > h \), we use the relationship \( T_d = f_e e_s^2 \) that is found in the previous studies on the migration of planetesimals with high eccentricities (e.g., Papaloizou & Larwood 2000; Cresswell et al. 2007; Cresswell & Nelson 2008; Bitsch & Kley 2010; Muto et al. 2011; Ida et al. 2020). The coefficient \( f_e \) is determined with the following considerations. By comparing Equation (32) in Papaloizou & Larwood (2000) with the empirical fitting formula in Cresswell & Nelson (2008), as well as the formula derived from a dynamical-friction model (Muto et al. 2011; Ida et al. 2020), we find a discrepancy of factor of two for \( f_e \). Therefore, we use the value derived in Papaloizou & Larwood (2000) with the default softening parameter as our default \( f_e \) and increase it by a factor of three to account for the current theoretical uncertainties. The resulting \( T_d \) is longer than that for \( e_s < h \).

Besides a longer damping timescale, the interaction between the IMBH and the inner sBH is also characteristically different. The orbital eccentricity of the sBH is now high, as we have seen in Figure 3. The resonance model based on perturbation theory and presented in the previous subsection becomes invalid. Therefore, we drop the 1D code and use instead the 2D \( N \)-body simulation package \texttt{rebound} (Rein & Liu 2012; Rein & Spiegel 2015) to study the gravitational interaction between the two BHs when \( a_I < 30R_s \). Since \texttt{rebound} does not include hydrodynamical effects, we implement an extra force term to the sBH to account for the hydrodynamical damping of \( e_s \) on the timescale of \( T_d \) (following Coleman & Nelson 2014). In addition, to include the effect of GW radiation, we imposed an extra friction on the IMBH and the sBH. The magnitude of this frictional force is calculated from the energy-loss timescale due to GW radiation (Peters & Mathews 1963).

We note that including the triple dynamics (using \texttt{rebound}) is crucial to correctly derive the eccentricity of the sBH, even in this late-evolutionary stage where GW radiation is strong. Although GW radiation now dominates the dynamics of the IMBH, it is not necessarily the case for the sBH. The reason for this is that the timescale of orbital circularisation due to GW radiation is proportional to the mass of the small body, and an sBH is much lighter than an IMBH. Therefore, the mean-motion resonance remains important, relative to the GW radiation, in determining the orbital eccentricity of the sBH. For a similar reason, the damping of eccentricity by gas is not negligible. In fact, the results shown later in this subsections corroborate the importance of the mean-motion torque and the gas torque imposed on the sBH.

In our 2D simulation, the initial loci of the sBH and the IMBH are \( a_s = 23R_s \) and \( a_I = 30R_s \), which are adopted from our 1D simulation. In this configuration, the two BHs are in the 3:2 resonance. To be consistent with the output of the 1D...
In our default damping timescale, we note that the pericenter of the sBH already enters the sensitive band of LISA (>1 mHz) eight years before the IMBH does and 10 years before the final coalescence between the IMBH and the SMBH. Therefore, LISA can also simultaneously detect an EMRI and an IMRI in this case. Similar to what we have found in the previous case with a short damping timescale, the sBH is eventually scattered to a much larger radius by the IMBH. Reentry of the sBH into the LISA band happens more than 4000 years later. Therefore, LISA could detect neither this event nor the earlier IMRI event.

5. Discussion and Conclusion

In this work, we studied the interaction between a gap-opening IMBH in an AGN disk and the sBHs embedded in the disk. We found that during its Type-II migration the IMBH could capture an sBH inside its orbit and form an EMRI-IMRI pair. Our simulations showed that the subsequent migration of the two BHs is synchronized until they reach a distance of about ∼10 R_S from the central SMBH. Consequently, the EMRI and the IMRI successively enter the LISA band within a timescale of four to eight years. The time delay depends on the effectiveness of the hydrodynamical damping of the orbital eccentricity of the sBH. When the damping effect is strong, the EMRI and the IMRI could even simultaneously appear in the LISA band for about one year before the EMRI is disrupted by the gravitational force of the IMBH.

Estimating the event rate of such EMRI-IMRI pairs is difficult given the many theoretical uncertainties, such as the number of sBHs inside an AGN accretion disk and the formation channel of the IMBHs. If the IMBHs are produced by successive mergers of the sBHs inside migration traps, then we can estimate the event rate in the following way. Recent theoretical models neglecting migration traps predict that the event rate of the EMRIs in AGNs is 10^{-4} per year (Pan et al. 2021). If a migration trap is present in the disk, then the sBHs will no longer form EMRIs but accumulate in the trap. Since on
average 10 sBHs are needed to form a gap-opening IMBH that can leave the migration trap (Section 2), the formation rate of the EMRI-IMRI pairs would be about 1–10^3 per year in this scenario.

Given the mutual interaction between the EMRI and the IMRI in the pair, we expect the GW signal to be different from either a single EMRI or a single IMRI. Some hints can be drawn from the previous studies on the perturbation of an EMRI by the nearby stars or sBHs. Amaro-Seoane et al. (2012), Flanagan & Hinderer (2012), Bonga et al. (2019), and Gupta et al. (2021) showed that the effect is the strongest when the perturber and the EMRI temporarily enter a resonance, which will result in a significant phase shift in the waveform of the EMRI. Gupta et al. (2022) further showed that such a phase shift, if ignored in the waveform model, will induce non-negligible biases in the estimated parameters of the EMRI. However, if properly accounted for, then the perturbed signal can reveal the mass and orbital parameters of the perturber (Spero & Gair 2021; Gupta et al. 2022). These earlier results also apply to our EMRI-IMRI pairs because they are locked in resonances for an extended period of time, as we have seen in Section 4.2. Moreover, because the EMRIs enter the LISA band earlier than the IMRIs, the EMRI signals could reveal the outer IMBHs and be used to infer their parameters, even before the IMRIs enter the LISA band.

Although we only run simulations with m_{IMBH} = 100 M_\odot, the results are applicable to more massive IMBHs as long as m_{IMBH} is not much larger than the local disk mass M_d \equiv 4\pi a_d^2. When m_{IMBH} \gg M_d, the IMBH cannot follow the viscous evolution of the disk because it will take a much longer time for the gas to significantly absorb the angular momentum of the IMBH (Syer & Clarke 1995). In this case, the migration timescale of the IMBH will be much longer than the T_g in Equation (4) and the Type-I migration timescale in Equation (1). Therefore, the IMBH cannot catch up with the sBHs migrating inside its orbit. The pair of an outer IMBH and an inner EMRI will not form in this case.

When m_{IMBH} \gg M_d, the slowly migrating IMBH, by opening a gap in the disk, would block the fast migrating sBHs outside the orbit of the IMBH. Therefore, a new type of trap forms at the outer boundary of the gap. We did not study this type of trap in this work because it disappears as soon as the inner IMBH enters the GW-radiation region, when the migration of the IMBH becomes faster than the Type-I migration of the sBHs (e.g., see the green-dotted line in Figure 3). Nevertheless, before the trap disappears, sBHs will accumulate inside it, interact with each other, and form binaries, just as they do in a conventional Type-I migration trap (Secunda et al. 2019). Therefore, we expect a positive correlation between the formation of SMBH-IMBH binaries and an enhancement of the merger rate of sBHs in AGNs. It is worth noting that such a trap exists not only around a massive IMBH but also around a secondary SMBH if there is one inside an AGN disk. In conclusion, we find that gap-opening IMBHs (or secondary SMBHs) in AGN disks could trap sBHs around their orbits. The traps could produce EMRI-IMRI pairs, which may appear simultaneously in the LISA band. Detecting such pairs can shed light on the formation and evolution of the compact objects in an AGN disk. A successful detection also requires more efforts in modeling the three-body dynamics and the corresponding GW signals in the region of strong gravity.

This work is supported by the National Key Research and Development Program of China grant No. 2021YFC2203002, and the National Science Foundation of China grants No. 11991053 and 11873022. The computation in this work was performed on the High Performance Computing Platform of the Centre for Life Science, Peking University. The authors would like to thank Camilo Fonteiclla and Jorge Cuadra for sharing their code.

ORCID iDs
Xian Chen https://orcid.org/0000-0003-3950-9317

References
Amaro-Seoane, P. 2018, LRR, 21, 4
Amaro-Seoane, P., Brem, P., Cuadra, J., & Armitage, P. J. 2012, ApJL, 744, L20
Amaro-Seoane, P., Gair, J. R., Freitag, M., et al. 2007, CQGra, 24, R113
Antoni, A., MacLeod, M., & Ramirez-Ruiz, E. 2019, ApJ, 884, 22
Armitage, P. J., & Natarajan, P. 2002, ApJL, 567, L9
Artymowicz, P., Lin, D. N. C., & Wampler, E. J. 1993, ApJ, 404, 653
Bartos, I., Kocsis, B., Haiman, Z., & Márka, S. 2017, ApJ, 835, 165
Baruteau, C., Cuadra, J., & Lin, D. N. C. 2011, ApJ, 726, 28
Basu, P., Mondal, S., & Chakrabarti, S. K. 2008, MNRAS, 388, 219
Bellobrowsky, J. M., Mac Low, M.-M., McKernan, B., & Ford, K. E. S. 2016, ApJL, 819, L17
Bitsch, B., & Kley, W. 2010, A&A, 523, A30
Bonga, P., Yang, H., & Hughes, S. A. 2019, PhRvL, 123, 101103
Chakrabarti, S. K. 1993, ApJ, 411, 610
Coleman, G. A. L., & Nelson, R. P. 2014, MNRAS, 445, 479
Cresswell, P., Dirksen, G., Kley, W., & Nelson, R. P. 2007, A&A, 473, 329
Cresswell, P., & Nelson, R. P. 2008, A&A, 482, 677
Derdzinski, A., & Mayer, L. 2022, MNRAS, 521, 4522
Duffell, P. C., & Chiang, E. 2015, ApJ, 812, 94
Fabj, G., Nasim, S. S., Caban, F., et al. 2020, MNRAS, 499, 2608
Fonteiclla, C., Haiman, Z., & Cuadra, J. 2019, MNRAS, 482, 4383
Ford, E. B., Kozinsky, B., & Rasio, F. A. 2000, ApJ, 535, 385
Flanagan, É. É., & Hinderer, T. 2012, PhRvL, 109, 071102
Gabor, J. M., & Bournaud, F. 2013, MNRAS, 434, 606
Gayathri, V., Bartos, I., Haiman, Z., et al. 2020, ApJL, 890, L20
Gerosa, D., & Fishbach, M. 2021, NatAs, 5, 749
Goldreich, P., & Tremaine, S. 1978, ApJ, 222, 850
Goldreich, P., & Tremaine, S. 1979, ApJ, 233, 857
Goldreich, P., & Tremaine, S. 1986, ApJ, 309, 846
Hopkins, P. F., & Hernquist, L. 2009, ApJ, 698, 1550
Iida, S., Muto, T., Matsumura, S., & Brassier, R. 2020, MNRAS, 494, 5666
Ivanov, P. B., Papaloizou, J. C. B., & Polnarev, A. G. 1999, MNRAS, 307, 79
Karas, V., & Subr, L. 2001, A&A, 376, 686
Kocsis, B., Haiman, Z., & Loeb, A. 2011, MNRAS, 417, L20
Kocsis, B., Yunes, N., & Loeb, A. 2012, MNRAS, 426, 2680
Kocsis, B., Yunes, N., & Loeb, A. 2011, PhRvD, 84, 024032
Leigh, N. W. C., Geller, A. M., McKernan, B., et al. 2018, MNRAS, 474, 5672
Levin, Y. 2003, arXiv:astro-ph/0307084
Lin, D. N. C., & Papaloizou, J. 1979a, MNRAS, 186, 799
Lin, D. N. C., & Papaloizou, J. 1979b, MNRAS, 188, 191
Lin, D. N. C., & Papaloizou, J. 1986, ApJ, 309, 846
Mastrobuono-Battisti, A., Perets, H. B., & Loeb, A. 2014, ApJ, 796, 40
McKernan, B., Ford, K. E. S., Bellobrowsky, J., et al. 2018, ApJ, 866, 66
McKernan, B., Ford, K. E. S., Lyra, W., et al. 2011, MNRAS, 417, L103
McKernan, B., Ford, K. E. S., Lyra, W., & Perets, H. B. 2012a, MNRAS, 425, 460
McKernan, B., Ford, K. E. S., Lyra, W., & Perets, H. B. 2012b, MNRAS, 425, 460

8
