The Scalar Sector in 331 Models

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Abstract

We calculate the exact tree-level scalar mass matrices resulting from symmetry breaking using the most general gauge-invariant scalar potential of the 331 model, both with and without the condition that lepton number is conserved. Physical masses are also obtained in some cases, as well as couplings to standard and exotic gauge bosons.
1 Introduction

The 331 model is an extension to the Standard Model in which the gauge group is $SU(3)_c \times SU(3)_L \times U(1)_N$. Some of its interesting features include that the third family of quarks is treated differently to the first two, that the number of families is required to be three (or a multiple of three) to ensure anomaly cancellation, and the existence of new phenomenology which would be apparent at reasonably low energies. In particular, a doublet of charged vector bosons carrying a lepton number $L = 2$ (known as bileptons) is predicted, as well as an additional neutral gauge boson, $Z'$.

Most analysis of the 331 model has centred on the phenomenology of the bileptons and $Z'$. While certain aspects of the scalar sector of such models have also been discussed several times (including supersymmetric extensions) these discussions have usually not considered a fully general potential for the scalar fields. The purpose of this work is therefore to calculate the exact tree-level mass matrices for the scalar particles using a potential which contains all possible gauge-invariant terms, both with and without those which violate lepton-number. In some cases the masses of the physical states will also be given, and coupling of the Higgs particles to both standard and exotic gauge bosons will also be briefly discussed.

We will be considering the original version of the 331 model, in which each lepton triplet is composed of $(\nu_l, l^-, l^+)_L$, where $l = e, \mu, \tau$. The Higgs content at first consisted of three triplets

\[ \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^+ \end{pmatrix} \sim (3, 0), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (3, 1), \quad \chi = \begin{pmatrix} \chi^- \\ \chi^{--} \end{pmatrix} \sim (3, -1) \]

However it was shown that in order to give the leptons realistic masses, a scalar sextet is also needed,

\[ S = \begin{pmatrix} \sigma^0_1 \\ s^-_2 \\ s^-_1 \\ \sigma^0_2 \\ \sigma^+ \quad \sigma^0_2 \end{pmatrix} \sim (6, 0) \]

An interesting feature of the 331 model is that that particles within a multiplet do not all carry the same value of lepton number, a fact which arises from having the charged lepton from each family in the same triplet as its anti-particle. The lepton number carried by the scalars can be obtained from inspection of the Yukawa couplings to fermions. The values obtained in this way are:

\[ L(\chi^-, \chi^{--}, \sigma^0_1, s^-_2, S^-_1) = +2 \]
\[ L(\eta^+, \rho^{++}, S^+_2) = -2 \]
\[ L(\eta^-_1, \rho^+, \rho^0, \chi^0, s^+_1, \sigma^0_2) = 0 \]
There are thus two ways in which lepton number can be violated in the scalar sector, either by allowing terms in the potential which do not conserve the values of lepton number assigned above, or by allowing the field $\sigma_0^b$ to develop a VEV, in which case lepton number is spontaneously broken. These possibilities will be discussed in section 5. Until that point we will be concerned only with the minimal model in which lepton number is conserved.

The formalism used will be based on that of Ref. [27]. The three triplets and the sextet will be made up of complex fields $\phi_x$, each field then composed of real fields $\phi_x = \frac{1}{\sqrt{2}}(\phi_{x_1} + i\phi_{x_2})$.

\[
\eta = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{a_1} + i\phi_{b_2} \\ \phi_{b_1} + i\phi_{b_2} \\ \phi_{c_1} + i\phi_{c_2} \end{pmatrix}, \quad \rho = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{d_1} + i\phi_{d_2} \\ \phi_{e_1} + i\phi_{e_2} \\ \phi_{f_1} + i\phi_{f_2} \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{g_1} + i\phi_{g_2} \\ \phi_{h_1} + i\phi_{h_2} \\ \phi_{k_1} + i\phi_{k_2} \end{pmatrix}
\]

(6)

\[
S = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{a_1} + i\phi_{a_2} \\ \phi_{b_1} + i\phi_{b_2} \\ \phi_{c_1} + i\phi_{c_2} \end{pmatrix} \begin{pmatrix} \phi_{a_1} + i\phi_{a_2} \\ \phi_{b_1} + i\phi_{b_2} \\ \phi_{c_1} + i\phi_{c_2} \end{pmatrix} \begin{pmatrix} \phi_{u_1} + i\phi_{u_2} \\ \phi_{v_1} + i\phi_{v_2} \\ \phi_{w_1} + i\phi_{w_2} \end{pmatrix}
\]

(7)

The mass matrices will then be calculated, using

\[
M_{ij}^2 = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j}
\]

(8)
evaluated at the chosen minimum. So that the particular set of values of VEVs chosen is actually a minimum, all first derivatives $\frac{\partial V}{\partial \phi_i}$ must be zero and the second derivatives are required to be non-negative. Where the mass matrices can be diagonalised, the eigenvalues and eigenstates will also be given.

## 2 Form of the Scalar Potential

Previous studies of the scalar sector of the 331 model by Tonasse [17] and recently by Nguyen, Nguyen and Long [22] have been based on the following potential:

\[
V_1(\eta, \rho, \chi, S) = \mu_1^2\eta^\dagger \eta + \mu_2^2\rho^\dagger \rho + \mu_3^2\chi^\dagger \chi + \lambda_1(\eta^\dagger \eta)^2 + \lambda_2(\rho^\dagger \rho)^2 + \lambda_3(\chi^\dagger \chi)^2 + \lambda_4(\eta^\dagger \eta)(\rho^\dagger \rho) + \lambda_5(\eta^\dagger \eta)(\chi^\dagger \chi) + \lambda_6(\rho^\dagger \rho)(\chi^\dagger \chi) + \lambda_7(\rho^\dagger \eta)(\rho^\dagger \rho) + \lambda_8(\chi^\dagger \eta)(\chi^\dagger \chi) + \lambda_9(\rho^\dagger \chi)(\chi^\dagger \rho) + f_1(\eta \rho \chi + H.c.) + \mu_2^2 Tr(S^\dagger S) + \lambda_{10} [Tr(S^\dagger S)]^2 + \lambda_{11} Tr[(S^\dagger S)^2] + [\lambda_{12}(\eta^\dagger \eta) + \lambda_{13}(\rho^\dagger \rho) + \lambda_{14}(\chi^\dagger \chi)] Tr(S^\dagger S) + f_2(\rho \chi S + H.c.)
\]
where the coefficients $f_1$ and $f_2$ have dimensions of mass.

Written in a different notation, the same potential was also used by Foot et al. \[9\] where the discrete symmetry

\[
\begin{align*}
\rho & \rightarrow i\rho \\
\chi & \rightarrow i\chi \\
\eta & \rightarrow -\eta \\
S & \rightarrow -S
\end{align*}
\]

was introduced. This was for the purpose of preventing terms which violate lepton number from appearing such as $SSS$ and $\eta S^\dagger \eta$. However, there are a number of additional possible gauge invariant terms which are not excluded by this symmetry. (The full potential is given by Liu and Ng in Ref. \[12\]) While it would be possible to choose a different symmetry which did prevent some of the extra terms, for example

\[
\begin{align*}
\eta & \rightarrow -\eta \\
\rho & \rightarrow -\rho \\
\chi & \rightarrow i\chi \\
S & \rightarrow -iS
\end{align*}
\]

such choices also exclude either $f_1\eta\rho\chi$ or $f_2\rho\chi S$ which are necessary to ensure that no continuous symmetries higher than $SU(3) \times U(1)$ exist, and thus avoid the generation of additional Goldstone bosons. \[18\] Therefore for this section, the original choice of discrete symmetry will be retained, and the potential extended to

\[
V_2(\eta, \rho, \chi, S) = V_1 + \lambda_{15}\eta^\dagger S S^\dagger \eta + \lambda_{16}\rho^\dagger S S^\dagger \rho + \lambda_{17}\chi^\dagger S S^\dagger \chi \\
+ (\lambda_{19}\rho^\dagger S\eta + \lambda_{20}\chi^\dagger S\chi\eta + \lambda_{21}\eta\eta SS + H.c.)
\]

Note that the $SU(3)$ invariant contractions for the last three terms are formed by:

\[
\begin{align*}
\epsilon_{ijk} \rho^\dagger S^i\rho^j\eta^k \\
\epsilon_{ijk} \chi^\dagger S^i\chi^j\eta^k \\
\epsilon_{ijk}\epsilon_{lmn} \eta^i S^j m S^k n
\end{align*}
\]

3 Calculation of the Mass Spectrum

The triplet Higgs fields develop VEVs as follows

\[
\langle \eta \rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}, \langle \rho \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}, \langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix}
\]
while for the sextet, only the $\sigma_2^0$ field develops a VEV

$$\langle \sigma_2^0 \rangle = v_4$$

All VEVs are taken to be real. (The possibility of CP-violation arising from the scalar sector has been investigated in detail by Gómez Dumm [16]).

Requiring this choice of VEVs to be a minimum of the potential leads to the following relations.

$$\mu_1^2 = -2\lambda_1 v_1^2 - \lambda_4 v_2^2 - \lambda_5 v_3^2 - f_1 \frac{v_2 v_3}{v_1} - \lambda_{12} v_4^2 + \frac{\lambda_{19}}{\sqrt{2}} \frac{v_2^2 v_4}{v_1}$$

$$\mu_2^2 = -2\lambda_2 v_2^2 - \lambda_4 v_2^2 - \lambda_6 v_2^2 - f_1 \frac{v_1 v_3}{v_2} - (\lambda_{13} + \frac{\lambda_{16}}{2}) v_4^2$$

$$\mu_3^2 = -2\lambda_3 v_3^2 - \lambda_5 v_3^2 - \lambda_6 v_3^2 - f_1 \frac{v_3 v_4}{v_3} - (\lambda_{14} + \frac{\lambda_{17}}{2}) v_4^2$$

$$\mu_4^2 = -(2\lambda_{10} + \lambda_{11}) v_4^2 - \lambda_{12} v_1^2 - (\lambda_{13} + \frac{\lambda_{16}}{2}) v_2^2 - (\lambda_{14} + \frac{\lambda_{17}}{2}) v_3^2$$

$$+ \frac{\lambda_{19}}{\sqrt{2}} \frac{v_1 v_2^2}{v_4} + \frac{\lambda_{20}}{\sqrt{2}} \frac{v_2 v_4^2}{v_4} - \lambda_{21} v_1^2 - f_2 \frac{v_2 v_3}{2v_4}$$

Using these values for the $\mu_i^2$, the mass matrices can now be calculated. The neutral CP-even mass matrix is the most complicated. In the basis formed by $(\phi_{a1} - v_1, \phi_{e1} - v_2, \phi_{k1} - v_3, \phi_{r1} - v_4)$ it may be written in the following form.
\[
\begin{pmatrix}
  m_{11} v_1^2 & m_{12} v_1 v_2 & m_{13} v_1 v_3 & m_{14} v_1 v_4 \\
  m_{12} v_1 v_2 & m_{22} v_2^2 & m_{23} v_2 v_3 & m_{24} v_2 v_4 \\
  m_{13} v_1 v_3 & m_{23} v_2 v_3 & m_{33} v_3^2 & m_{34} v_3 v_4 \\
  m_{14} v_1 v_4 & m_{24} v_2 v_4 & m_{34} v_3 v_4 & m_{44} v_4^2
\end{pmatrix}
\begin{pmatrix}
  v_1^2 v_3 \\
  v_2^2 v_3 \\
  v_1 v_2^2 v_3 \\
  v_1^2 v_2 v_3
\end{pmatrix}
\begin{pmatrix}
  v_1 v_2 v_3 \\
  v_1 v_2 v_3 \\
  v_1 v_2 v_3 \\
  v_1 v_2 v_3
\end{pmatrix}\]

\[
-\frac{f_1}{v_1 v_2 v_3} \begin{pmatrix}
  0 & 0 & 0 & 0 \\
  0 & v_2^2 v_4 & v_2 v_3 v_4 & -v_2 v_4 v_4 \\
  0 & v_2 v_3 v_4 & v_2^2 v_4 & -v_2 v_4 v_4 \\
  0 & -v_2 v_4 v_4 & -v_2 v_4 v_4 & v_2 v_4 v_4
\end{pmatrix}
\]

\[
-\frac{f_2}{\sqrt{2} v_2 v_3 v_4} \begin{pmatrix}
  v_2^2 v_4 & 0 & 0 & -v_1 v_4 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  -v_1 v_4 & 0 & 0 & v_1^2\end{pmatrix}
\]

where

\[
m_{11} = 2 \lambda_1 \\
m_{12} = m_{21} = (\lambda_4 - \frac{\lambda_{19} v_4}{\sqrt{2} v_1}) \\
m_{13} = m_{31} = (\lambda_5 + \frac{\lambda_{20} v_4}{\sqrt{2} v_1}) \\
m_{14} = m_{41} = (\lambda_1 + 2 \lambda_{21}) \\
m_{22} = 2 \lambda_2 v_2^2 \\
m_{23} = m_{32} = \lambda_6 \\
m_{24} = m_{42} = (\lambda_{13} + \frac{\lambda_{16} v_4}{2} - \frac{\lambda_{19} v_4}{\sqrt{2} v_1}) \\
m_{33} = 2 \lambda_3 \\
m_{34} = m_{43} = (\lambda_{14} + \frac{\lambda_{17} v_4}{2} + \frac{\lambda_{20} v_4}{\sqrt{2} v_1}) \\
m_{44} = (2 \lambda_{10} + \lambda_{11})
\]

This matrix will lead to the existence of four neutral CP-even particles, denoted $H_{1,2,3,4}^0$, the masses of which can only be obtained by making some fairly severe approximations.
The neutral CP-odd matrix takes the simpler form:

\[
\begin{pmatrix}
 \frac{-f_1}{v_1v_2v_3} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
 \frac{-f_2}{\sqrt{2}v_2^2v_3v_4} & 0 & 0 & 0 \\
 0 & \frac{v_3^2v_4^2}{v_2^2v_3^2} & \frac{v_2^2v_3^2}{v_2^2v_3^2} & \frac{-v_2^2v_3^2}{v_2^2v_3^2} \\
 0 & v_2v_3v_4 & v_2v_3v_4 & \frac{-v_2v_3v_4}{v_2v_3v_4} \\
 0 & v_2v_3v_4 & \frac{-v_2v_3v_4}{v_2v_3v_4} & \frac{v_2^2v_3^2}{v_2^2v_3^2} \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
 \lambda_9 v_3^2 & 0 & 0 & v_1v_4 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 v_1v_4 & 0 & 0 & v_1^2 \\
\end{pmatrix}
\]

(37)

and this leads to two massless particles $G_1^0, G_2^0$ and two physical ones, $A_1^0, A_2^0$.

There is further an additional CP-even and CP-odd state, degenerate in mass, resulting from the $\phi_s$ field, which carry a lepton number $L = 2$, and thus do not mix with the other neutral states.

\[
H_5^0 = \phi_{s_1}
\]

\[
A_3^0 = \phi_{s_2}
\]

\[
m_{H_5^0}^2 = m_{A_3^0}^2 = -\lambda_{11}v_4^2 + \lambda_{15}v_1^2 - \frac{f_2}{\sqrt{2}} \frac{v_2v_3}{v_4} - \frac{\lambda_{16}v_2^2}{2} - \frac{\lambda_{17}v_3^2}{2}
\]

In the singly-charged sector, there is mixing between the $L = 0$ fields $\phi_b, \phi_d^*$ and $\phi_d^*$, and also between the $L = -2$ fields $\phi_c, \phi_b^*$ and $\phi_b^*$. The mixing between $\phi_b, \phi_d^*$ and $\phi_d^*$ is given by:

\[
l_1 \begin{pmatrix} v_2^2 & v_1v_2 & 0 \\ v_1v_2 & v_1^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + l_2 \begin{pmatrix} v_1^2 & 0 & v_1v_4 \\ 0 & 0 & 0 \\ v_1v_4 & 0 & v_1^2 \end{pmatrix} + l_3 \begin{pmatrix} 0 & 0 & 0 \\ 0 & v_1^2 & -v_2v_4 \\ 0 & -v_2v_4 & v_2^2 \end{pmatrix}
\]

(41)

where

\[
l_1 = \lambda_7 - f_1 \frac{v_3}{v_1v_2} + \lambda_{19} \frac{v_4}{v_1}
\]

\[
l_2 = \lambda_{15} - 4\lambda_{21} - \lambda_{20} \frac{v_3^2}{v_1v_4}
\]

\[
l_3 = -\lambda_{16} - f_2 \frac{v_3}{v_2v_4} + \lambda_{19} \frac{v_1}{v_4}
\]

(42) (43) (44)
which leads to a massless scalar $G_1^+$ and two physical particles of mass:

$$m^2_{H_{1,4}^+} = l_1(v_1^2 + v_2^2) + l_2(v_1^2 + v_4^2) + l_3(v_2^2 + v_4^2)$$

$$+ \pm \frac{1}{2} \left\{ l_4(v_1^2 + v_2^2) + l_5(v_1^2 + v_4^2) + l_6(v_2^2 + v_4^2) \right\}^2$$

$$- 4(v_1^2 + v_2^2 + v_4^2)(l_4l_5v_1^2 + l_4l_6v_2^2 + l_5l_6v_4^2)^2$$

The mixing between $\phi_c$, $\phi_b^*$ and $\phi_u^*$ is

$$l_4 \begin{pmatrix} v_3^2 & v_1v_3 & 0 \\ v_1v_3 & v_1^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + l_5 \begin{pmatrix} v_2^2 & 0 & v_1v_4 \\ 0 & 0 & 0 \\ v_1v_4 & 0 & v_1^2 \end{pmatrix} + l_6 \begin{pmatrix} 0 & 0 & 0 \\ 0 & v_4^2 & -v_3v_4 \\ 0 & -v_3v_4 & v_3^2 \end{pmatrix}$$

where

$$l_4 = \lambda_8 - f_1 \frac{v_2}{v_1v_3} - \lambda_20 \frac{v_4}{v_1}$$

$$l_5 = \lambda_{15} + \lambda_{19} \frac{v_2^2}{v_1v_4} - 4\lambda_{21}$$

$$l_6 = -f_2 \frac{v_2}{v_3v_4} + \lambda_{17} + \lambda_{20} \frac{v_1}{v_4}$$

again leading to a massless state $G_2^+$ and two physical particles

$$m^2_{H_{1,4}^+} = l_4(v_1^2 + v_3^2) + l_5(v_1^2 + v_4^2) + l_6(v_3^2 + v_4^2)$$

$$+ \pm \frac{1}{2} \left\{ l_4(v_1^2 + v_3^2) + l_5(v_1^2 + v_4^2) + l_6(v_3^2 + v_4^2) \right\}^2$$

$$- 4(v_1^2 + v_3^2 + v_4^2)(l_4l_5v_1^2 + l_4l_6v_3^2 + l_5l_6v_4^2)^2$$

Lastly, mixing between the doubly-charged fields is given in the $\phi_f,\phi_f^*,\phi_u^*,\phi_u$ basis, by:

$$l_7 \begin{pmatrix} v_3^2 & v_2v_3 & 0 & 0 \\ v_2v_3 & v_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + l_8 \begin{pmatrix} \sqrt{2}v_4^2 & 0 & v_2v_4 & 0 \\ 0 & 0 & 0 & 0 \\ v_2v_4 & 0 & \frac{1}{2}v_2^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$+ l_9 \begin{pmatrix} \sqrt{2}v_4^2 & 0 & 0 & -v_2v_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + l_{10} \begin{pmatrix} 0 & \sqrt{2}v_4^2 & 0 & -v_3v_4 \\ 0 & 0 & 0 & 0 \\ 0 & -v_3v_4 & \frac{1}{2}v_3^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$+ l_{11} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \sqrt{2}v_4^2 & 0 & v_3v_4 \\ 0 & 0 & 0 & 0 \\ 0 & v_3v_4 & 0 & \frac{1}{2}v_3^2 \end{pmatrix} + l_{12} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & v_4^2 & 0 & v_2^2 \\ 0 & 0 & v_4^2 & 0 \end{pmatrix}$$
where

\[ l_7 = \left( \lambda_9 - f_1 \frac{v_2 v_3}{v_1} + \frac{f_1}{\sqrt{2} v_2 v_3} \right) \] (52)

\[ l_8 = \left( \frac{\lambda_{16}}{\sqrt{2}} + \lambda_{19} \frac{v_1}{v_4} \right) \] (53)

\[ l_9 = \left( \frac{\lambda_{16}}{\sqrt{2}} + \lambda_{19} \frac{v_1}{v_4} - f_2 \frac{v_3}{v_2 v_4} \right) \] (54)

\[ l_{10} = \left( -\frac{\lambda_{17}}{\sqrt{2}} - \lambda_{20} \frac{v_1}{v_4} - f_2 \frac{v_2}{v_3 v_4} \right) \] (55)

\[ l_{11} = \left( \frac{\lambda_{17}}{\sqrt{2}} - \lambda_{20} \frac{v_1}{v_4} \right) \] (56)

\[ l_{12} = \left( \lambda_{11} - \lambda_{12} \frac{v_1^2}{v_4^2} \right) \] (57)

This matrix has zero determinant and thus there will exist a doubly charged Goldstone boson of each sign, \( G^{++} \) and three massive particles \( H_1^{++}, H_2^{++} \) and \( H_3^{++} \), all having a lepton number \( L = -2 \).

To summarise, symmetry breaking will lead to eight Goldstone bosons, \( G_1^0, G_2^0, G_1^\pm, G_2^\pm \) and \( G^{\pm \pm} \), which become incorporated into the \( Z^0, Z'^0, W'^+, Y'^+ \) and \( Y'^{\pm \pm} \) gauge bosons respectively. The physical states consist of five CP-even neutral scalars \( H_1^0, H_2^0, H_3^0, H_4^0, H_5^0 \), three neutral CP-odd states \( A_1^0, A_2^0, A_3^0 \), four singly-charged particles of each charge \( H_1^+, H_2^+, H_3^+ \) and \( H_4^+ \) and three doubly-charged ones \( H_1^{+ \pm}, H_2^{+ \pm}, H_3^{+ \pm} \).

### 4 Couplings to Gauge Bosons

After the breaking of \( SU(3)_L \times U(1)_N \) symmetry the gauge bosons are formed as follows:

\[ \gamma = \sin \theta_W W_3 - \cos \theta_W (\sin \varphi W_8 - \cos \varphi B) \] (58)

\[ Z = -\cos \theta_W W_3 - \sin \theta_W (\sin \varphi W_8 - \cos \varphi B) \] (59)

\[ Z' = \cos \varphi W_8 + \sin \varphi B \] (60)

\[ W'^+ = \frac{1}{\sqrt{2}} (W_1 - iW_2) \] (61)

\[ Y'^- = \frac{1}{\sqrt{2}} (W_4 - iW_5) \] (62)

\[ Y'^{--} = \frac{1}{\sqrt{2}} (W_6 - iW_7) \] (63)
where $W_\alpha$ and $B$ represent the gauge fields of $SU(3)_L$ and $U(1)_N$ respectively, and

$$\sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} = \frac{g_N}{\sqrt{g^2 + 4g_N^2}}$$  \hspace{1cm} (64)
$$\sin \varphi = \sqrt{3\tan \theta_W} = \frac{\sqrt{3g_N}}{\sqrt{g^2 + 3g_N^2}}$$  \hspace{1cm} (65)

The three coupling constants are related to each other by

$$\frac{1}{g'^2} = \frac{1}{g^2 N} + \frac{3}{g^2}$$  \hspace{1cm} (66)

but it should be noted that this relation will clearly be altered if the covariant derivative (eq. 67) is written with an extra factor multiplying $g_N$. Compared to the Standard Model, there is now an extra neutral gauge boson ($Z'$) and the two bileptons ($Y^-, Y^{--}$).

Couplings of the gauge bosons to the various Higgs states can then be calculated using these definitions and the covariant derivatives for the triplets:

$$D_\mu \phi = \partial_\mu \phi + ig \left( [W_\mu \cdot \lambda] \phi + g_N N_\phi B_\mu \phi \right)$$  \hspace{1cm} (67)

where $W_\mu \cdot \lambda = W_\mu^a \lambda^a$, with $Tr[\lambda^a \lambda^b] = 2\delta^{ab}$.

Considering the CP-even states, all four physical particles are composed of mixtures of $\phi_1^a - v^1_1$, $\phi_2^a - v^1_2$, $\phi_1^k - v^3_4$ and $\phi_1$ - $v^4_4$. However, since in the 331 model $v_3 > v_1, v_2, v_4$ it can be assumed that the lightest state, $H_1^0$ is made up predominantly of $\phi_1^a - v^1_1$, $\phi_2^a - v^1_2$, and $\phi_1^k - v^4_4$ only.

Using this approximation, the following quartic couplings are found.

$$g \left( H_1^0 H_1^0 \gamma \gamma \right) = 0$$  \hspace{1cm} (69)
$$g \left( H_1^0 H_1^0 ZZ \right) \simeq \frac{1}{2} \left( g^2 + g'^2 \right)$$  \hspace{1cm} (70)
$$g \left( H_1^0 H_1^0 W^+ W^- \right) \simeq \frac{g^2}{2}$$  \hspace{1cm} (71)

This state therefore resembles the minimal Standard Model Higgs boson.

Some other values for Higgs-gauge boson couplings have also been calculated.

$$g \left( H_1^+ H_1^- ZZ \right) = g \left( H_2^+ H_2^- ZZ \right) = \frac{1}{2} \left( g^2 - g'^2 \right)^2$$  \hspace{1cm} (72)
$$g \left( H_1^+ H_1^- W^+ W^- \right) = g \left( H_2^+ H_2^- W^+ W^- \right) = \frac{g^2}{2}$$  \hspace{1cm} (73)
$$g \left( H_3^+ H_3^- Y^+ Y^- \right) = g \left( H_4^+ H_4^- Y^+ Y^- \right) = \frac{g^2}{2}$$  \hspace{1cm} (74)
5 Lepton number violation

Until this point we have been considering a minimal version of the 331 model in which lepton number is conserved. There are two ways of extending the model so that this is no longer the case. Firstly, $\sigma_0$ can develop a VEV (denoted $v_5$) satisfying

\[
v_5^2 = v_4^2 - \frac{1}{\lambda_{11}} \left[ \lambda_{15} v_1^2 + \lambda_{16} v_2^2 - \frac{\lambda_{17}}{2} v_3^2 + \frac{\lambda_{19}}{\sqrt{2}} v_1 v_2 \right]
\]

(75)

In this case, lepton number is spontaneously broken, and there therefore arises an additional Goldstone boson - the majoron. In the notation used here, the field $\phi_{s_2}$ becomes massless if $\langle \phi_s \rangle = v_5$. This situation, however, is equivalent to the well-known triplet majoron model, which is ruled out experimentally by Z lineshape measurements, in that the existence of such a particle would contribute to the invisible decay width of the Z and this is not observed. [28]

The other possibility is to extend the potential to include terms which, although gauge-invariant, do not conserve lepton number, that is

\[
V_3(\eta, \rho, \chi, S) = V_2 + f_3 \eta S^\dagger \eta + f_4 S^3 S + \lambda_{22} \chi^4 \eta^\dagger \eta
\]

\[
+ \lambda_{23} \eta^\dagger S \rho + \lambda_{24} \chi S S + H.c.
\]

(76)

The $SU(3)$ contractions for these terms are formed by

\[
\begin{align*}
\epsilon_{ijk} \epsilon_{lmn} S^i_{jm} S^k_{ln} & \\
\chi^4_{ij} \rho^j_{ij} & \\
\epsilon_{ijk} \epsilon_{lmn} \chi^4_{ij} \rho_{jm} S^k_{ln}
\end{align*}
\]

(77)

It is still possible with this potential to maintain $v_5 = 0$ if the condition

\[
f_3 v_1^2 - 3 f_4 v_2^2 + \lambda_{23} v_1 v_2 v_3 - \sqrt{2} \lambda_{24} v_2 v_3 v_4 = 0
\]

(78)

is imposed. In this scenario, neutrinos are massless at tree-level, but develop a majorana mass at the one loop level. [10] With regard to the scalar sector, the effect of the extra terms is to lead to mixing between particles of different lepton number.

Previously, the charged scalars made up of the $L = 0$ fields $\phi_b, \phi_d$ and $\phi_q$ did not mix with those from the $|L| = 2$ fields $\phi_c, \phi_g$ and $\phi_p$. Now however, mixing between the two arises, which in the $(\phi_b, \phi_d^*, \phi_q^*, \phi_g, \phi_p)$ basis may be written as

\[
\begin{pmatrix}
M_{bdq}^2 & M_{bdдcgp}^2 \\
M_{bdдcgp}^2 & M_{cgp}^2
\end{pmatrix}
\]

(79)
where $M_{aekr}^2$ is given by equation (26), $M_{aekr}^2$ by equation (40) and the mixing terms $M_{bdq-egp}^2$ by

\[
\begin{pmatrix}
\sqrt{2} f_3 v_4 + \sqrt{2} v_1 v_2 v_3 & \lambda_22 v_1 v_2 - \lambda_22 v_2 v_3 & \sqrt{2} f_3 v_4 + \lambda_22 v_2 v_3 \\
\lambda_22 v_1 v_3 - \lambda_22 v_3 v_4 & \lambda_22 v_1 - \lambda_22 v_4' & -\lambda_22 v_1 v_3 + \lambda_22 v_4 v_4 \\
\sqrt{2} f_3 v_4 + \sqrt{2} v_1 v_2 v_3 & -\lambda_22 v_1 v_2 + \lambda_22 v_4 v_4 & 3\sqrt{2} f_4 v_4 + \lambda_22 v_4 v_4
\end{pmatrix}
\] (80)

Similarly, with the minimal potential $V_2$ the neutral scalars arising from the $L = 0$ fields $\phi_A, \phi_e$ and $\phi_k$ did not mix with those from the $L = 2$ $\phi_s$ field, but now mixing occurs between both the CP-even states and the CP-odd.

In the CP-even basis ($\phi_a_1 - v_1, \phi_e_1 - v_2, \phi_k_1 - v_3, \phi_r_1 - v_4, \phi_s_1$) the mass matrix takes the form

\[
\begin{pmatrix}
M_{aekr}^2 & M_{aekr-s}^2 \\
M_{aekr-s}^2 & M_s^2
\end{pmatrix}
\] (81)

where $M_{aekr}^2$ is given by equation (26), $M_{aekr-s}^2$ by equation (40) and the mixing term $M_{aekr-s}^2$ by

\[
\begin{pmatrix}
f_3 v_1 + \sqrt{2} v_1 v_2 v_3 \\
\sqrt{2} v_1 v_3 - \sqrt{2} \lambda_24 v_3 v_4 \\
\sqrt{2} v_1 v_2 - \sqrt{2} \lambda_24 v_4 v_4 \\
-6 f_4 v_4 - \sqrt{2} \lambda_24 v_4 v_4
\end{pmatrix}
\] (82)

In the CP-odd basis ($\phi_a_2, \phi_e_2, \phi_k_2, \phi_r_2, \phi_s_2$) the mass matrix becomes

\[
\begin{pmatrix}
M_{aekr}^2 & M_{aekr-s}^2 \\
M_{aekr-s}^2 & M_s^2
\end{pmatrix}
\] (83)

where $M_{aekr}^2$ is given by equation (57), $M_{aekr-s}^2$ by equation (40) and the mixing term $M_{aekr-s}^2$ by

\[
\begin{pmatrix}
f_3 v_1 + \sqrt{2} v_1 v_2 v_3 \\
-\sqrt{2} v_1 v_3 + \sqrt{2} \lambda_24 v_3 v_4 \\
-\sqrt{2} v_1 v_2 + \sqrt{2} \lambda_24 v_4 v_4 \\
6 f_4 v_4 + \sqrt{2} \lambda_24 v_4 v_4
\end{pmatrix}
\] (84)

The additional terms have no effect on the doubly-charged scalars.

## 6 Conclusion

A scalar potential for the 331 model more general than those previously considered in detail has been studied and the scalar mass matrices resulting from symmetry breaking calculated. In some cases, the exact masses for the physical states have been obtained, and the couplings to gauge bosons calculated. Such a potential leads to a very wide variety of scalar bosons.
Extensive studies have been made regarding the possible discovery of various types of Higgs bosons at future collider experiments. The most studied have been the CP-even $H^0$, which occurs in the Standard Model (SM), and those states which occur in the Minimal Supersymmetric Standard Model (MSSM), $H_1^0, H_2^0, A^0$ and $H^\pm$.

A mass limit of 88 GeV has been obtained for the SM $H^0$ by searching for the process $e^+e^- \rightarrow ZH^0$ at LEP, while limits of 71 GeV have been given for both the lightest CP-even state $H_1^0$ and CP-odd state $A^0$ of the MSSM. The current limit for $H^\pm$ is 60 GeV. Doubly-charged states have also received some attention.

The phenomenology of both scalar and vector bileptons has been reviewed by Cuypers and Davidson who noted that the mass of neutral as well as singly and doubly-charged scalar bileptons are all constrained by their non-observation in Z decay to be greater than 45 GeV. Future $e^-e^-$ and $e^+e^-$ collider experiments give the best hope of detecting these particles, providing their mass is less than half the centre-of-mass energy of the collider.

Clearly the phenomenology of the scalar sector of the 331 model is very rich and worthy of further study.

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