3D Turbulent Reconnection: 20 Years After

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Abstract. A theory of 3D turbulent reconnection was published 20 years ago. At that time it was suggested that the fast reconnection requires X-point Petschek-type configuration of magnetic field in the reconnection region, as well as plasma effects that would stabilize this configuration. On the contrary, the turbulent reconnection theory (i) identified 3D Alfvénic turbulence as the cause of fast reconnection, (ii) postulated the ubiquitous nature of turbulent reconnection in astrophysical high-Reynolds number environments, and (iii) identified Y-point volume reconnection as the typical magnetic reconnection on length scales where the magnetohydrodynamic (MHD) approximation is valid. In this article we briefly summarize the status of the 3D turbulent reconnection theory. We show that other alternative models of reconnection have evolved toward supporting the essential features of the original turbulent reconnection theory. We also provide numerical and observational evidence supporting the turbulent reconnection and its consequences, e.g., the breakdown of classical magnetic flux freezing in highly conducting turbulent fluids.

1. Magnetic reconnection in astrophysics

Magnetic fields are ubiquitous in astrophysical systems and they critically affect the dynamics and properties of magnetized plasmas on all length scales, including those much larger than any relevant plasma scales, e.g., the ion inertial length. In fact, the extensively studied magnetic fields of Earth’s magnetosphere are an exception in the astrophysical context in terms of the involved length scales.

It is widely believed in astrophysical studies, that magnetic fields in highly conducting plasmas are nearly perfectly frozen into the fluid and retain their topology all the time \cite{10, 2}. The concept of flux-freezing is at the heart of many astrophysical theories, e.g., the theory of star formation in magnetized interstellar medium (ISM).

The theory of 3D turbulent reconnection was introduced by \cite{3} (henceforth LV99). This model was followed by several subsequent theoretical studies, for instance, \cite{4} (henceforth ELV11), \cite{5}, and \cite{6}. The LV99 theory has been supported by numerical simulations in non-relativistic (see \cite{7, 8, 9, 10, 11, 12, 13, 14}) as well as relativistic settings (see \cite{13, 14}). In addition, different pieces of observational evidence support the predictions of the 3D turbulent reconnection theory of LV99.

2. Turbulence in astrophysical fluids

2.1. Observational evidence

Observations of the ISM strongly support its turbulent state (see \cite{15, 16}), see Figure \ref{fig:1} (see \cite{17} for a review). This is also clear from the \textit{in situ} measurements of the solar wind fluctuations \cite{18} and via
Figure 1. Big power law in the sky from (15) extended to length scales of tens of parsecs using WHAM data. From (16).

studies of non-thermal broadening of spectral lines as well as measures obtained by other techniques (see [19]. Turbulence naturally arises from magnetized astrophysical plasmas generally with very large Reynolds numbers. Indeed, the length scales of the motions are large, while the motions of charged particles in the direction perpendicular to magnetic fields are of the order of the Larmor radius. As a result, laminar plasma flows are prey to numerous linear and finite-amplitude instabilities that induce turbulent motions.

Turbulence can be driven by an external energy source, such as supernovae in the ISM ([20][21]), merger events and active galactic nuclei outflows in the intracluster medium (ICM) ([22][23][24]), and baroclinic forcing behind shock waves in interstellar clouds. In addition, magneto-rotational instability (MRI) in accretion disks ([25][26]), kink instability of twisted flux tubes in the solar corona ([27][28]), etc. can drive turbulence. In some cases, magnetic reconnection can also be the driver.

Observations show that turbulence is ubiquitous in all astrophysical plasmas. The spectrum of electron density fluctuations in Milky Way is presented in Figure 1 as one dramatic piece of evidence for cosmic turbulence. Similar spectra are observed in solar wind ([18] and [29]), molecular clouds ([30]), and the ICM ([31]). New techniques for studying turbulence, e.g. the Velocity Channel Analysis (VCA) and Velocity Coordinate Spectrum (VCS) techniques ([32][33][34], have provided important insight into the velocity spectra of turbulence in molecular clouds (see [30][35]). Galactic and extragalactic atomic hydrogen gas ([36][37][38] see also the review by [17], where a compilation of velocity and density spectra obtained with contemporary HI and CO data is presented).
2.2. Alfvénic turbulence: eddies perpendicular to magnetic field

The Goldreich-Sridhar (1995) model of MHD turbulence predicts the Kolmogorov spectrum of motions that are perpendicular to the direction of magnetic field. This dynamics suggests that the magnetic field does not resist such mixing motions. This is possible if the field lines do not get entangled by such motions, which, in its turn means that the reconnection of magnetic field lines belonging to the eddy happens over the eddy-turnover time. This is exactly the prediction of LV99 theory. An important consequence of LV99 picture is that magnetic eddies should be aligned not with the mean magnetic field but with the magnetic field of the eddies. Note that this notion of the local magnetic field, which is currently an accepted corner stone of the MHD turbulence theory, was not in the original GS95 formulation of the turbulence theory. The MHD turbulence theory in LV99 identified an important intrinsic connection between the MHD turbulence and turbulent magnetic reconnection.

Adopting the picture of eddies perpendicular to the local magnetic field of the eddies, in terms of perpendicular motions we expect to have the Komogorov scaling, i.e. \( u_\ell \sim l_\perp^{1/3} \), and also obtain the relation between the parallel and perpendicular scales of the eddies. The so-called critical balance in the picture of eddies follows naturally. Indeed, as the eddy provides mixing of magnetic field lines in the perpendicular direction over the time scale \( l_\perp / u_\ell \), it sends out an Alfvén wave with the same period. This is exactly

\[
\ell_\parallel V_A \sim \ell_\perp u_\ell, \tag{1}
\]

where \( \ell_\parallel \) and \( \ell_\perp \) are the eddy scales parallel and perpendicular to the local direction of magnetic field (LV99, 39; 40).

It is assumed in GS95 that the energy is injected at the scale \( L_i \) with the injection velocity equal to the Alfvén velocity \( V_A \). If the injection velocity \( u_L \) is smaller than \( V_A \), then the eddies on smaller length scales get more elongated. For \( M_A = u_L / V_A < 1 \), the eddy description is valid only for the strong turbulence on length scales smaller than \( l_{\text{trans}} = L M_A^2 \).

2.3. MHD turbulence in compressible and partially ionized media

The compressible MHD turbulence can be presented by the superposition of three cascades of basic MHD modes, namely, Alfvénic, slow, and fast. For magnetic reconnection one should concentrate on Alfvénic turbulence as this component is the most important for the theory of reconnection. In non-relativistic MHD turbulence, the backreaction of slow and fast magnetosonic modes (41; 42; 43) on Alfvénic cascade is insignificant (41; 44; 45), and therefore the Alfvénic turbulence is not much affected by the presence of compressible modes.

In partially ionized astrophysical plasmas in e.g., the early universe, interstellar medium, protoplanetary disks, solar atmosphere, MHD turbulence is subjected to the damping effects due to the presence of neutrals. The damping effects in a partially ionized medium mainly arise from the collisional friction between ions and neutrals, i.e., ion-neutral collisional damping and the viscosity in neutral fluid, i.e., neutral viscous damping (45; 46). When the damping effects are significant, MHD turbulence is suppressed. The damping scale where the MHD turbulence is dissipated can be determined by comparing the turbulent energy cascade rate with the damping rate. The general expressions that describe the damping scales in different conditions were obtained in (47).

The scaling relations of relativistic Alfvénic turbulence were first derived in (48), and in terms of the spectral slope and anisotropy they coincide with those in GS95 theory. These relations were numerically tested later in (49). For relativistic compressible turbulence theory, the earlier studies of relativistic MHD turbulence (see 50; 51; 52; 53; 54; 55) did not reveal significant differences between the non-relativistic and relativistic turbulence. However, more recent studies in (13; 56) provided the decomposition of relativistic turbulent motions into Alfvén, slow, and fast modes and found that the coupling between compressible and incompressible turbulence is stronger than that in the non-relativistic case.
3. Analytical model of turbulent reconnection

The 3D model of turbulent reconnection proposed by LV99 generalizes the classical Sweet-Parker model. There are two regions with uniform laminar magnetic fields separated by a thin current sheet. The magnetic fields are frozen in through the two regions, and the violation of flux freezing takes place over a thin slot $\Delta$, determined by Ohmic diffusion. Thus the speed of reconnection is,

$$ V_{rec} \approx \frac{\eta}{\Delta}. \quad (2) $$

The Sweet-Parker model deals with the steady state reconnection, and thus the plasma in the diffusion region must be ejected from the edge of the current sheet at the Alfvén speed $V_A$. Therefore, the reconnection speed is limited by the conservation of mass condition

$$ V_{rec2} \approx \frac{V_A \Delta}{L_x}, \quad (3) $$

where $L_x$ is the lateral extend of the current sheet. Combining Eq. (1) and (3) one gets the Sweet-Parker reconnection speed, which is reduced from $V_A$ by the square root of the Lundquist number, $S \equiv L_x V_A/\eta$, i.e.

$$ V_{rec,SP} = V_A S^{-1/2}. \quad (4) $$

The Lundquist number in astrophysical conditions is very large, and therefore the reconnection rate is negligible. The disparity between $L_x$ and $\Delta$ makes the Sweet-Parker model unable to explain most of the observed astrophysical reconnection events, e.g. Solar flares. A way to deal with this problem was suggested by (59), who proposed that in special conditions the reconnection configurations may have magnetic field lines converging to the reconnection zone at a sharp angle making $L_x$ and $\Delta$ comparable. The corresponding configuration is known as X-point reconnection as opposed to the Sweet-Parker extended current sheet Y-type reconnection. Since the pioneering work of Petschek, the research exploring X-point reconnection was the dominant trend in the attempts of obtaining fast reconnection. However, as it was discussed in LV99 the major problem of X-point reconnection is how to preserve the X-point configuration over astrophysically large scales and in the presence of external driving of motions, e.g. turbulence.

Instead, LV99 identified fast reconnection in the presence of the Y-type magnetic configurations. The authors showed a way to make $\Delta$ macroscopic and even astrophysically large. The LV99 proposal was based on property of MHD turbulence that they also quantified, namely, the magnetic field line wandering. The corresponding model of magnetic reconnection is illustrated in Figure 2. In LV99 model, the outflow thickness $\Delta$ is determined by the large-scale magnetic field wandering that depends on the level of turbulence. Therefore, the rate of reconnection in LV99 model is determined by the level of MHD turbulence. To quantify the reconnection rate, the scaling relations of Alfvénic turbulence in §2 should be used.

On length scales less than the injection scale of turbulence $L_i$, the mean squared magnetic field wandering is equal to:

$$ \langle y^2 \rangle \approx \frac{27 x^3}{L_i} \left( \frac{u_L}{V_A} \right)^4, \quad x < L_i \quad (5) $$

which provides a superdiffusive dependence of $\langle y^2 \rangle \sim x^3$ with $x$ as the measured distance along the magnetic field. If the scale of the current sheet is larger than $L_i$, i.e. $L_x \gg L_i$, the magnetic field wandering obeys the usual random walk scaling. The corresponding random walk step is $L_x/L_i$ with the mean squared displacement per step equal to $L_x^2 (u_L/V_A)^4$. Therefore,

$$ \langle y^2 \rangle^{1/2} \approx (L_i x)^{1/2} (u_L/V_A)^2, \quad x > L_i. \quad (6) $$

The reconnection at low Alfvén Mach numbers is astrophysically important, and this motivated LV99 to generalize the MHD turbulence theory for arbitrary Alfvén Mach numbers.
Figure 2. Upper plot: Sweet-Parker model of reconnection. The thickness of the outflow $\Delta$ is limited by Ohmic diffusivity that takes place on microscopic scales. The disparity between $\Delta$ and the astrophysically large scale $L_x$ makes the reconnection slow. Lower plot: LV99 model of turbulent reconnection. The width of outflow $\Delta$ is determined by macroscopic field line wandering, and it can be comparable to $L_x$ for trans-Alfvénic turbulence. (46).

which provides the other limit for the magnetic field wandering. Combining Eqs. (5) and (6), it is possible to derive the thickness of the outflow $\Delta$ in the aforementioned two regimes. Then, using the mass conservation Eq. (3), LV99 obtained

$$V_{rec} \approx V_A \min \left[ \left( \frac{L_x}{L_i} \right)^{1/2}, \left( \frac{L_i}{L_x} \right)^{1/2} \right] M_A^2,$$

where $V_A M_A^2$ is proportional to the turbulent eddy speed. When $M_A \sim 1$, the reconnection rate reaches a large fraction of $V_A$. Importantly, when using Eq. (7), one should keep in mind that the value of the guide magnetic field, i.e. the field component shared by two reconnecting fluxes, does not affect the reconnection speed. Indeed, both turbulent velocity and magnetic field can be decomposed into components in the direction of the guide field and perpendicular to it. The field wandering in the direction perpendicular to the guide field is affected by the velocity perturbations in the same direction. The guide field does not affect the bending of magnetic field. The outflow velocity is, however, is determined only by $V_A$ corresponding to the perpendicular component of magnetic field.

The LV99 theory of turbulent reconnection has provided a number of predictions. (1) First of all, the turbulent reconnection was identified as a generic process that takes place everywhere in magnetized turbulent fluids. A violation of flux freezing follows from that. (2) Second, LV99 showed that the speed of reconnection varies depending on the level of turbulence, which enables the theory to explain diverse observations. (3) Third, the independence on plasma parameters makes the turbulent reconnection a generic reconnection process in different astrophysical settings.

(1) Flux freezing violation: Richardson dispersion

The “flux freezing principle” (1) has become a powerful tool to study astrophysical problems (2, 60). For instance, if magnetic field were frozen into the fluid in astrophysical turbulent dynamos, the tangled structure of the generated magnetic field would quench its further growth.

The Richardson dispersion manifests a gross violation of the flux freezing in magnetized turbulent fluids. This numerically confirmed phenomenon (see (4, 2) provides another way of re-deriving LV99 rate
of turbulent reconnection. It was initially introduced in hydrodynamic turbulence (61). Using this theory one can predict that the separation between two particles obeys the equation $d/dt[l(t)] \sim v(l) \sim \alpha l^{1/3}$, where $\alpha$ is proportional to the cube-root of the energy cascading rate. The corresponding solution $l(t) = \left[ l(0)^{2/3} + \alpha(t-t_0) \right]^{1/3}$ describes the Richardson dispersion, i.e. the mean square separation between particles increasing in proportion to the cube of time, i.e. $\langle l^2 \rangle \sim t^3$.

In terms of motions perpendicular to the local direction of magnetic field, the scaling of velocities in GS95 picture corresponds to the Kolmogorov one. Therefore we expect that the plasma particles separate in accordance to the Richardson law over the inertial range of turbulence. The Richardson dispersion is a numerically proven phenomenon (see §4.2), and it was used in ELV11 to re-derive the LV99 rate of reconnection.

(2) Magnetic field wandering

For magnetized fluids one can trace magnetic fields and study their separation. This effect was discussed in the cosmic ray literature as the cause of the perpendicular diffusion of cosmic rays in Galactic magnetic field (62). Dealing with magnetic reconnection we have discussed the separation of magnetic field lines. Figure 3 illustrates the spread of magnetic field lines as traced by test particles in the perpendicular direction. Eq. (5) exhibits two remarkable results. First of all, the squared separation between magnetic field lines increases in proportion to the cube of the distance measured along the magnetic field lines. The other result is that the square of the separation between magnetic field lines increases with the fourth power of Alfvén Mach number, i.e. $\sim M_A^4$. The LV99 prediction of magnetic field line superdiffusion in space was successfully tested by (63) and later by (64) and (65) (see Figure 3). The latter study also confirmed the $M_A^4$ dependence.

We would like to stress again that magnetic wandering cannot be understood by assuming that magnetic field lines preserve their identify in turbulent flows. Magnetic field lines wondering in turbulent magnetized flow constantly reconnect, inducing the exchange of plasmas and magnetic field. It was decided in ELV11 to term this new type of dispersion, the Richardson diffusion in space, as opposed to the original Richardson dispersion that is a time-dependent process.

(3) Irrelevance of plasma effects in large-scale turbulent reconnection

While plasma effects play a crucial role in reconnection of magnetic reversals near electron- and ion-scales, they are totally negligible and irrelevant to reconnection of magnetic structures at scales $\ell \gg \rho_i$, where $\rho_i$ is the ion gyroradius. This has been demonstrated in different ways, i.e. with either the generalized Ohm’s law (5) or the full Vlasov-Maxwell-Landau kinetic equations (66). This conclusion does not mean that the large-scale turbulent reconnection in a weakly collisional plasma such as the
Figure 4. Dependence of the reconnection speed $V_{\text{rec}}$ on the injection power $P_{\text{inj}}$ (left) and the injection scale $k_{\text{inj}}$ (right). Different colors correspond to different turbulence driving methods (black - Fourier driving in velocity, blue and red - real space driving in velocity and magnetic field, respectively). The dotted line corresponds to the Sweet-Parker reconnection rate for models with $\eta_u = 10^{-3}$. A unique red symbol shows the reconnection rates for model with driving in velocity performed with higher resolution ($512x1024x512$) and resistivity coefficient reduced to $\eta_u = 5 \cdot 10^{-4}$. From [8].

solar wind can be described by ideal MHD equations. Although an ideal Ohm’s law holds, there are not enough collisions to make the ion distribution function close to a local Maxwellian and to make the ion pressure tensor an isotropic function of local density and temperature. The approximation of ideal MHD should, however, be a suitable model to investigate some of the basic effects of turbulence at inertial-range scales. As the incompressible shear-Alfvén description is applicable on large scales, quantitative measures of field-line stochasticity and turbulent Richardson dispersion should carry over with little change to this collisionless plasma environment.

4. Testing of turbulent reconnection and flux freezing violation

4.1. Testing LV99 theory with MHD codes

Numerical two-dimensional studies of turbulent reconnection in a periodic box were done by [67, 68]. The results indicated a strong effect of turbulence on the reconnection process. Unfortunately, the numerical setup precluded the calculation of long-term reconnection speed, and there were no studies on how the properties of turbulence affect reconnection. More recently, [69] studied the effects of small-scale turbulence on two-dimensional reconnection. However, no significant effects of turbulence on reconnection were observed. In what follows we consider realistic 3D magnetic reconnection. Indeed, any 2D configuration significantly restricts the magnetic field freedom, making impossible, for example, field line wandering. Moreover, magnetized turbulence is very different in 2D and 3D†.

The LV99 model is intrinsically 3D, requiring significant computational resources and sufficient resolution to resolve all length scales from large ones where turbulence is injected to dissipation ones where magnetic resistivity operates. The first extensive testing of the theoretical LV99 model with numerical simulations was performed by [7], a decade after the LV99 theory was published. The tests were carried out using MHD compressible code in 3D domain with open boundary conditions in the subAlfvénic turbulence regime. Some additional tests in terms of different turbulence driving was published in [8]. The authors performed numerical experiments by changing three parameters, the power

†In fact, no 2D magnetized turbulence model was proposed until today, while for the 3D turbulence we could list a number of existing theoretical models (see, e.g. [70, 71, 44].
Figure 5. Dependence of the reconnection speed $V_{rec}$ on the resistivity $\eta$ for models with weak (diamonds) and strong (circles) guide field. From (7).

$P_{inj}$ of turbulence driving, length scale $k_{inj}$ of driving, and resistivity coefficient $\eta$. Figure 4 presents the measured dependence of the reconnection speed $V_{rec}$ on the turbulence parameters $P_{inj}$ and $k_{inj}$. Even though relatively low resolution was used, the simulations confirmed the theoretical predictions by LV99 on the dependence of reconnection rate on the turbulence parameters. More importantly, the numerical studies by (7) proved that the reconnection is indeed fast in the presence of turbulence, since it does not depend on the resistivity $\eta$, as shown in Figure 5. Moreover, the same dependence was observed in different conditions with weak and strong guide fields $B_z$. In (7), they also tested the role of the anomalous resistivity intended to mimic plasma effects and found that there is no essential dependence of the reconnection rate on anomalous plasma effects as well.

As expected in the LV99 model, the current sheet is broad with individual currents distributed widely within a 3D volume, and the turbulence within the reconnection region is similar to the turbulence within a statistically homogeneous volume. The structure of the reconnection region was analyzed in more details by (72) based on the numerical work by (7). The results supported LV99 picture about the reconnection region being broad, the magnetic shear being more or less coincident with the outflow zone, and the turbulence within it being similar to that in a homogeneous system.

4.2. Flux freezing violation test

As we discussed earlier, the LV99 theory of turbulent reconnection is closely related to the gross violation of flux freezing in turbulent environments. A direct test of the temporal Richardson dispersion of magnetic field lines was performed recently by (9). In this experiment, stochastic fluid trajectories were tracked backward in time to determine which field lines at earlier times would arrive to the point of interest. In the flux-freezing situation, the one to one mapping is expected for different times, but the Richardson dispersion introduces a very different behavior. To test the idea, many time frames of an MHD simulation were stored so that the equations for the trajectories could be integrated backward. The results of this study are presented in Figure 6. It shows the trajectories of the arriving magnetic field lines, which are widely dispersed backward in time resembling a spreading plume of smoke and are very different from a single “frozen-in” line. The quantitative results of the time-dependent separation are consistent with the temporal Richardson dispersion behavior.
4.3. Self-driven turbulent reconnection
The LV99 theory assumes that the level of turbulence is given. In fact, turbulence can also be produced by the outflow from a reconnection zone, as well as the instabilities of current sheets. This is the case of the self-driven reconnection. It was first addressed in LV99 and further discussed in (73). Turbulence in self-driven reconnection is generated by reconnection itself. The developed turbulence, in return, influences the reconnection process, resulting in its enhanced rate.

The problem of self-driven reconnection was approached by numerical simulations within last several years. The first numerical simulations of turbulence generated in spontaneous magnetic reconnection were presented in (10) with an incompressible MHD code. Later works on numerical simulations of stochastic reconnection were done by (11) and (74), both using compressible MHD codes. The most
extensive studies of the reconnection-driven turbulence were carried out by (12) (see Fig. 7). They investigated the dependence of the properties of developed turbulence on the plasma $\beta$ parameter. They reported Kolmogorov power spectra of generated velocity fluctuations, which also exhibit scale-dependent Goldreich-Sridhar anisotropy at later times in large-$\beta$ models. Their results demonstrated that the turbulence statistics are similar to that of strong MHD turbulence.

(75) addressed the issue whether the tearing instability is important for generating turbulence. They considered two instabilities, tearing mode and Kelvin-Helmholtz instability. For each analyzed model they determined the magnetic (in the case of tearing mode) and velocity (in the case of Kelvin-Helmholtz instability) shear location. The obtained results indicated that the tearing mode growth is significantly suppressed at all studied wave numbers. In Figure 8, the statistical distribution of the growth rates of tearing mode (left panel) and Kelvin-Helmholtz instability (right panel) at three different perturbation wave numbers are shown. Strong suppression of tearing mode can be explained by the fact, that its growth rate actually decreases with the wave number $k$, while for Kelvin-Helmholtz instability it increases. It could also be attributed to the presence of the transverse component of magnetic field, which is generated by turbulence. The study demonstrates that a significant number of cells within current sheets are stable due to the strong transverse magnetic field component.

5. Observations of turbulent reconnection

Turbulent reconnection provides a natural explanation for the variations of solar activity. Indeed, the reconnection speed that can be inferred varies dramatically. This is difficult to explain on the basis of any plasma reconnection process. On the contrary, as we discussed earlier, the LV99 predictions suggest that the rate of magnetic reconnection varies with the level of turbulence, and this level of turbulence can be affected by the reconnection itself. Using the observational data, (76) provided the observational evidence in favor of LV99 model, namely, thick current sheets consistent with the theoretical predictions. Their observational results were reanalyzed later by taking into account the nature of turbulent driving (see (9; 77)) and even better correspondence to the LV99 predictions was reported. A very distinct prediction in LV99 is the spread of magnetic reconnection from one active reconnection region to adjacent reconnection regions. Accordingly, the turbulence induced by the reconnection in one flaring region is expected to induce reconnection in adjacent regions. In (78) and (79), the authors reported this effect.

The events of turbulent reconnection were also compared with the in situ measurements of the solar wind. There are extensive observations of strong narrow current sheets in the solar wind (80).
However, within the LV99 picture these are small-scale events representing the microphysics, but not much affecting the large-scale reconnection. The theory deals with very large-scale reconnection events in the solar wind, often associated with interplanetary coronal mass ejections and magnetic clouds or occasionally magnetic disconnection events at the heliospheric current sheet (81, 80). The outflows associated with these events have reconnection widths up to nearly $10^3$ of the ion gyroradius $\rho_i$, and exhibit a prolonged, quasi-stationary regime with reconnection lasting for several hours. The “current sheet” producing such large-scale reconnection contains many small-scale intense current sheets embedded in a diffuse turbulence background of weaker current. A successful comparison between the reconnection events in High-Speed Solar Wind and those in MHD turbulence simulations was performed in (82).

The violation of flux freezing required by the LV99 model of turbulent reconnection can be demonstrated in the case of the heliospheric current sheet (HCS). By analyzing the HCS data, (5) showed its turbulent nature and argued that the LV99 magnetic reconnection can explain the properties of the HCS. In addition, the Parker’s spiral model (83) of the interplanetary magnetic field is a classical example used to demonstrate the concept of magnetic flux freezing. However, the deviations from this model are notable. (84) interpreted the observed deviations arising from a quasi-continuous magnetic reconnection. Their interpretation is consistent with the results of (5). The magnetic slippage through the fluid due to the pervasive turbulence in the near-ecliptic solar wind leads to a less tightly wound spiral and a stronger radial field strength than those in the Parker model, in agreement with the observations in (84). This is closely related to the reconnection diffusion proposed by (63) and (85).

6. Astrophysical implications of turbulent reconnection

6.1. Nonlinear turbulent dynamo, reconnection diffusion and star formation

The turbulent dynamo is the most promising mechanism that accounts for the cosmic magnetism (86). For the so-called “small-scale” turbulent dynamo, which operates on length scales smaller than the driving scale of turbulence, the nonlinear stage is the most astrophysically important. It gives rise to both the amplification of magnetic field strength and the increase of the correlation length. Numerical studies show that the nonlinear dynamo has a universal linear-in-time growth of magnetic energy with a low growth rate as a small fraction of the constant turbulent energy cascading rate (87, 88, 89, 90). In fact, the dynamo efficiency is determined by the interaction between the turbulent dynamo and the diffusion of magnetic fields. In earlier theoretical studies, only the microscopic diffusion of magnetic fields due to plasma effects, e.g., resistive diffusion, ambipolar diffusion, was considered (91, 92). However, their theoretical predictions cannot explain the numerical findings on the low dynamo efficiency and the spectral shape of magnetic fluctuations.

As demonstrated by the analytical study by (93), the reconnection diffusion of magnetic fields facilitated by the turbulent reconnection dominates over microscopic diffusion processes in the nonlinear stage of dynamo. As discussed in (63, 94, 95), the Richardson dispersion describes the magnetic field dynamics based on the Lagrangian approach, while the Eulerian approach in describing this dynamics is known as the “reconnection diffusion”. The name stresses the importance of turbulent reconnection that enables this diffusion process. After incorporating the reconnection diffusion, (93) derived a small energy growth rate as $3/38$ of the turbulent energy cascading rate for the nonlinear dynamo. This result is in a good agreement with earlier numerical measurements (88, 90). As a result of reconnection diffusion, the growing magnetic energy spectrum peaks at the dynamo driving scale, where the growing magnetic energy reaches equipartition with the turbulent energy. A comparison between the analytical prediction by (93) on the magnetic energy spectrum and the numerical measurements by (86) confirms that the reconnection diffusion plays a crucial role in the nonlinear turbulent dynamo (see Fig. 9).

The nonlinear dynamo theory incorporating reconnection diffusion has important astrophysical implications for studying the role of magnetic fields for the first star formation (93) and the acceleration of Galactic cosmic rays (47). (47) found that the low efficiency of nonlinear dynamo due to reconnection diffusion well explains the magnetic field evolution and structure in shock simulations (96) and X-ray
Figure 9. Upper panel: illustration of the magnetic (solid line) and turbulent (dashed line) energy spectra for the nonlinear dynamo based on the theory by (93). Lower panel: numerical measurements by (86). From (93).

observations of supernova remnants (97).

The reconnection diffusion is also an important effect changing the classical theory of star formation. The standard theory is based on the assumption that magnetic field lines preserve their identify and the diffusion of charged particles perpendicular to magnetic field lines is restricted. Therefore, the mass loading of magnetic field lines does not change. However, the LV99 theory suggests that this is not true for turbulent magnetized fluids. MHD turbulence is expected to induce the diffusion of matter perpendicular to magnetic field. The process of reconnection connects magnetic fields with different mass loading and plasma pressures. As a result, plasmas stream along magnetic field lines to equalize the pressure. In the process, the portions of magnetic flux tubes with higher magnetic pressure expand as plasma pressure increases due to the flow of plasma along magnetic field lines. In other words, the process of diffusion driven by turbulent reconnection does not depend on the degree of ionization of the matter. In the absence of gravity, the outcome of reconnection diffusion is to make magnetic fields and plasmas more homogeneously distributed through the turbulent volume. Figure 10 presents a toy model of two adjacent magnetized eddies. The magnetic flux tubes moving with the eddies reconnect and exchange plasmas and magnetic fields. This induces turbulent diffusion of both magnetic fields and plasmas perpendicular to the mean magnetic field. In real turbulence, the above process takes place at every length scale. The concept of reconnection diffusion has been applied to different aspects of star formation, from the loss of magnetic support of molecular clouds to the problem of magnetic braking of accretion disks (see 98; 99).
Figure 10. Reconnection diffusion: exchange of magnetic flux with entrained matter. Illustration of the mixing of matter and magnetic fields due to reconnection as two magnetic flux tubes of different eddies interact. In real turbulent cascade, such interactions proceed at every length scale of turbulent motions. From (95).

6.2. Energetic particle acceleration, flares and bursts of reconnection

Particle acceleration induced by magnetic reconnection was discussed before the appearance of LV99 model (e.g., (100)). The problem was that the Sweet-Parker reconnection is too slow to transfer an appreciable amount of energy to particles. The Petschek model faces the same difficulty as only a tiny part of magnetic field lines undergo reconnection. In this respect, the LV99 model with its volume reconnection offers a great opportunity to explore particle acceleration. In turbulent reconnection regions magnetic reconnection results in shrinking of magnetic loops and the charged particles entrained on magnetic loops get accelerated. It is important that the process takes place even in incompressible fluids, as the volume available for particles moving with magnetic field lines decreases as the magnetic field lines shrink. This acceleration process in the turbulent reconnection region was introduced in (101) (henceforth GL05). The process is illustrated in the left panel of Figure 11 that is taken from (63). The process of acceleration is the first-order Fermi acceleration, and it increases the parallel component of particle momentum as illustrated in the right panel of Figure 11. A similar acceleration processes was discussed later in the case of 2D tearing reconnection in (102). In 2D tearing reconnection, one has to consider magnetic islands, and the shrinking of magnetic field lines in 2D islands is limited by them getting circular. In 3D turbulent reconnection, the magnetic loops can shrink all the way with no constraints arising from the extraneous dimensional constraints.

The new mechanism of particle acceleration by reconnection has been invoked in many studies (see (104; 105; 106; 107; 108; 109; 110; 111), see also (112) for a review). In particular, it was discussed in the context of acceleration of energetic particles in pulsars (e.g. (107; 108; 109; 113)) and relativistic jets of active galactic nuclei (AGNs) (e.g. (114)). Most of the studies referred to the 2D tearing reconnection. However, the turbulent reconnection is the necessary feature of realistic astrophysical settings, and therefore the corresponding 3D turbulent reconnection is more relevant for particle acceleration in astrophysical environments. The particle acceleration in a large-scale turbulent current sheet was numerically studied by (115) with 3D MHD simulations and the injection of 10,000 test particles. The results are presented in Figure 12. The exponential growth of particle energy is shown in the figure. During this stage the acceleration rate scales as $E^{-0.4}$ (116). We note that these results are difficult to reproduce with PIC simulations, as most of PIC simulations still do not have enough particles to reproduce the turbulent behavior of fluid, which is the key physical ingredient of astrophysical magnetic reconnection.

The fast turbulent reconnection acceleration is a widespread phenomenon. In particular, it has been
Figure 11. Left: Shrinking magnetic loop within the turbulent reconnection region with a particle gaining higher \( p_\parallel \) as it follows the magnetic field. From (63). Right: A particle with a large Larmor radius interacts with converging magnetized flows and its \( p_\perp \) increases as a result. From (103).

Figure 12. Particle kinetic energy distributions for 10,000 protons injected in the simulated turbulent reconnection region. The color indicates the velocity component that is accelerated (red or blue for parallel or perpendicular, respectively). The energy is normalized by the rest proton mass. From (115).

discussed in (117) as a source of anomalous cosmic rays observed by Voyagers and in (118) as a source of the observed cosmic ray anisotropies. It was also discussed for the environments around black holes (GL05, 119, 120, 121, 116, 122).

In a magnetically dominated medium, the release of magnetic energy via reconnection results in the outflow that induces turbulence in astrophysical high-Reynolds number plasma. This in turn inevitably increases the rate of reconnection and magnetic energy dissipation, leading to a reconnection instability (73). The applications of the theory range from solar flares to gamma ray bursts (GRBs) (see 123). It was the basis of the Internal-Collision-induced MAgnetic Reconnection and Turbulence (ICMART) model of GRBs by (124). They showed that the model can overcome several difficulties of the traditional internal shock model (125, 126, 127, 128, 129) and well interpret the lightcurves and spectra of GRBs (130, 131, 132, 133). (134) used the theoretical advances in relativistic turbulent reconnection to improve the ICMART model. They also modified the ICMART model by identifying the kink instability as the most plausible mechanism of creating the magnetic configurations prone to reconnection and triggering the turbulent reconnection. In addition, the magnetic energy release and particle acceleration arising
from turbulent magnetic reconnection associated with AGNs and galactic black hole binaries (GHBs) has also been explored using the model in GL05. (120) revisited the aforementioned model and extended the study to investigating the gamma-ray flare emission of these sources. They confirmed the predictions in GL05. The study by (121) suggests that the results in (120) are robust and depend on properties of turbulent reconnection rather than the detailed accretion physics.

7. Turbulent and tearing reconnection

7.1. Comparison of turbulent and tearing reconnection models

Tearing reconnection was discussed in the literature prior to the recent surge of interest in the process (see 135). It can act as a trigger for turbulent reconnection process in the settings where the reconnecting magnetic fields are initially laminar. Most studies of tearing reconnection were carried out in 2D (136; 137; 138; 139; 140; 141; 142; 143; 144; 145; 146; 147; 148; 74; 149; 150; 151; 152; 153; 154; 155), and this is a serious theoretical deficiency. The reason that the 2D tearing reconnection model is adopted by the reconnection community is that in 3D the particle-in-cell (PIC) simulations do not yet have enough particles to reproduce fluid/turbulent behavior. Therefore, for most PIC simulations the regime of MHD-like turbulent reconnection is very difficult to achieve.

In 2D MHD numerical simulations, the transfer from Sweet-Parker to tearing reconnection was observed in the 2D current sheet starting with a particular Lundquist number larger than $S \approx 10^4$. The resulting reconnection does not depend on the fluid resistivity. The study of tearing reconnection momentarily eclipsed the earlier mainstream research based on the Petschek-type X point reconnection (156; 157; 102). Conceptually, the tearing reconnection, which invokes a cascade of mergers that increases the size of plasmoids, is closer to the turbulent reconnection compared with the Petschek X-point reconnection. Tearing introduces stochasticity into reconnection and brings the mainstream models of magnetic reconnection closer to the LV99 turbulent reconnection model. In particular, similar to the LV99 model, the tearing reconnection presents volume-filling Y-type reconnection with a broad outflow. This allows some ideas, e.g. first-order Fermi acceleration of cosmic rays, developed within the framework of the LV99 theory to be transferred to the case of tearing reconnection.

LV99 model may not be applicable to 2D configurations (4), and we do not expect to see the turbulent reconnection dominating the 2D reconnection process. Numerical tests in a number of recent studies (8; 9; 11; 10; 12; 14) show that the transfer to turbulent reconnection is inevitable in 3D. This transfer is also inevitable in 3D and large-Lundquist number astrophysical systems. Indeed, the mass conservation constraint requires that one has to increase the outflow region thickness $\Delta$ in proportion to $L_x$. The Reynolds number $Re$ of the outflow, approximately $\Delta V / \nu$ with $\nu$ as the viscosity, grows with the Lundquist number $S$. As $Re$ increases the flow gets turbulent.

Recently, an extensive literature has developed suggesting that reconnection in MHD turbulence is necessarily induced by tearing instability (see 158; 159; 160; 161; 162; 163; 164; 165). The authors explored the possible modifications of the properties of MHD turbulence through this instability. They believed that for high Reynolds and Lundquist numbers, the microscale current sheets become tearing unstable, and this instability affects the turbulence on length scales smaller than some critical scale but larger than the dissipation scale of turbulence. This idea is based on a dubious assumption that the properties of laminar tearing instability can be directly applied to turbulent environments. The modification of turbulence spectrum does not affect the LV99 picture, which does not directly depend on the particular GS95 scaling and also holds for an arbitrary spectrum of MHD turbulence (see LV99, Appendix D). However, we feel that it is a serious misconception to talk about the “reconnection-mediated turbulence” or a modification of MHD turbulence “due to reconnection” at scales smaller than some critical scale. It implies that the fast magnetic reconnection does not occur in the inertial range on scales larger than this critical length. For example, (158) regarded it as a surprise that in MHD turbulence “the anisotropic, current-sheetlike eddies become the sites of magnetic reconnection before the formal Kolmogorov dissipation scale is reached” and they argued that such fast reconnection sets in only because “Sweet-Parker current sheets above a certain critical aspect ratio ... are violently unstable
to the formation of multiple magnetic islands, or plasmoids.” This contradicts to what we know about turbulent reconnection and related processes, i.e. Richardson dispersion, flux freezing violation, that are discussed above. Indeed, fast reconnection on all length scales is an essential part of the MHD turbulent cascade, which is required for the dynamical consistency of MHD turbulence.

7.2. Reconnection diffusion, turbulent resistivity, and turbulent ambipolar diffusion
LV99 predicted dramatic changes of the dynamics of magnetic fields in turbulent fluids compared to their laminar counterparts. These changes may be erroneously associated with the concept of “turbulent resistivity”. The description of the phenomena in MHD turbulence based on the concept of “turbulent resistivity” can be very problematic. If we know the reconnection rate, e.g. from LV99, then an eddy-resistivity can always be adjusted to achieve the required rate. Although the manually tuned reconnection rate is numerically satisfactory, the model itself is unphysical. For instance, the required large-eddy resistivity is expected to smooth out all magnetic structures on length scales below the injection scale. Obviously, this contradicts to the physical reality, where turbulent magnetic fluctuations are observed on all length scales down to the dissipation micro-scale. All the relevant MHD turbulence physics, including the Richardson diffusion, magnetic helicity conservation, is also lost within the turbulent resistivity approach.

In partially ionized gas, the concept “turbulent ambipolar diffusion” was developed to explain the magnetic flux removal from molecular clouds. The concept is based on the idea that turbulence can create gradients in neutrals, and those can accelerate the overall pace of ambipolar diffusion. The simulations in were claimed as the numerical support for the “turbulent ambipolar diffusion”. However, in 3D if there is no fast turbulent reconnection, the turbulent mixing and thus the “turbulent ambipolar diffusion” is not possible. Moreover, reported no dependence of the “turbulent ambipolar diffusion” on the ambipolar diffusion rate. Both facts taken together suggest that in turbulent partially ionized gas the reconnection diffusion dominates over the ambipolar diffusion for the magnetic field diffusion process.

8. Past, present, and future of turbulent reconnection theory
It has been 20 years since the introduction of the 3D turbulent reconnection model. The tearing reconnection model shares many common features with the original LV99 model of turbulent reconnection, and the two models can complement each other in some instances. For example, the tearing reconnection can be the initial transient stage of magnetic reconnection before turbulence is developed. Numerical simulations testify that the steady-state 3D reconnection is generally turbulent and astrophysically more relevant.

The turbulent reconnection is an essential part of the magnetic field dynamics in turbulent fluids. The turbulent reconnection theory is a part of the theories of MHD turbulence, turbulent dynamo, magnetic field stochasticity, etc. It has induced significant changes in understanding fundamental astrophysical processes and interpreting observational phenomena involving magnetic fields, including cosmic ray acceleration, star formation, and non-thermal radiation associated with high-energy astrophysical systems. Similar to other turbulence induced processes, it is important to note that turbulent reconnection does not depend on plasma microphysics, and this justifies the MHD treatment of turbulent fluids in numerical studies of reconnection.

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