Perturbative renormalization factors for bilinear and four-quark operators for Kogut-Susskind fermions on the lattice

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Abstract

Renormalization factors for bilinear and four-quark operators with the Kogut-Susskind fermion action are perturbatively calculated to one-loop order in the general covariant gauge. Results are presented both for gauge invariant and non-invariant operators. For four-quark operators the full renormalization matrix for a complete set of operators with two types of color contraction structures are worked out and detailed numerical tables are given.

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1 Introduction

Calculation of weak matrix elements of hadrons is one of the central subjects in numerical simulation of lattice QCD. An important ingredient in such studies is the value of renormalization factors that relate operators on the lattice to those in the continuum. In this article we present a one-loop evaluation of renormalization factors for bilinear and four-quark operators for the Kogut-Susskind (KS) fermion action. The results have been used in our recent work for the pion decay constant [1] and the $K^0 - \bar{K}^0$ mixing matrix [2].

Perturbative calculation of renormalization factors for KS fermions has been developed in several previous studies [3, 4, 5, 6, 7]. In particular Daniel and Sheard [5, 6] calculated the renormalization factors for KS bilinear operators [5] and a subset of four-quark operators [6] in the Feynman gauge. For applications in numerical simulations, however, their results need to be extended in several directions. (1) Weak operators for KS fermions are generally extended in space and time. The calculation of Daniel and Sheard has been carried out for the operators which are made gauge invariant through insertion of gauge link variables between quark and anti-quark fields. In recent numerical simulations, however, an alternative method of evaluating matrix elements of gauge non-invariant operators without link insertions on gauge fixed configurations has been employed [8]. In fact, whether the two types of operators yield consistent results is the question we have recently addressed [1, 2]. Analyzing this problem requires the renormalization factors for gauge non-invariant operators as well as for gauge invariant ones. We have therefore carried out the calculation for both types of operators. (2) Four-quark operators of form $O_2 = (\bar{q}_a^1 q_b^2)(\bar{q}_c^3 q_d^4)$ with $a, b$ the color indices generally mix with those of form $O_1 = (\bar{q}_a^1 q_b^2)(\bar{q}_c^3 q_d^4)$ under renormalization. We evaluate the full renormalization factor for the two sets of operators, while the previous work of Sheard [6] listed explicit results only for $O_2$ mixing with itself and with $O_1$. (3) Renormalization factors for lattice operators generally take larger values than those for continuum operators due to contribution of gluon tadpoles. Lepage and Mackenzie [9] have argued that the tadpole contributions can be removed by a rescaling of quark and gluon fields. We work out the renormalization factors for the rescaled operators and examine to what extent their values are reduced by rescaling. (4) In addition to the extensions above we have carried out the calculation in the general covariant gauge which allows us to check the gauge parameter independence of the results for gauge invariant operators.

For bilinear quark operators a calculation similar to ours has been reported recently by Patel and Sharpe [10]. Our results are in agreement with theirs and also with those of Ref. [5] for gauge invariant operators. Patel and Sharpe have extended their calculation to four-quark operators [11]. For the gauge non-invariant operators which are relevant for the $K$ meson $B$ parameter their values fully agree with our results. We also find agreement with the results of Sheard [6] for gauge invariant four-quark operators when a comparison is possible.
We should mention that we do not treat penguin operators in this article. Calculation of their renormalization factors is technically feasible, which should be pursued in future investigations.

This paper is organized as follows. In Sec. 2 KS operators whose renormalization factors we evaluate are defined and a general strategy for one-loop calculations is summarized following the method of Daniel and Sheard. Results for quark bilinear operators are given in Sec. 3. Those for four-quark operators are described in Sec. 4 where the relation between lattice and continuum operators is illustrated for the case of $\Delta S = 2$ operator relevant for extraction of the $K^0 - \bar{K}^0$ mixing matrix. Analytical expressions for one-loop amplitudes are summarized in Appendices A, B and C.

2 Formalism

2.1 Quark operators

The KS fermion action is given by

$$S = a^4 \sum_n \left[ \frac{1}{2a} \sum_\mu \eta_\mu(n) \left( \bar{\chi}(n)U_\mu(n)\chi(n + \hat{\mu}) - \bar{\chi}(n + \hat{\mu})U_\mu^\dagger(n)\chi(n) \right) + m\bar{\chi}(n)\chi(n) \right],$$

(1)

where $a$ is the lattice spacing, $U_\mu(n)$ denotes the gauge link variable, $\eta_\mu(n) = (-1)^{n_1 + \cdots + n_{\mu - 1}}$ and $\chi$ and $\bar{\chi}$ are the single component KS fermion fields. For the construction of four-component Dirac fields we employ the coordinate-space method of taking a linear combination of the $\chi$ fields over a hypercube. The Dirac field $Q(2N)$ defined for each hypercube $2N \in (2\mathbb{Z})^4$ is given by

$$Q(2N)_{\alpha i} = \frac{1}{8} \sum_A (\gamma_A)_{\alpha i} \chi(2N + A),$$

(2)

where $\alpha$ and $i$ are the Dirac and flavor indices, and

$$\gamma_A = \gamma^A_1 \gamma^A_2 \gamma^A_3 \gamma^A_4$$

(3)

with $A$ running over the vertices of a hypercube ($i.e., A_\mu = 0$ or 1, $\mu = 1, \cdots, 4$).

The bilinear quark operator we use is defined by

$$O_{SF} = \bar{Q}(\gamma_S \otimes \xi_F)Q.$$  

(4)

Here $\gamma_S = \gamma^S_1 \gamma^S_2 \gamma^S_3 \gamma^S_4$ and $\xi_F = \gamma^F_1 \gamma^F_2 \gamma^F_3 \gamma^F_4$ with the components $S_\mu$ and $F_\mu$ either 0 or 1. They act on spinor and flavor indices of the Dirac field $Q$, and represent the spin and
flavor $SU(4)$ content of the bilinear operator. In terms of the $\chi$ field this operator can be written as

$$O_{SF} = \frac{1}{16} \sum_{AB} \bar{\chi}_A (\gamma_S \otimes \xi_F)_{AB} \chi_B \ ,$$

where we write $\chi_A$ instead of $\chi(2N+A)$ for simplicity and

$$(\gamma_S \otimes \xi_F)_{AB} = \frac{1}{4} \text{tr}(\gamma_A^\dagger \gamma_S \gamma_B^\dagger \gamma_F^\dagger) \ .$$

The operator (3) is not gauge invariant. In order to make it gauge invariant we insert the average of products of gauge link variables along all possible shortest paths connecting the cite $2N + A$ and $2N + B$. Denoting the link factor by $U_{AB}$ we then define the gauge invariant bilinear operator by

$$O_{SF} = \frac{1}{16} \sum_{AB} \bar{\chi}_A (\gamma_S \otimes \xi_F)_{AB} U_{AB}^a \chi_b^a \ ,$$

with $a$ and $b$ the color indices.

We consider two types of four-quark operators differing in the color contraction structure defined by

$$O_2 = \bar{Q}^a (\gamma_{S_1} \otimes \xi_{F_1}) Q^a \cdot \bar{Q}^b (\gamma_{S_2} \otimes \xi_{F_2}) Q^b \ ,$$

$$O_1 = \bar{Q}^a (\gamma_{S_1} \otimes \xi_{F_1}) Q^a \cdot \bar{Q}^b (\gamma_{S_2} \otimes \xi_{F_2}) Q^a \ ,$$

with $a$ and $b$ the color indices. In the method of Ref. [14] for calculating weak matrix elements with KS fermions the operator $O_2$ yields an amplitude with two color contractions after fermion integration and $O_1$ with a single color contraction. For this reason we shall refer to $O_2$ and $O_1$ as color two-loop and one-loop operators. These two operators generally mix under renormalization.

The expression of the operators above in terms of the $\chi$ field is given by

$$O_2 = \left( \frac{1}{16} \right)^2 \sum_{ABCD} \bar{\chi}_A (\gamma_{S_1} \otimes \xi_{F_1})_{AB} \chi_B^a \cdot \bar{\chi}_C (\gamma_{S_2} \otimes \xi_{F_2})_{CD} \chi_D^a \ ,$$

$$O_1 = \left( \frac{1}{16} \right)^2 \sum_{ABCD} \bar{\chi}_A (\gamma_{S_1} \otimes \xi_{F_1})_{AB} \chi_B^a \cdot \bar{\chi}_C (\gamma_{S_2} \otimes \xi_{F_2})_{CD} \chi_D^a \ ,$$

where we again ignored the hypercube label $2N$ in $\chi(2N+A)$ for simplicity. To make these operators gauge invariant we insert gauge link factors according to

$$O_2 = \left( \frac{1}{16} \right)^2 \sum_{ABCD} \bar{\chi}_A (\gamma_{S_1} \otimes \xi_{F_1})_{AB} \chi_B^a \cdot \bar{\chi}_C (\gamma_{S_2} \otimes \xi_{F_2})_{CD} \chi_D^a \cdot U_{AB}^a U_{CD}^{cd} \ ,$$

$$O_1 = \left( \frac{1}{16} \right)^2 \sum_{ABCD} \bar{\chi}_A (\gamma_{S_1} \otimes \xi_{F_1})_{AB} \chi_B^a \cdot \bar{\chi}_C (\gamma_{S_2} \otimes \xi_{F_2})_{CD} \chi_D^a \cdot U_{AD}^{ad} U_{CB}^{cb} \ .$$
For the color two-loop operators the length of paths for the link factors $U_{AB}$ and $U_{CD}$ is fixed since $(\gamma_S \otimes \xi_F)_{AB}$ is non-vanishing only for $A + B = S + F$(mod2). On the other hand, the length varies for the factor $U_{AD}$ and $U_{CB}$ that appear in the color one-loop operator.

2.2 Feynman rules

We adopt the general covariant gauge with a gauge parameter $\alpha$ in our perturbative calculations. The gluon propagator is given by

$$D_{\mu\nu}(k)_{IJ} = \frac{\delta_{IJ} \delta_{\mu\nu}}{\sum_\beta \frac{4}{a^2} \sin^2(ak_\beta/2)} - (1 - \alpha) \frac{\delta_{IJ} \cdot \frac{4}{a} \cdot \sin(ak_\mu/2) \cdot \sin(ak_\nu/2)}{\left[\sum_\beta \frac{4}{a^2} \sin^2(ak_\beta/2)\right]^2},$$

(14)

and the KS fermion propagator takes the form

$$S(p, -q)_{ab} = \delta_{ab} \sum_\mu \frac{\bar{\delta}(p - q + \pi \bar{\mu}/a)}{\sum_\mu \frac{1}{a^2} \sin^2 ap + m^2},$$

(15)

where

$$\bar{\mu} = \sum_{\nu=1}^{\mu-1} \hat{\nu},$$

(16)

and

$$\bar{\delta}(p) = (2\pi)^4 \sum_n \delta(p + \frac{2\pi}{a} n).$$

(17)

The one-gluon vertex arising from the action is given by

$$V_\mu(p, -q; k)_{ab} = -iga(T^I)_{ab} \cdot \cos a(p + k/2)_\mu \cdot \bar{\delta}(p - q + k + \pi \bar{\mu}/a),$$

(18)

with $T^I$ the $SU(3)$ generators normalized by $\text{Tr}(T^I T^J) = \delta_{IJ}/2$, and the two-gluon vertex by

$$V_{\mu\nu}(p, -q; k_1, k_2)_{ab} = iag \frac{1}{2} \{T^I, T^J\}_{ab} \cdot \sin a(p + \frac{k_1 + k_2}{2})_\mu \times \bar{\delta}(p - q + k_1 + k_2 + \pi \bar{\mu}/a),$$

(19)

where $p, q$ are the incoming fermion momenta and $k_1, k_2$ the gluon momenta.

Vertices of bilinear operators (7) have the form,

$$M_{SF}^{(0)}(p, -q)_{ab} = \delta_{ab} \frac{1}{16} \sum_{AB} e^{iap A - iaq B} (\gamma_S \otimes \xi_F)_{AB},$$

(20)

$$M_{SF}^{(1)}(p, -q; k)_{ab} = -iga(T^I)_{ab}$$
where the superscript in parentheses denote the number of emitted gluons. The function $f^\mu_{(AB)}(ak)$ is defined by

$$f^\mu_{(AB)}(ak) = \frac{1}{12} \sum_{\mu \neq \nu} e^{(B - A)_{\mu\nu}} \theta^{(1)}_{\mu\nu}(ak)$$

(23)

with

$$\theta^{(1)}_{\mu\nu}(ak) = \frac{1}{2} (ak_{\mu} \hat{\mu} - \theta^{(1)}_{\mu\nu}(ak)),$$

$$\theta^{(2)}_{\mu\nu}(ak) = \frac{1}{2} (ak_{\mu} \hat{\mu} + ak_{\nu} \hat{\nu})$$

$$\theta^{(3)}_{\mu\nu}(ak) = \sum_{\rho=1}^{4} ak_{\rho} \hat{\rho} - \theta^{(1)}_{\mu\nu}(ak),$$

$$\theta^{(4)}_{\mu\nu}(ak) = \sum_{\rho=1}^{4} ak_{\rho} \hat{\rho} - \theta^{(2)}_{\mu\nu}(ak)$$

(24)

At the one-loop level the two-gluon vertex appears only through gluon tadpole diagrams. Thus we only need the expression for an equal color index $a = b$ and for the gluon momenta $k_1 = -k_2 \equiv k$. In this case we find

$$g_{(AB)}^{\mu\nu}(ak, -ak) = 1 \quad \text{for} \quad \mu = \nu$$

$$= e^{iak \cdot (\Delta_{\mu} + \Delta_{\nu})} \left[ 6 + 2 \sum_{\rho \neq \mu\nu} e^{iak \cdot \Delta_{\rho}} + 2e^{iak \cdot \sum_{\rho \neq \mu\nu} \Delta_{\rho}} \right] + \text{h.c} \quad \text{for} \quad \mu \neq \nu$$

(25)

with $\Delta_{\mu} = (B - A)_{\mu\hat{\mu}}$.

Vertices for four-quark operators are given by a product of those for the bilinear operators except that the color and site indices have to be interchanged appropriately.

### 2.3 Procedure of calculation

Let us consider a one-loop diagram of a bilinear operator with two external fermion lines. The corresponding amplitude generated by the Feynman rules above are written in terms of momenta $p$ taking values in the range $-\pi/a < p \leq \pi/a$. Since the Dirac field $Q(2N)$ is defined on sites with even coordinates the physical momentum $\tilde{p}$ for quarks is related to $p$ through $p = \tilde{p} + C\pi/a$ where the vector $C$ ($C_{\mu} = 0$ or $1$) represents the spin-flavor content of quarks. We extract renormalization factors from Feynman amplitudes evaluated at vanishing
physical momenta $\vec{p} = 0$ for external fermion lines. We therefore set the external fermion momenta to $p = C\pi/a$ and $D\pi/a$. In this case the spin-flavor part of the tree amplitude $M_{SF}^{(0)}$ takes the form

$$
(\gamma_S \otimes \xi_F)_{CD} = \frac{1}{16} \sum_{AB} (-1)^{A+C+B-D} (\gamma_S \otimes \xi_F)_{AB},
$$

which shows that the calculation of renormalization factors requires a conversion of spin-flavor Dirac structure from the ‘single-bar’ basis, in which the Feynman rules are given, to the ‘double-bar’ basis defined in (26).

We employ the technique developed by Daniel and Sheard[5, 6] to carry out the conversion. The general form of one-loop amplitudes that results is given by

$$
\sum_{MNM'N'} \int_{-\pi/a}^{\pi/a} \frac{d^4k}{(2\pi)^4} \cdot A_{MNM'N'}(ak)(\gamma_{MSN} \otimes \xi_{M'F N'})_{CD},
$$

where $A_{MNM'N'}$ is a function of loop momenta $k$. The product of Dirac matrices $(\gamma_{MSN} \otimes \xi_{M'F N'})$ can be reexpanded in terms of the basis \{$(\gamma_S \otimes \xi_F); S_{\mu}, F_{\mu} = 0, 1$\};

$$
(\gamma_{MSN} \otimes \xi_{M'F N'}) = \sum_{S'F'} C_{MN'M'^N'}^{SF'F}(\gamma_{S'F'} \otimes \xi_{F'}).
$$

The contribution of (27) to the renormalization factor of $O_{SF}$ is given by

$$
\sum_{MNM'N'} C_{MN'M'^N'}^{SF'F} \int_{-\pi/a}^{\pi/a} \frac{d^4k}{(2\pi)^4} A_{MNM'N'}(ak). \quad (29)
$$

In most cases the decomposition (28) is too tedious to work out analytically. We generate tables of $C_{MN'M'^N'}^{SF'F}$ on a computer and combined them with tables of one-loop integrals $A_{MNM'N'}$, separately evaluated with the Monte Carlo integration routine VEGAS, to calculate the sum (29).

Our calculations are carried out for massless quark. In this case the amplitudes for the diagrams which have counterparts in the continuum perturbation theory contain infrared divergent terms of form

$$
\int_{-\pi/a}^{\pi/a} \frac{d^4k}{(2\pi)^4} \frac{1}{\left[\sum_{\mu} 4 \sin^2(k_{\mu}a/2)\right] \cdot \left[\sum_{\nu} 4 \sin^2(k_{\nu}a/2)\right]},
$$

where the first factor in the denominator arises from the gluon propagator and the second from the massless quark propagator. To regularize the divergence we supply a finite mass $\kappa$ to the gluon propagator. The integral then takes the value

$$
\int_{-\pi/a}^{\pi/a} \frac{d^4k}{(2\pi)^4} \frac{1}{\left[\sum_{\mu} 4 \sin^2(k_{\mu}a/2)\right] \cdot \left[\sum_{\nu} 4 \sin^2(k_{\nu}a/2) + (a\kappa)^2\right]},
$$
\[\begin{align*}
\frac{1}{16\pi^2} [-2 \cdot \log(a\kappa) + F_{0000} - \gamma_E + 1] + O(\alpha) \quad ,
\end{align*}\]

with \(F_{0000} = 4.36923(1)\) and \(\gamma_E = 0.577216 \cdots\).

The infrared regularization above is different from that of Daniel and Sheard who added the mass term \((a\kappa)^2\) to both the quark and gluon propagators, in which case the finite part of the integral is given by \(F_{0000} - \gamma_E\). We prefer not to adopt their regulator since it leads to a violation of fermion number conservation in continuum perturbation theory. We also note that the dependence on the gluon mass should cancel out between the renormalization factors in the continuum and on the lattice as long as one employs the same infrared regularization in the two cases.

Evaluation of one-loop amplitudes for four-quark operators are much more cumbersome since they contain a product of two spin-flavor Dirac matrices. The calculational procedure, however, is essentially the same as for bilinear operators.

### 3 Bilinear operators

The one-loop renormalization of the quark bilinear operator \(\mathcal{O}_{SF}\) on the lattice can be written as

\[\mathcal{O}_{SF}^{\text{lat}}(1) = \sum_{S'F'} \left( \delta_{SS'} \delta_{FF'} + \frac{g^2}{16\pi^2} z_{SF,S'F'}^{\text{lat}} \right) \mathcal{O}_{SF}^{\text{lat}}(0) \quad ,\]

where the superscript \(j\) on \(\mathcal{O}_{SF}^{\text{lat}}(j)\) refer to the number of loops. The one-loop diagrams are shown in Fig. 1 and analytic expressions of the amplitudes are collected in Appendix A. Defining the coefficient \(z^{\text{cont}}\) for the continuum operators in a similar manner the one-loop relation between the lattice and continuum operators are given by

\[\mathcal{O}_{SF}^{\text{cont}}(1) = \sum_{S'F'} \left[ \delta_{SS'} \delta_{FF'} + \frac{g^2}{16\pi^2} (z_{SF,S'F'}^{\text{cont}} - z_{SF,S'F'}^{\text{lat}}) \right] \mathcal{O}_{SF}^{\text{lat}}(1) \quad .\]

The continuum renormalization factor for massless quarks can be expressed as

\[z_{SF,S'F'}^{\text{cont}} = \delta_{SS'} \delta_{FF'} \cdot \gamma_S \log(\mu/\kappa) + \delta_{S'S'} \delta_{FF'} C_S^{\text{cont}} \quad .\]

We used the same infrared regularization as for the lattice, \(i.e.\) a finite gluon mass \(\kappa\) is given to the gluon propagator. The anomalous dimension of the operator \(\gamma_S\) is given by

\[\gamma_S = \frac{8}{3} \cdot (\sigma_S - 1) \quad ,\]
with $\sigma_S = (4, 1, 0, 1, 4)$ for the spin structure $\gamma_S = (I, \gamma_\mu, \gamma_{\mu\nu}, \gamma_{\mu5}, \gamma_5)$. The finite constant $C_{S}^{\text{cont}}$ depends on the continuum regularization and renormalization schemes. For the $\overline{\text{MS}}$ subtraction scheme it takes the values

$$C_{S}^{\text{cont}} = \begin{cases} 
(10/3, \ 0, \ 2/3, \ 0, \ 10/3) & \text{for NDR} \\
(14/3, \ 0, \ -2/3, \ 0, \ 14/3) & \text{for DREZ}
\end{cases}$$

(36)

for $\gamma_S = (I, \gamma_\mu, \gamma_{\mu\nu}, \gamma_{\mu5}, \gamma_5)$, where NDR refers to the naive dimensional regularization with an anti-commuting $\gamma_5$ and DREZ to the variant of the dimensional reduction defined in Ref. [15].

For the renormalization factor on the lattice we find

$$z_{SF,S'}^{\text{lat}} = -\delta_{SS'}\delta_{FF'}\gamma_S \log(a\kappa) + C_{SF,S'}^{\text{lat}}.$$  

(37)

The logarithmically divergent term arises from the diagrams in Fig. 1(a) and (d). It takes the same form for gauge invariant and non-invariant operators, and is independent of the gauge parameter $\alpha$. Comparing (34) and (37) we find that the gluon mass $\kappa$ cancels out between $z_{SF,S'}^{\text{cont}}$ and $z_{SF,S'}^{\text{lat}}$ as it should be.

The finite coefficient $C_{SF,S'}^{\text{lat}}$ has the following properties. (1) The coefficients have the same value for the two operators with the spin-flavor structure $(\gamma_S \otimes \xi_F)$ and $(\gamma_{5S} \otimes \xi_{5F})$ (see Ref. [10]). (2) Our explicit calculation shows that the Landau gauge part of $C_{SF,S'}^{\text{lat}}$ coming from the $(\sin k/2 \cdot \sin k'/2)$ term of the gluon propagator is diagonal in spin and flavor. Their values are the same for the gauge invariant and non-invariant operators. (3) The remaining part of the coefficient generally mixes different spin-flavor structures. Chiral $U(1)$ symmetry of the KS action, however, places a restriction that operators of even distance with $\sum_{\mu=1}^{4}(S_\mu + F_\mu)(\text{mod} \ 2) = 0, 2, 4$ do not mix with those having odd distance ($\sum_{\mu=1}^{4}(S_\mu + F_\mu)(\text{mod} \ 2) = 1, 3$). In fact there are only a few non-vanishing off-diagonal elements (see Table. 1(b) below). Furthermore their values are the same for gauge invariant and non-invariant operators.

Numerical values of the diagonal constants $C_{SF,SF}^{\text{lat}}$ are tabulated in Table. 1(a) for both gauge invariant (first column) and non-invariant (second column) operators. The results for the gauge non-invariant operators are given in the Landau gauge. The non-vanishing off-diagonal elements are tabulated in Table. 1(b). Numerical accuracy is within 0.1%. For the gauge invariant operators we have numerically checked the independence of results on the gauge parameter $\alpha$. Our results confirm those of Daniel and Sheard (Table 4 and 5 in Ref. [15]) after correcting our $C_{SF,SF}^{\text{lat}}$ by $-\delta_{SS'}\delta_{FF'}\gamma_S/2$ to take into account the difference in the regularization of infrared divergence. They are in a complete agreement with the recent calculation of the same quantity reported by Patel and Sharpe (Table 6 and 7 in Ref. [10]). The relationship between our coefficients $C_{SF,SF}^{\text{lat}}$ and theirs $C_{SF,SF}^{\text{PS}}$ is

$$C_{SF,SF}^{\text{PS}} = \frac{3}{4} \left[ \delta_{SS'}\delta_{FF'} \left( C_{S}^{\text{cont}} + \gamma_S \log \pi \right) - C_{SF,S'}^{\text{lat}} \right] ,$$

(38)

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where \( C_{\text{cont}} \) are the finite continuum renormalization factor for the DRE\( \Xi \) scheme given by (36).

We observe in Table. 1(a) that the coefficients for gauge non-invariant operators have a similar magnitude while those for gauge invariant ones show a substantial variation from operator to operator. This stems from the fact that the gauge invariant operators receive contribution of gluon tadpoles whose magnitude increases with the distance of the operator \( \Delta_{SF} = \sum_{\mu} (S_{\mu} + F_{\mu})(\text{mod } 2) \). Lepage and Mackenzie [9] have argued that the tadpole contributions can be removed by a rescaling of fields which, for KS fermion action, takes the form

\[
\chi \rightarrow \sqrt{u_0} \chi, \quad \bar{\chi} \rightarrow \sqrt{u_0} \bar{\chi}, \quad U_{\mu} \rightarrow u_0^{-1} U_{\mu},
\]

where \( u_0 \) represents the tadpole renormalization of link variables. A gauge-invariant choice for \( u_0 \) is given by

\[
u_0 = \left[ \frac{1}{3} \langle \text{Tr} U_P \rangle \right]^{1/4} = 1 - \frac{1}{12} g^2 + O(g^4)
\]

with \( \langle \text{Tr} U_P \rangle \) the plaquette average. For gauge-invariant bilinear operators the rescaling amounts to a multiplication by a factor \( u_0^{1-\Delta_{SF}} \). The renormalization factors for the rescaled operators are obtained by subtracting \( 4\pi^2 (1 - \Delta_{SF})/3 \) from the second column of Table. 1(a). The results listed in the third column of Table. 1(a) show that rescaled gauge invariant operators receive much less renormalization, and that their magnitude becomes less dependent on the flavor of operators. For gauge non-invariant operators without insertion of link variables the rescaling factor is universally given by \( u_0 \). The rescaling reduces the magnitude of the correction without spoiling the weak flavor dependence already apparent for the original operators.

### 4 Four-quark operators

#### 4.1 Lattice result

One-loop diagrams which contribute to the renormalization of the color two-loop four-quark operators \( \mathcal{O}_2 \) defined in Sec. 2 are shown in Fig. 2 and those for color one-loop operators \( \mathcal{O}_1 \) in Fig. 3. In these figures horizontal lines at the four-quark vertices signify contraction of spin-flavor quantum numbers, while dotted lines represent link factors and flow of color indices.

For the diagrams of Fig. 2(a)–(e) for the color two-loop operator evaluation of momentum and Dirac matrix parts are the same as those of the diagrams in Fig. 4 for the bilinear operator. The color factor is also the same for these diagrams. For the diagrams of Fig. 2(f)–(h), on the other hand, the color factor takes the form \( \sum_I (T^I)_{ab} (T^I)_{a'b'} \) which has to be
decomposed into the color one- and two-loop basis. This can be done by the $SU(3)$ identity,

$$\sum_I (T^I)_{ab}(T^I)_{a'b'} = -\frac{1}{6} \delta_{ab} \delta_{a'b'} + \frac{1}{2} \delta_{ab} \delta_{a'b'} .$$  \hspace{1cm} (41)

Such a rearrangement is also generally needed for the diagrams of color one-loop operators in Fig. 3 in addition to manipulation of momentum and Dirac parts. Analytic expressions for all the diagrams are summarized in Appendices B and C.

Due to the mixing of color one- and two-loop operators the renormalization factor for the four-quark operators $O_i$ takes a $2 \times 2$ matrix form,

$$O_i^{\text{lat}} (1) = \left( \delta_{ij} + \frac{g^2}{16\pi^2} z_{ij}^{\text{lat}} \right) O_i^{\text{lat}} (0) , \quad i, j = 1, 2 .$$  \hspace{1cm} (42)

Since the operators $O_i = \bar{Q}(\gamma S_i \otimes \xi F_i)Q \cdot \bar{Q}(\gamma S_2 \otimes \xi F_2)Q$ further depend on the pair of spin-flavor indices $sf = (S_1 F_1)(S_2 F_2)$, each element $z_{ij}^{\text{lat}}$ is a matrix $z_{ij}^{\text{lat}} = \{z_{ij;sf,s'f'}\}$. Treating the infrared divergence as in the case of bilinear operators we find that this matrix can be written as

$$z_{ij;sf,s'f'}^{\text{lat}} = -\delta_{f'f} z_{ij;ss'}^{\text{lat}} \cdot \log(\alpha \kappa) + C_{ij;sf,s'f'}^{\text{lat}} .$$  \hspace{1cm} (43)

In Table 2–4 we list the numerical values of the matrices $\gamma_{ij;ss'}^{\text{lat}}$ and $C_{ij;sf,s'f'}^{\text{lat}}$ for the operators with the spin-flavor structure

$$sf = (\gamma_\mu \otimes \xi_5)(\gamma_\mu \otimes \xi_5) , \quad (\gamma_\mu_5 \otimes \xi_5)(\gamma_\mu_5 \otimes \xi_5) \quad \text{(Table 2 and 6)} ,$$

$$\gamma_\mu \otimes \xi_5)(\gamma_\mu_5 \otimes \xi_5) , \quad (\gamma_\mu \otimes I)(\gamma_\mu \otimes \xi_5) \quad \text{(Table 3 and 7)} ,$$

$$\gamma_\mu \otimes I)(\gamma_\mu_5 \otimes \xi_5) , \quad (\gamma_\mu_5 \otimes I)(\gamma_\mu \otimes \xi_5) \quad \text{(Table 4 and 8)} ,$$

$$\gamma_\mu \otimes I)(\gamma_\mu_5 \otimes \xi_5) , \quad (\gamma_\mu_5 \otimes I)(\gamma_\mu \otimes \xi_5) \quad \text{(Table 5 and 9)} .$$  \hspace{1cm} (44)

The results for the gauge invariant operators are in Table 2–4 and those for the gauge non-invariant operators are in Table 3–9. The anomalous dimension matrix $\gamma_{ij;ss'}^{\text{lat}}$ is gauge independent and takes the same value for gauge invariant and non-invariant operators. The results of $C_{ij;sf,s'f'}^{\text{lat}}$ for the gauge non-invariant operators are for the Landau gauge.

The one-loop renormalization coefficients for the operators having the spin-flavor structure $(\gamma S_5 \otimes \xi F_5)(\gamma S'5 \otimes \xi F'5)$ are the same as those for $(\gamma S \otimes \xi F)(\gamma S' \otimes \xi F')$. Hence the tables also cover the renormalization factor for the operators obtained by the interchange $\gamma_\mu \leftrightarrow \gamma_\mu_5$, $I \leftrightarrow \xi_5$. These operators are the most relevant for calculation of matrix elements of the effective weak Hamiltonian. The gauge non-invariant operators with the spin-flavor structure $(\gamma S_5 \otimes \xi F_5)(\gamma S' \otimes \xi F')$ are renormalized in the same way as those for $(\gamma S \otimes \xi F)(\gamma S' \otimes \xi F')$ at least to one loop order, and similarly for the operators $(\gamma S \otimes \xi F)(\gamma S'5 \otimes \xi F'5)$ and $(\gamma S \otimes \xi F)(\gamma S' \otimes \xi F')$[10]. It is not known if this property persists at higher orders. We also note that the operators of even distance do not mix with those of odd distance due to $U(1)$ chiral symmetry, similar to the case of bilinear operators.
The numerical accuracy is within the level of ±0.001 for the majority of elements in the tables, increasing to ±0.01 for large elements whose magnitude is $O(10)$. This accuracy should be sufficient for practical applications. Reducing errors is quite computer time consuming because of a very large number of lattice integrals ($\sim 360$) which have to be computed.

We have checked the results in two ways. (i) For gauge invariant operators the Landau gauge part proportional to $1 - \alpha$ has to vanish. This has been confirmed numerically. (ii) One can rewrite the color one-loop operator as a linear combination of color two-loop operators through the Fierz transformation given by

$$
(\gamma_S \otimes \xi_F)_{AB}(\gamma_{S'} \otimes \xi_{F'})_{A'B'} = \frac{1}{16} \sum_{DE} (\gamma_S \gamma_D \otimes \xi_E \xi_{F'})_{AB'}(\gamma_{S'} \gamma_D \otimes \xi_E \xi_F)_{A'B}.
$$

(45)

The results for the color two-loop operators can then be used to evaluate the renormalization factors for the color one-loop operators. The results obtained in this way agree with those of a direct calculation of the color one-loop operators.

The matrix elements for the mixing of gauge invariant color two-loop operators with color one- and two-loop operators have been computed previously by Sheard[6]. He took a basis quite different from ours, and we found it difficult to make a full comparison. For those cases where we can compare, however, our results are in agreement with his results. Also the results in Table. 6 for gauge non-invariant operators agree with those of Patel and Sharpe[11].

Let us finally consider improvement of four-quark operators by factoring out tadpole renormalizations through rescaling of fields as suggested by Lepage and Mackenzie[9]. For the color two-loop operators the rescaling (39) yields a simple form for the improved operators,

$$
O_{2,sf}^{imp} = u_0^{2 - \Delta_{sf}} O_{2,sf}
$$

where $\Delta_{sf} = \Delta_{S_1F_1} + \Delta_{S_2F_2}$ for gauge invariant operators having the spin-flavor structure $sf = (\gamma_{S_1} \otimes \xi_{F_1})(\gamma_{S_2} \otimes \xi_{F_2})$ with $\Delta_{SF} = \sum_{\mu} (S_{\mu} + F_{\mu})(\mod 2)$ the distance of bilinear operators, while $\Delta_{sf} = 0$ for the gauge non-invariant operators. For the color one-loop operators, on the other hand, rescaling is not straightforward since the link insertion factors have lengths ranging from 0 to 4. To handle this case we recall the Fierz formula (45) and rewrite the color one-loop operators in terms of color two-loop operators as

$$
O_{1,sf} = \sum_{s'f'} F_{sf,s'f'} O_{2,s'f'}
$$

(47)

with $F_{sf,s'f'}$ numerical constants. The rescaled color one-loop operators can then be defined as

$$
O_{1,sf}^{imp} = \sum_{s'f'} F_{sf,s'f'} u_0^{2 - \Delta_{s'f'}} O_{2,s'f'}
$$

(48)
For the gauge invariant choice \( u_0 \) for the tadpole factor the finite renormalization factors for rescaled four-quark operators are listed in Table 2–9 where those elements changed by rescaling are given after a slush (/) symbol. As one can see in the tables the rescaling indeed reduces the magnitude of the renormalization correction.

### 4.2 Relation with continuum operators

In order to obtain the physical values of weak matrix elements the renormalization factor on the lattice obtained in the previous section has to be combined with those in the continuum. In this section we illustrate the procedure for the \( K \) meson \( B \) parameter relevant for the \( K^0 - \bar{K}^0 \) mixing matrix.

The \( K \) meson \( B \) parameter \( B_K \) in the continuum theory is defined by

\[
B_K = \frac{\langle \bar{K}^0 | \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{s} \gamma_\mu (1 - \gamma_5) d | K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2}.
\]

In the method of Ref. [14] for calculating weak matrix elements with KS fermions, the operator in the numerator is replaced by the sum of the following four operators

\[
\mathcal{V}_1 = \bar{S}^a (\gamma_\mu \otimes \xi_5) D^b \cdot \bar{S}^a (\gamma_\mu \otimes \xi_5) D^a,
\]

\[
\mathcal{V}_2 = S^a (\gamma_\mu \otimes \xi_5) D^a \cdot S^b (\gamma_\mu \otimes \xi_5) D^b,
\]

\[
\mathcal{A}_1 = \bar{S}^a (\gamma_{\mu 5} \otimes \xi_5) D^b \cdot \bar{S}^b (\gamma_{\mu 5} \otimes \xi_5) D^a,
\]

\[
\mathcal{A}_2 = \bar{S}^a (\gamma_{\mu 5} \otimes \xi_5) D^a \cdot \bar{S}^b (\gamma_{\mu 5} \otimes \xi_5) D^b,
\]

(50)

where \( S \) and \( D \) are the KS quark fields introduced for \( s \) and \( d \) quark separately, \( a \) and \( b \) the color indices, and the quark fields in the first current are to be contracted with \( \bar{K}^0 \) and those in the second current with \( K^0 \). The choice of flavor \( \xi_5 \) in these operators corresponds to the use of \( \bar{D}(\gamma_5 \otimes \xi_5) S \) for creating the external \( K^0 \) and \( \bar{K}^0 \) in the Nambu-Goldstone channel associated with \( U(1) \) chiral symmetry of the KS fermion action.

The renormalization factor for these operators can be read off from Table 2 for gauge invariant operators and from Table 6 for gauge non-invariant operators. As can be seen, the four operators not only mix among themselves but also with a large number of others having different spin-flavor structures. We note that the extra operators all have the flavor matrix \( \xi_F \neq \xi_5 \). Since the \( K^0 \) and \( \bar{K}^0 \) mesons are created with the flavor \( \xi_5 \), the matrix element of the extra operators should vanish in the continuum limit where a restoration of \( SU(4) \) flavor symmetry is expected. The renormalization factor for some of the extra operators are numerically not small, however. Whether they yield negligibly small contributions at the current range of inverse lattice spacing \( 1/a \sim 2 - 3 \text{GeV} \) has to be checked through actual simulations. For simplicity we disregard the mixing with the extra operators in the following. Extensions to the general case is straightforward.
Denoting the four operators (50) as \( \{ \mathcal{O}^{\text{lat}}_{\alpha}; \alpha = 1, \ldots, 4 \} = \{ \mathcal{V}_1, \ldots, \mathcal{A}_2 \} \), we find that the \( 4 \times 4 \) anomalous dimension matrix \( \gamma_{\text{lat}}^{\alpha\beta} \) is given by

\[
\gamma_{\text{lat}}^{\alpha\beta} = \begin{pmatrix}
9 & -3 & -7 & -3 \\
0 & 0 & -6 & 2 \\
-7 & -3 & 9 & -3 \\
-6 & 2 & 0 & 0
\end{pmatrix},
\]

which takes the same form for the gauge invariant and non-invariant cases. The finite part \( C_{\text{lat}}^{\alpha\beta} \) takes the values,

\[
C_{\text{lat}}^{\alpha\beta} = \begin{pmatrix}
-18.915 & -4.772 & -5.253 & -2.251 \\
0 & -60.000 & -4.502 & 1.501 \\
-5.253 & -2.251 & -19.513 & -2.977 \\
-4.502 & 1.501 & 0 & 0
\end{pmatrix}
\]

for the gauge invariant operator, and

\[
C_{\text{lat}}^{\alpha\beta} = \begin{pmatrix}
37.446 & -2.913 & -5.253 & -2.251 \\
0 & 28.706 & -4.502 & 1.501 \\
-5.253 & -2.251 & 37.976 & -4.504 \\
-4.502 & 1.501 & 0 & 24.464
\end{pmatrix}
\]

for the gauge non-invariant operator in the Landau gauge. The result for the gauge non-invariant operator (53) is in agreement with those of Ref. [11]. The second and the fourth row of (52) has previously been calculated by Sheard, with which our results agree. For the rescaled operators discussed in Sec. 4.1 the diagonal elements of \( C_{\text{lat}}^{\alpha\beta} \) changes to \( (7.403, -7.361, 6.805, 0) \) for the gauge invariant operators and to \( (11.127, 2.387, 11.657, -1.854) \) for the gauge non-invariant operators (off-diagonal elements are not affected).

The \( \Delta S = 2 \) continuum operators are given by

\[
\begin{align*}
\mathcal{L}_1 &= \bar{s}^a \gamma_{\mu}(1 - \gamma_5)d^b \cdot \bar{s}^b \gamma_{\mu}(1 - \gamma_5)d^a, \\
\mathcal{L}_2 &= \bar{s}^a \gamma_{\mu}(1 - \gamma_5)d^a \cdot \bar{s}^b \gamma_{\mu}(1 - \gamma_5)d^b,
\end{align*}
\]

with the one-loop renormalization taking the form

\[
\mathcal{L}_i^{(1)} = \left[ \delta_{ij} + \frac{g^2}{16\pi^2} (\gamma_{ij}^{\text{cont}} \cdot \log(\mu/\kappa) + C_{ij}^{\text{cont}}) \right] \mathcal{L}_j^{(0)},
\]

where the anomalous dimension matrix \( \gamma_{ij}^{\text{cont}} \) is given by

\[
\gamma_{ij}^{\text{cont}} = \begin{pmatrix}
2 & -6 \\
-6 & 2
\end{pmatrix}.
\]
For the finite part $C_{ij}^{cont}$ we find that

$$C_{ij}^{cont} = c_{ij}^{cont}$$

(57)

with

$$c = \begin{cases} 
11/12 & \text{for NDR[17]} \\
7/12 & \text{for DREZ} 
\end{cases}$$

(58)

In order to relate the lattice operators to those in the continuum let us define a $2 \times 4$ matrix $M_{i\alpha}$ by

$$M = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

(59)

with which one can write $L^{(0)}_i = M_{i\alpha}O^{\text{lat}(0)}_{\alpha}$. Using the intertwining property $M_{i\alpha}\gamma^{\text{lat}}_{\alpha\beta} = \gamma^{\text{cont}}_{ij}M_{j\beta}$ it is easy to see that the one-loop renormalization relation between the continuum and lattice operators is given by

$$L^{(1)}_i = M_{i\alpha} \left[ \delta_{\alpha\beta} + \frac{g^2}{16\pi^2} (\gamma^{\text{lat}} \log(\mu a) + c\gamma^{\text{lat}} - C^{\text{lat}})_{\alpha\beta} \right] O^{\text{lat}(1)}_{\beta}$$

(60)

The numerator of the $B_K$ parameter with one-loop renormalization correction equals the sum $\langle \bar{K}^0 | L^{(1)}_1 | K^0 \rangle + \langle \bar{K}^0 | L^{(1)}_2 | K^0 \rangle$.

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Appendix A One-loop amplitudes for bilinear operators

The one-loop amplitudes corresponding to the diagrams of Fig. 1 for the external quark momenta \( p = C\pi/a, D\pi/a \) and color indices \( a, b \) are as follows (the common factor \( 4/3 \cdot \delta_{ab} \cdot g^2/(16\pi^2) \) is not included).

\[
G_1^{(a)} = (\sigma_S - (1 - \alpha)) x (\bar{\sigma} S \otimes \xi_F)_{CD} + \sum_{\mu\rho\sigma M N} X_{MN}^{\mu\rho\sigma} (\gamma_{\mu\rho MN} \sigma_{\mu} \otimes \xi_{MN}^{F})_{CD}
\]

\[
- (1 - \alpha) \sum_{M} X_{M} (\gamma_{MSM} \otimes \xi_{FMF})_{CD}
\]

\[
G_1^{(b)} = \sum_{MN\mu\rho} Y_{MN}^{\mu\rho} |S + F|_{\mu} (\gamma_{\mu\rho5 MN} \otimes \xi_{5 MFN})_{CD} + \sum_{MN\mu} X_{MN}^{\mu} |S + F|_{\mu} (\gamma_{\mu5 MN} \otimes \xi_{5 MFN})_{CD}
\]

\[
- 4(1 - \alpha) \sum_{MN\mu} Y_{MN}^{\mu} |S + F|_{\mu} (\gamma_{\mu5 MN} \otimes \xi_{5 MFN})_{CD}
\]

\[
G_1^{(c)} = \frac{1}{8} (3 + \alpha) Z_{0000} (\gamma_S \otimes \xi_F)_{CD}
\]

\[
G_1^{(d)} = \left[ -x\alpha - \frac{1}{8} (1 + \alpha) Z_{0000} - R \right] (\gamma_S \otimes \xi_F)_{CD}
\]

\[
G_1^{(e)} = \left[ -\frac{1}{8} (3 + \alpha) \Delta_{SF} Z_{0000} - 2(1 - \alpha) T_{\Delta_{SF}} \right] (\gamma_S \otimes \xi_F)_{CD}
\]

where \( \sigma_S = (4, 1, 0, 1, 4) \) for \( S = (I, \gamma_{\mu}, \gamma_{\mu\rho}, \gamma_{\mu5}, \gamma_5) \), \( x = -2 \log a\kappa + F_{0000} - \gamma_E + 1 \), \( |S + F|_{\mu} = (S_\mu + F_\mu) \) (mod 2), and \( \Delta_{SF} = \sum_{\mu} |S + F|_{\mu} \).

We use the following notations to simplify the expressions for the loop integrals which appear in the amplitudes above:

\[
s_\mu = \sin \phi_\mu , \quad \bar{s}_\mu = \sin \phi_\mu / 2 , \quad B = [4 \sum_{\mu} s_{\mu}^2]^{-1} ,
\]

\[
c_\mu = \cos \phi_\mu , \quad \bar{c}_\mu = \cos \phi_\mu / 2 , \quad F = [\sum_{\mu} s_{\mu}^2]^{-1} ,
\]

\[
\int_{\phi} = 16\pi^2 \int_{-\pi}^{\pi} \frac{d^4 \phi}{(2\pi)^4} ,
\]

where \( \phi \) is the loop momentum and \( B \) and \( F \) originate from gluon and fermion propagators. In terms of these symbols the loop integrals are defined as follows.

\[
X_{MN}^{\mu\rho\sigma} = \int_{\phi} \left[ \bar{c}_\mu s_\rho s_\sigma E_{M}(\phi) E_{N}(\phi) B F^2 - \frac{1}{4} \delta_{\rho\sigma} \delta_{M0} \delta_{N0} B^2 \right]
\]

\[
X_{M} = \int_{\phi} \left[ E_{M}(\phi) E_{M}(\phi) - \delta_{M0} \right] B^2
\]

\[
Y_{MN}^{\mu\rho} = \int_{\phi} i\bar{c}_\mu S_\rho B F \cdot \frac{1}{12} \sum_{j=1}^{4} \sum_{\sigma \neq \mu} E_{M}(\theta_{i\sigma}^{(j)}) E_{N}(\theta_{i\sigma}^{(j)})
\]

16
\[ Y^\mu_{MN} = \int_\phi i\bar{s}_\mu B^2 \frac{1}{12} \sum_{j=1}^{4} \sum_{\sigma \neq \mu} E_M(\theta^{(j)}\mu\sigma)E_N(-\theta^{(j)}\mu\sigma) \]  

(70)

\[ Z_{0000} = \int_\phi B \]  

(71)

\[ T_\Delta = \int_\phi 2s_1^2s_2^2B^2 \times (0, 0, 1, 2 + c_3, 3 + 2c_3 + c_3c_4) \]  

for \( \Delta = (0, 1, 2, 3, 4) \)  

(72)

\[ R = \int_\phi \left[ c_1(2s_1^2 - \frac{1}{F})(-2 - 2s_1^2 + \frac{1}{4B})BF^2 - B^2 \right] \]  

(73)

where \( E_M(\phi) \) and \( \theta_{\mu\nu} \) are given by

\[ E_M(\phi) = \prod_\mu \frac{1}{2} \left( e^{-i\phi_\mu/2} + (-1)^{\bar{M}_\mu} e^{i\phi_\mu/2} \right) \]  

(74)

\[ \bar{M}_\mu = \sum_{\nu \neq \mu} M_\nu \]  

(75)

\[ \theta^{(1)}_{\mu\nu} = \frac{1}{2} \phi_\mu \hat{\nu} \]  

\[ \theta^{(2)}_{\mu\nu} = \frac{1}{2} \phi_\mu \hat{\mu} + \phi_\nu \hat{\nu} \]  

\[ \theta^{(3)}_{\mu\nu} = \sum_{\rho=1}^{4} \phi_\rho \hat{\rho} - \theta^{(1)}_{\mu\nu} \]  

\[ \theta^{(4)}_{\mu\nu} = \sum_{\rho=1}^{4} \phi_\rho \hat{\rho} - \theta^{(2)}_{\mu\nu} \]  

(76)

The integrals \( X^{\mu\rho\sigma}_{MN} \) and \( Y^{\mu\nu}_{MN} \) were evaluated numerically by Daniel and Sheard\[5\]. We have confirmed their results except for some sign reversals for \( Y^{11}_{MN} \) in their Table. 3. For the calculation of the integral we employed the Monte Carlo integration routine VEGAS.

Appendix B One-loop amplitudes for color two-loop four-quark operators

Analytic expressions for one-loop diagrams in Fig. 2 are listed below for color two-loop four-quark operators of a general spin-flavor structure \((\gamma_S \otimes \xi_F)(\gamma_{S'} \otimes \xi_{F'})\). The amplitudes for the diagrams Fig. 2(a)–(e) are products of tree and one-loop bilinear amplitudes. The diagrams in Fig. 2(f)–(h) cannot be factorized in this way. They take the following form, where we drop the common factor \( \sum_I(T^I)_{ab}(T^I)_{a'b'} \) and \( g^2/16\pi^2 \). External fermion lines have momenta \( p = C\pi/a, D\pi/a, C'\pi/a, D'\pi/a \) and color indices \( a, b, a', b' \) as specified in Fig. 2.
where \( S + F \mid_{\mu} (\gamma_{\mu 5 M SL} \otimes \xi_{5 MF L})_{CD} (\gamma_{S' N \mu} \otimes \xi_{F' N} - \gamma_{\mu 5 NS' \mu} \otimes \xi_{F' N})_{C'D'}
\)
\[= |S' + F'|_{\mu} (\gamma_{SN \mu} \otimes \xi_{F N} - \gamma_{\mu 5 NS \mu} \otimes \xi_{F N})_{CD} (\gamma_{5 M S'L} \otimes \xi_{5 MF'L})_{C'D'} \]
\[= (1 - \alpha) \sum_{\mu \rho KLMN} U_{\mu \rho L MN} \left\{ |S + F|_{\mu} (\gamma_{\mu 5 M SL} \otimes \xi_{5 MF L})_{CD} (\gamma_{S' N \mu} \otimes \xi_{F' N} - \gamma_{\mu 5 NS' \mu} \otimes \xi_{F' N})_{C'D'}
\right\}
\]
\[= (1 - \alpha) \sum_{\mu \rho KLMN} U_{\mu \rho L MN} \left\{ |S + F|_{\mu} (\gamma_{\mu 5 M SL} \otimes \xi_{5 MF L})_{CD} (\gamma_{S' N \mu} \otimes \xi_{F' N} - \gamma_{\mu 5 NS' \mu} \otimes \xi_{F' N})_{C'D'}
\right\}
\]

\[= (1 - \alpha) \sum_{\mu \rho KLMN} U_{\mu \rho L MN} \left\{ |S + F|_{\mu} (\gamma_{\mu 5 M SL} \otimes \xi_{5 MF L})_{CD} (\gamma_{S' N \mu} \otimes \xi_{F' N} - \gamma_{\mu 5 NS' \mu} \otimes \xi_{F' N})_{C'D'}
\right\}
\]

where \(|S + F|_{\mu} = (S_{\mu} + F_{\mu})(\mod 2)\). The four types of loop integrals are defined by

\[U_{\mu \rho L MN}^{\mu} = \int_{\phi} i \bar{c}_{\mu} s_{\rho} B F \frac{1}{12} \sum_{\nu \neq \mu} \sum_{j=1}^{4} E_{L} (\theta_{\mu \nu}^{(j)}) E_{M} (\phi - \theta_{\mu \nu}^{(j)}) E_{N} (-\phi) \]

\[U_{\mu L MN}^{\mu} = \int_{\phi} i 2 \bar{s}_{\mu} B^2 \frac{1}{12} \sum_{\nu \neq \mu} \sum_{j=1}^{4} E_{L} (\theta_{\mu \nu}^{(j)}) E_{M} (\phi - \theta_{\mu \nu}^{(j)}) E_{N} (-\phi) \]

\[V_{\mu KLMN}^{\mu} = \int_{\phi} B \frac{1}{12} \sum_{\nu \neq \mu} \sum_{i=1}^{4} E_{K} (\theta_{\mu \nu}^{(i)}) E_{L} (\phi - \theta_{\mu \nu}^{(i)}) \]
\begin{align*}
&\sum_{\lambda\neq\mu,j=1}^{4} E_M(-\theta^{(j)}_{\mu\lambda})E_N(-\phi + \theta^{(j)}_{\mu\lambda}) 
&\times \frac{1}{12} \sum_{\lambda\neq\mu,i=1}^{4} E_K(\theta^{(i)}_{\mu\lambda})E_L(\phi - \theta^{(i)}_{\mu\lambda}) 
&\times \frac{1}{12} \sum_{\sigma\neq\nu,j=1}^{4} E_M(-\theta^{(j)}_{\nu\sigma})E_N(-\phi + \theta^{(j)}_{\nu\sigma}) .
\end{align*}

The numerical values of the last three integrals reported in Tables 4 and 5 in Ref. 8 have some minus signs missing. We have corrected the sign and evaluated the values of \(U_{LMN}\) which are not listed in Ref. 8.

### Appendix C One-loop amplitudes for color one-loop four-quark operators

Analytic expressions for one-loop diagrams in Fig. 8 are listed below for color one-loop four-quark operators of a general spin structure \((\gamma_S \otimes \xi_F)(\gamma_{S'} \otimes \xi_{F'})\). We set the momenta of the external fermion lines to \(p = C\pi/a, D\pi/a, C'\pi/a, D'\pi/a\) and assign the color indices \(a, b, a', b'\) as shown in Fig. 8. The amplitudes of the diagrams Fig. 8(a), (c), and (d) are the same as those for the color two-loop operators except for the color factor which takes the form \(\sum_f (T^I)_{ab} (T^I)_{a' b'} \) for the diagram (a) and \(\delta_{ab} \delta_{a'b'}\) for the diagrams (c) and (d). Other amplitudes take the following form, where we drop the common factor \(g^2/16\pi^2\).

\begin{equation}
G^{(b)} = \frac{4}{3} \delta_{ab} \delta_{a'b'} \left[ -\sum_{\mu\rho MN} \frac{1}{2} (Y^{\mu,\rho}_{M[M\rho 5N]} + Y^{\mu,\rho}_{N[M\rho 5M]} ) \right] \\
\times \left[ (\gamma_{\mu \rho M} \otimes \xi_{MF})_{CD} (\gamma_{S' N} \otimes \xi_{F' N})_{C'D'} + (\gamma_{SM \rho} \otimes \xi_{FM})_{CD} (\gamma_{S' N} \otimes \xi_{NF'})_{C'D'} \\
+ (\gamma_{SN} \otimes \xi_{FN})_{CD} (\gamma_{\mu \rho M} \otimes \xi_{MF'})_{C'D'} + (\gamma_{SN} \otimes \xi_{NF})_{CD} (\gamma_{SM \rho} \otimes \xi_{F'M})_{C'D'} \right] \\
- 4(1 - \alpha) \sum_{\mu M} Y^{\mu}_{M[M\rho 5M]} \left( (-1)^{M(S+F)} + (-1)^{\bar{M}(S'+F')} \right) \\
\times (\gamma_{SM} \otimes \xi_{FM})_{CD} (\gamma_{S'M} \otimes \xi_{F'M})_{C'D'} \right]
\end{equation}

\begin{equation}
C^{(c)} = \frac{4}{3} \delta_{ab} \delta_{a'b'} \\
- 2Z_{0000} (\gamma_S \otimes \xi_F)_{CD} (\gamma_{S'} \otimes \xi_{F'})_{C'D'}
\end{equation}
\begin{align}
&+\sum_{\mu} \frac{1}{4} Z_{0000}((-1)^{(S+F)}_{\mu} + (-1)^{(S'+F')}_{\mu})(\gamma_{\mu\delta S} \otimes \xi_{\mu S'F})_{CD}(\gamma_{\mu\delta S'} \otimes \xi_{\mu S'F'}_{C'D'}) \\
&+(1 - \alpha) \frac{1}{2} Z_{0000}(\gamma_S \otimes \xi_F)_{CD}(\gamma_{S'} \otimes \xi_{F'})_{C'D'} \\
&-(1 - \alpha) \sum_{\mu} Z_{0000} \frac{1}{2}((-1)^{(S+F)}_{\mu} + (-1)^{(S'+F')}_{\mu})(\gamma_{\mu\delta S} \otimes \xi_{\mu S'F})_{CD}(\gamma_{\mu\delta S'} \otimes \xi_{\mu S'F'}_{C'D'}) \\
&+(1 - \alpha) \frac{1}{2} \sum_{\mu \neq \nu, M} T_{\mu\nu}^{\mu\nu} \left\{ \\
&\quad -\left((-1)^{M-(S+F)} + (-1)^{M-(S'+F')}\right)(\gamma_{MS} \otimes \xi_{MF})_{CD}(\gamma_{MS'} \otimes \xi_{MF'})_{C'D'} \\
&\quad +2\left((-1)^{(S+F)_{\mu} + M-(S+F)} + (-1)^{(S'+F')_{\mu} + M-(S'+F')}\right) \\
&\quad \times (\gamma_{\mu5MS} \otimes \xi_{\mu5MF})_{CD}(\gamma_{\mu5MS'} \otimes \xi_{\mu5MF'})_{C'D'} \\
&\quad -\left((-1)^{(S+F)_{\mu} + (S+F)_{\mu} + M-(S+F)} + (-1)^{(S'+F')_{\mu} + (S'+F')_{\mu} + M-(S'+F')}\right) \\
&\quad \times (\gamma_{\mu\sigma MS} \otimes \xi_{\mu\sigma MF})_{CD}(\gamma_{\mu\sigma MS'} \otimes \xi_{\mu\sigma MF'})_{C'D'} \right\} \right) \\
\end{align}

\begin{align}
G_{(f)} &= \\
\sum_{\mu \sigma MN} (X_{\mu,\sigma}^{MN} + \frac{x}{4} \delta_{\rho\sigma} \delta_{M0} \delta_{N0}) \left[ \\
&\quad \frac{4}{3} \delta_{ab} \delta_{a'b} \left\{ \\
&\quad (\gamma_{\mu0MS} \otimes \xi_{MF})_{CD}(\gamma_{S'N\sigma M} \otimes \xi_{F'N})_{C'D'} + (\gamma_{SM\rho\mu} \otimes \xi_{FM})_{CD}(\gamma_{\mu0NS'} \otimes \xi_{NF'})_{C'D'} \right\} \\
&\quad -\sum_{I} (T_{I})_{ab'}(T_{I})_{a'b} \left\{ \\
&\quad (\gamma_{\mu0MS} \otimes \xi_{MF})_{CD}(\gamma_{\mu0NS'} \otimes \xi_{NF'})_{C'D'} + (\gamma_{SM\rho\mu} \otimes \xi_{FM})_{CD}(\gamma_{S'N\sigma M} \otimes \xi_{F'N})_{C'D'} \right\} \right] \\
&\quad -\sum_{M} \left[ (X_{M} + x \delta_{MO})(\gamma_{SM} \otimes \xi_{FM})_{CD}(\gamma_{S'M} \otimes \xi_{MF})_{C'D'} \right] \\
&\quad -\frac{4}{3} \delta_{ab} \delta_{a'b} \left((-1)^{M-(S+F)} + (-1)^{M-(S'+F')}\right) + \sum_{I} (T_{I})_{ab'}(T_{I})_{a'b}(1 + (-1)^{M-(S+F+S'+F')}) \right]\right) \\
&\quad \sum_{\mu \sigma MN} \frac{1}{2} \left(U_{[\mu\delta L]MN}^{\mu\rho} - U_{[\mu\delta M]LN}^{\mu\rho}\right) \left\{ \\
&\quad (\gamma_{LSS\rho\mu} \otimes \xi_{LFN})_{CD}(\gamma_{S'M} \otimes \xi_{MF})_{C'D'} + (\gamma_{SM} \otimes \xi_{FM})_{CD}(\gamma_{LS'N\rho\mu} \otimes \xi_{LF'}N)_{C'D'} \right\} \\
&\quad +\left(\gamma_{\mu\rho NSL} \otimes \xi_{NFLC}ight)_{CD}(\gamma_{MS} \otimes \xi_{MF})_{C'D'} + \left(\gamma_{MS} \otimes \xi_{MF})_{CD}(\gamma_{\mu0NSL} \otimes \xi_{NFL})_{C'D'} \right}\right) \\
\end{align}
$\alpha$-$\alpha$

\sum_{\mu L M N} \frac{1}{2} \left( U_{[\mu 5 L] M N} - U_{[\mu 5 M L] N} \right) \left\{ \begin{array}{c} \langle \gamma_{S M} \otimes \xi_{F M} \rangle_{CD} \langle \gamma_{L S N} \otimes \xi_{L F N} \rangle_{C^D'} + \langle \gamma_{M S L} \otimes \xi_{M F L} \rangle_{C^D'} \\
+ \langle \gamma_{L S N} \otimes \xi_{L F N} \rangle_{CD} \langle \gamma_{S M} \otimes \xi_{F M} \rangle_{C^D'} + \langle \gamma_{M S L} \otimes \xi_{M F L} \rangle_{C^D'} \end{array} \right\} ) \right\}

(87)

$T_{\mu \nu} = \int \phi \bar{\phi} s_{\mu} s_{\nu} B^2 \left[ \frac{1}{6} \sum_{j=1}^{3} E_M (\psi_{(j)}^{(1)}) E_M (\psi_{(j)}^{(2)}) + \frac{1}{2} \delta_{M 0} \right] , \quad (89)$

where

$\psi_{(1)}^{(1)} = \phi_{\rho} \hat{\rho} , \quad \psi_{(2)}^{(2)} = \phi_{\sigma} \hat{\sigma} , \quad \psi_{(3)}^{(3)} = \phi_{\rho} \hat{\rho} + \phi_{\sigma} \hat{\sigma} , \quad (90)$

the components $\mu, \nu, \rho,$ and $\sigma$ are all different.
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[17] To obtain this result, we followed the procedure of Buras and Weisz ( A. Buras and P. Weisz, Nucl. Phys. B333 (1990) 69 ) and decomposed the one loop amplitudes for four-quark operators as a sum of two terms, one proportional to $\Gamma \otimes \Gamma = \gamma_\mu(1-\gamma_5) \otimes \gamma_\mu(1-\gamma_5)$ and the other proportional to $E = \mathcal{R}_{\alpha \beta} \Gamma \otimes \Gamma \mathcal{R}_{\beta \alpha}$ with $\mathcal{R}_{\alpha \beta} = \gamma_\alpha \gamma_\beta \otimes I - I \otimes \gamma_\alpha \gamma_\beta$. 

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The operator $E$ is independent of the left-left operator $\Gamma \otimes \Gamma$ in general dimensions, vanishing in four dimensions. It can be ignored at one loop, but has to be retained at two loop order and beyond.
Table 1: One-loop perturbative corrections for bilinear operators. Operators of form ($\gamma_S \otimes \xi_F$) and ($\gamma_{SF} \otimes \xi_{F5}$) receive the same one-loop corrections. The components $\mu$, $\nu$, $\rho$ and $\sigma$ are all different and not summed. Values of anomalous dimension $\gamma_S$ are also listed. (a) Diagonal elements of the renormalization factor for gauge invariant (first column) and non-invariant (second column) operators. The third and fourth column shows the values for rescaled operators as discussed in text. (b) Off-diagonal elements of the correction which take the same values for gauge invariant and non-invariant operators. The first and second columns of operators specifies the row and column of the mixing matrix.

(a)

| operator | $\gamma$ | invariant | non-invariant | inv(rescaled) | non-inv(rescaled) |
|----------|---------|-----------|---------------|---------------|--------------------|
| 1 ($I \otimes I$) | 8 | 55.585 | 55.585 | 42.426 | 42.426 |
| 2 ($I \otimes \xi_5$) | 8 | -47.783 | 12.813 | -8.304 | -0.346 |
| 3 ($I \otimes \xi_{\mu}$) | 8 | 14.844 | 27.077 | 14.844 | 13.918 |
| 4 ($I \otimes \xi_{\mu\nu}$) | 8 | -29.948 | 14.405 | -3.629 | 1.246 |
| 5 ($I \otimes \xi_{\mu\nu\rho}$) | 8 | -10.569 | 17.583 | 2.589 | 4.423 |
| 6 ($\gamma_{\mu} \otimes I$) | 0 | 0.000 | 12.232 | 0.000 | -0.927 |
| 7 ($\gamma_{\mu} \otimes \xi_{5}$) | 0 | -30.000 | 14.353 | -3.682 | 1.193 |
| 8 ($\gamma_{\mu} \otimes \xi_{\mu}$) | 0 | 19.693 | 19.693 | 6.533 | 6.533 |
| 9 ($\gamma_{\mu} \otimes \xi_{\mu\nu}$) | 0 | -13.388 | 14.764 | -0.228 | 1.605 |
| 10 ($\gamma_{\mu} \otimes \xi_{\mu\nu\rho}$) | 0 | -13.409 | 14.743 | -0.249 | 1.584 |
| 11 ($\gamma_{\mu} \otimes \xi_{\mu\nu\rho\sigma}$) | 0 | -45.988 | 14.608 | -0.651 | 1.448 |
| 12 ($\gamma_{\mu} \otimes \xi_{5}$) | 0 | 4.519 | 16.752 | 4.519 | 3.593 |
| 13 ($\gamma_{\mu} \otimes \xi_{5\nu\lambda}$) | 0 | -29.651 | 14.702 | -3.332 | 1.543 |
| 14 ($\gamma_{\mu\nu} \otimes I$) | -8/3 | -14.623 | 13.529 | -1.464 | 0.369 |
| 15 ($\gamma_{\mu\nu} \otimes \xi_{\mu}$) | -8/3 | -0.428 | 11.804 | -0.428 | -1.355 |
| 16 ($\gamma_{\mu\nu} \otimes \xi_{\nu}$) | -8/3 | -29.668 | 14.685 | -3.349 | 1.525 |
| 17 ($\gamma_{\mu\nu} \otimes \xi_{\nu\lambda}$) | -8/3 | 7.728 | 7.728 | -5.430 | -5.430 |
| 18 ($\gamma_{\mu\nu} \otimes \xi_{\nu\lambda\rho}$) | -8/3 | -14.200 | 13.952 | -1.041 | 0.792 |
| 19 ($\gamma_{\mu\nu} \otimes \xi_{\nu\lambda\rho\sigma}$) | -8/3 | -45.390 | 15.206 | -5.911 | 2.046 |

(b)

| operator | mixed operator |
|----------|----------------|
| 9 ($\gamma_{\mu} \otimes \xi_{5\nu\lambda}$) | ($\gamma_{\mu} \otimes \xi_{\mu}$) | -4.504 |
| 11 ($\gamma_{\mu} \otimes \xi_{5\mu\nu}$) | ($\gamma_{\mu} \otimes \xi_{5\nu\lambda}$) | 0.860 |
| 13 ($\gamma_{\mu} \otimes \xi_{5\nu}$) | ($\gamma_{\mu} \otimes \xi_{6\mu\nu}$), ($\gamma_{\mu} \otimes \xi_{5\nu\lambda}$) | 1.980 |
| 16 ($\gamma_{\mu\nu} \otimes \xi_{\lambda}$) | ($\gamma_{\mu\nu} \otimes \xi_{\mu}$), ($\gamma_{\mu\nu} \otimes \xi_{\nu}$) | 0.902 |
Table 2: Anomalous dimensions and finite corrections for gauge invariant four-quark operators with the spin-flavor structure $VV = (\gamma_\mu \otimes \xi_5)(\gamma_\mu \otimes \xi_5)$ and $AA = (\gamma_\mu \otimes \xi_5)(\gamma_\mu \otimes \xi_5)$. Subscripts 1 or 2 attached to the operators refer to the the number of color loops. Operators listed at the top row mixes with those on the first column with the numerical coefficients given (a common factor $g^2/16\pi^2$ is removed). The values after '/' are the finite corrections for the rescaled operators. The two rows of numerical values for each operator in the first column are for the color one-loop operator (first row) and for the color two-loop operator (second column). All indices of operators are summed with different indices not taking equal values.

| mixed operator | $VV_1$ finite | $VV_2$ finite | $AA_1$ finite | $AA_2$ finite |
|---------------|---------------|---------------|---------------|---------------|
| 1 ($I \otimes \xi_\mu)(I \otimes \xi_\mu$) | 1.725 | -19.114/ -5.954 | -5.573 |
| 2 ($I \otimes \xi_\mu)(I \otimes \xi_\nu$) | -1.575 | -2.412 | 1.857 |
| 3 ($I \otimes \xi_{\sigma\sigma})(I \otimes \xi_\nu$) | -0.624 | -0.485 | 1.112 |
| 4 ($I \otimes \xi_{\sigma\sigma})(I \otimes \xi_\xi$) | 0.208 | -0.208 | -0.370 |
| 5 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\mu$) | -0.252 | -0.235 | -0.067 |
| 6 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.022 | 0.078 | 0.022 |
| 7 ($\gamma_\mu \otimes \xi_{\sigma\sigma})(\gamma_\mu \otimes \xi_\xi$) | -0.876 | -1.310 | 0.565 |
| 8 ($\gamma_\mu \otimes \xi_{\sigma\sigma})(\gamma_\mu \otimes \xi_\xi$) | -1.029 | 0.436 | -0.188 |
| 9 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\xi$) | 20.774/7.615 | -3.675 | -3.150 |
| 10 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\xi$) | -2.569 | -1.575 | 1.050 |
| 11 ($\gamma_\mu \otimes \xi_{\sigma\sigma})(\gamma_\mu \otimes \xi_\sigma$) | 0.319 | 0.485 | 0.416 |
| 12 ($\gamma_\mu \otimes \xi_{\sigma\sigma})(\gamma_\mu \otimes \xi_\xi$) | 0.706 | 0.208 | -0.138 |
| 13 ($\gamma_\mu \otimes \xi_{\sigma\sigma})(\gamma_\mu \otimes \xi_\xi$) | 0.022 | 0.044 | 0.130 |
| 14 ($\gamma_\mu \otimes \xi_{\sigma\sigma})(\gamma_\mu \otimes \xi_\xi$) | -0.439 | -0.376 | 1.511 |
| 15 ($\gamma_\mu \otimes \xi_{\sigma\sigma})(\gamma_\mu \otimes \xi_\xi$) | -1.188 | 0.125 | -0.872 |
| 16 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes I$) | 0.428 | -0.086 | -1.107 |
| 17 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes I$) | -0.142 | -0.396 | 0.369 |
| 18 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes I$) | 9 -18.915/7.403 | -7 -5.253 | -6 -4.502 |
| 19 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\mu$) | -3 -4.772 | -60.000/ -7.361 | -3 -2.251 | 2 1.501 |
| 20 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\mu$) | -1.289 | -1.104 | 2.683 |
| 21 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.552 | 0.368 | -1.615 |
| 22 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | 0.174 | 0.149 | 0.399 |
| 23 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | 0.074 | -0.049 | 0.573 |
| 24 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | 0.153 | -0.134 |
| 25 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -1.139 | -2.774 | 2.342 |
| 26 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -1.189 | 0.924 | -0.780 |
| 27 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.232 | -0.049 | 0.074 |
| 28 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.086 | -0.478 | -0.428 |
| 29 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.396 | 0.159 | -0.142 |
| 30 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | 7 -5.253 | -6 -4.502 | 9 -19.513/6.805 |
| 31 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -3 -2.251 | 2 1.501 | -3 -2.977 |
| 32 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | 2.342 | 1.007 | -2.774 |
| 33 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.780 | -1.584 | 0.924 |
| 34 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | 0.224 | 0.174 | 0.656 |
| 35 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.074 | 0.074 | 0.218 |
| 36 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.249 | 2.683 | -1.289 | 1.104 |
| 37 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -1.615 | -0.552 | 0.368 |
| 38 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | 0.224 | 0.174 | 0.149 |
| 39 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.074 | 0.074 | -0.049 |
| 40 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | 2.342 | -6 2.763 | -1.381 | 0.921 | -3.157 |
| 41 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.485 | -0.416 | -1.465 |
| 42 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | 0.208 | 0.138 | -2.733 |
| 43 ($\gamma_\mu \otimes \xi_\mu)(\gamma_\mu \otimes \xi_\nu$) | -0.222 | 0.764 | -0.686 | -0.254 |
| #  | Expression                                          | Value          | Value          | Value          |
|----|-----------------------------------------------------|----------------|----------------|----------------|
| 26 | $(\gamma_{\mu \nu} \otimes \xi_{\rho})(\gamma_{\mu \nu} \otimes \xi_{\rho})$ | $-19.434/ -6.274$ | $-5.960$       | $4.145$        |
|    |                                                     | $-3.000$       | $1.986$        | $-1.381$       |
| 27 | $(\gamma_{\mu \nu} \otimes \xi_{\rho})(\gamma_{\mu \nu} \otimes \xi_{\sigma})$ | $0.485$        | $1.944$        | $0.624$        |
|    |                                                     | $0.208$        | $-0.648$       | $-0.208$       |
| 28 | $(\gamma_{\mu \nu} \otimes \xi_{\nu})^5(\gamma_{\mu \nu} \otimes \xi_{\mu})^5$ | $-0.003$       | $-0.052$       | $-0.044$       |
|    |                                                     | $-0.168$       | $-0.022$       | $0.014$        |
| 29 | $(\gamma_{\mu \nu} \otimes \xi_{\nu})^5(\gamma_{\mu \nu} \otimes \xi_{\nu})^5$ | $0.918$        | $-0.900$       | $-0.771$       |
|    |                                                     | $-0.675$       | $-0.385$       | $0.257$        |
| 30 | $(\gamma_{\mu \nu} \otimes \xi_{\sigma})^5(\gamma_{\mu \nu} \otimes \xi_{\nu})^5$ | $0.031$        | $-0.095$       | $0.067$        |
|    |                                                     | $0.067$        | $0.052$        | $-0.528$       |
| 31 | $(\gamma_{\mu \nu} \otimes \xi_{\sigma})^5(\gamma_{\mu \nu} \otimes \xi_{\rho})^5$ | $-0.022$       | $0.022$        | $0.176$        |
| 32 | $(\gamma_{\mu \nu} \otimes \xi_{\sigma})^5(\gamma_{\mu \nu} \otimes \xi_{\sigma})^5$ | $1.157$        | $-0.416$       | $-1.704$       |
|    |                                                     | $-0.385$       | $-0.832$       | $0.568$        |
Table 3: Renormalization factors for the gauge invariant operators $VA = (\gamma_\mu \otimes \xi_5)(\gamma_{5\mu} \otimes \xi_5)$. Matrix elements are arranged in the same way as in Table. 2.

| mixed operator | $VA_1$ | $VA_2$ |
|----------------|--------|--------|
| $1$ (\gamma_\mu \otimes I)(\gamma_{5\mu} \otimes I) | 0.228 | | 
| $2$ (\gamma_\mu \otimes \xi_5)(\gamma_{5\mu} \otimes \xi_5) | -0.142 | -19.319/6.998 |
| $3$ (\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_{5\mu} \otimes \xi_{\mu\nu}) | -3 -3.560 | -29.999/ -3.680 |
| $4$ (\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_{5\mu} \otimes \xi_{\mu\rho}) | 0.174 | 0.149 |
| $5$ (\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_{5\mu} \otimes \xi_{\nu\rho}) | 0.074 | -0.049 |
| $6$ (\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_{5\mu} \otimes \xi_{\mu\nu}) | 0.402 | | 
| $7$ (\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_{5\mu} \otimes \xi_{\mu\nu}) | -1.344 | -1.152 |
| $8$ (\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_{5\mu} \otimes \xi_{\mu\nu}) | -0.576 | 0.384 |
| $9$ (\gamma_{5\mu} \otimes I)(\gamma_{\sigma} \otimes I) | -0.082 | -0.164 |
| $10$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | -7 -5.253 | -6 -4.502 |
| $11$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | -3 -2.251 | 2 1.501 |
| $12$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | 2.342 | -0.780 |
| $13$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | 0.224 | 0.074 |
| $14$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | 0.083 | 0.249 |
| $15$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | 2.412 | -0.804 |
| $16$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | 0.307 | -0.324 |
| $17$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | 0.174 | 0.149 |
| $18$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | 0.224 | 0.074 |
| $19$ (\gamma_{5\mu} \otimes \xi_{\mu\nu})(\gamma_{\sigma} \otimes \xi_{\mu\nu}) | 0.074 | -0.049 |
Table 4: Renormalization factors for the gauge invariant operator $VV = (\gamma^\mu \otimes I)(\gamma^\mu \otimes \xi_\lambda)$. Matrix elements are arranged in the same way as in Table 2.

| mixed operator | $\gamma V V_1$ finite | $\gamma V V_2$ finite |
|----------------|-----------------------|-----------------------|
| 1 $(\gamma^\mu \otimes I)(\gamma^\mu \otimes \xi_5)$ | 9 $-19.319/0.999$ | $-29.999/-3.680$ |
| 2 $(\gamma^\mu \otimes \xi_5)(\gamma^\mu \otimes I)$ | $-3 -3.560$ | $-0.142$ |
| 3 $(\gamma^\mu \otimes \xi_\mu\nu)(\gamma^\mu \otimes \xi_\nu\sigma\lambda)$ | $0.051$ | $-0.153$ |
| 4 $(\gamma^\mu \otimes \xi_\mu\nu)(\gamma^\mu \otimes \xi_\nu\sigma\lambda)$ | $0.225$ | $0.253$ |
| 5 $(\gamma^\mu \otimes \xi_\mu\nu)(\gamma^\mu \otimes \xi_\nu\sigma\lambda)$ | $0.078$ | $-0.084$ |
| 6 $(\gamma^\mu \otimes \xi_\mu\nu)(\gamma^\mu \otimes \xi_\nu\sigma\lambda)$ | $-1.344$ | $-1.152$ |
| 7 $(\gamma^\mu \otimes \xi_\nu\rho)(\gamma^\nu \otimes \xi_\nu\rho)$ | $0.174$ | $0.149$ |
| 8 $(\gamma^\mu \otimes \xi_\nu\rho)(\gamma^\nu \otimes \xi_\nu\rho)$ | $0.074$ | $-0.049$ |
| 9 $(\gamma_\nu \otimes \xi_5)(\gamma_\nu \otimes \xi_5)$ | $-7 -5.253$ | $-6 -4.502$ |
| 10 $(\gamma_\nu \otimes \xi_\lambda)(\gamma_\nu \otimes I)$ | $-3 -2.251$ | $2 1.501$ |
| 11 $(\gamma_\nu \otimes \xi_\nu\rho)(\gamma_\nu \otimes \xi_\nu\rho)$ | $-0.191$ | $-0.164$ |
| 12 $(\gamma_\nu \otimes \xi_\nu\rho)(\gamma_\nu \otimes \xi_\nu\rho)$ | $-0.082$ | $0.054$ |
| 13 $(\gamma_\nu \otimes \xi_\mu\nu)(\gamma_\nu \otimes \xi_\mu\nu)$ | $2.412$ | $-0.324$ |
| 14 $(\gamma_\nu \otimes \xi_\mu\nu)(\gamma_\nu \otimes \xi_\mu\nu)$ | $-0.804$ | $0.074$ |
| 15 $(\gamma_\nu \otimes \xi_\mu\nu)(\gamma_\nu \otimes \xi_\mu\nu)$ | $2.342$ | $-0.780$ |
| 16 $(\gamma_\nu \otimes \xi_\mu\nu)(\gamma_\nu \otimes \xi_\mu\nu)$ | $0.091$ | $-0.253$ |
| 17 $(\gamma_\nu \otimes \xi_\mu)(\gamma_\nu \otimes \xi_\mu)$ | $0.324$ | $0.084$ |
| 18 $(\gamma_\nu \otimes \xi_\mu)(\gamma_\nu \otimes \xi_\mu)$ | $-0.174$ | $-0.149$ |
| 19 $(\gamma_\nu \otimes \xi_\mu)(\gamma_\nu \otimes \xi_\mu)$ | $-0.074$ | $0.049$ |
| 20 $(\gamma_\nu \otimes \xi_\mu)(\gamma_\nu \otimes \xi_\mu)$ | $0.224$ | $-0.074$ |
| 21 $(\gamma_\nu \otimes \xi_\mu)(\gamma_\nu \otimes \xi_\mu)$ | $-0.173$ | $0.078$ |
Table 5: Renormalization factors for the gauge invariant operator $VA = (\gamma_{\mu} \otimes I)(\gamma_{\mu5} \otimes \xi_5)$ and $AV = (\gamma_{\mu5} \otimes I)(\gamma_{\mu} \otimes \xi_5)$. Matrix elements are arranged in the same way as in Table. 4

| mixed operator | $VA_1$ finite | $VA_2$ finite | $AV_1$ finite | $AV_2$ finite |
|---------------|---------------|---------------|---------------|---------------|
| 1 $(I \otimes \xi_{\mu})(\gamma_{\mu} \otimes \xi_{\mu5})$ | 1.511         | -0.139        | -0.376        |
| 2 $(I \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_5)$ | -0.872        | -0.188        | 0.125         |
| 3 $(I \otimes \xi_{\sigma})(\gamma_{\mu5} \otimes \xi_{\mu})$ | 0.130         | 0.052         | 0.044         |
| 4 $(I \otimes \xi_{\sigma})(\gamma_{\mu5} \otimes \xi_5)$ | -0.213        | 0.022         | -0.014        |
| 5 $(\gamma_{\mu5} \otimes \xi_{\mu})(I \otimes \xi_{\mu5})$ | 0.485         | 0.416         | 0.319         |
| 6 $(\gamma_{\mu5} \otimes \xi_{\mu})(I \otimes \xi_5)$ | 0.208         | -0.138        | 0.706         |
| 7 $(\gamma_{\mu5} \otimes \xi_{\mu})(I \otimes \xi_{\mu5})$ | -3.675        | -3.150        | 20.774/7.615  |
| 8 $(\gamma_{\mu5} \otimes \xi_{\mu})(I \otimes \xi_5)$ | -1.575        | 1.050         | -2.569        |
| 9 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_5)$ | 19.513/6.805  | -7 -5.253     | -6 -4.502     |
| 10 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes I)$ | 9 -2.977       | -3 -2.251     | 2 1.501       |
| 11 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | 0.142         | -0.396        | 0.159         |
| 12 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -0.174        | 0.656         | -0.224        |
| 13 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -0.074        | -0.218        | 0.074         |
| 14 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -1.007        | -2.774        | 2.342         |
| 15 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -1.584        | 0.924         | -0.780        |
| 16 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | 0.174         | 0.149         | 0.224         |
| 17 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | 0.074         | -0.049        | -0.074        |
| 18 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -1.289        | -1.104        | 2.683         |
| 19 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -0.552        | 0.368         | -1.615        |
| 20 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | 0.086         | -1.107        | 0.428         |
| 21 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -0.396        | 0.369         | -0.142        |
| 22 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -0.086        | -0.139        | -0.049        |
| 23 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -1.615        | -0.552        | 0.368         |
| 24 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | 0.242         | -0.774        | 0.924         |
| 25 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -0.416        | -0.051        | 0.402         |
| 26 $(\gamma_{\mu5} \otimes \xi_{\mu})(\gamma_{\mu5} \otimes \xi_{\mu5})$ | -0.832        | 0.568         | -0.385        |

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|   | \((\gamma_{\mu\nu} \otimes \xi_\mu)(\gamma_{\mu\nu} \otimes \xi_\sigma)\) | \((\gamma_{\mu\nu} \otimes \xi_\mu)(\gamma_{\mu\nu} \otimes \xi_\rho)\) | \((\gamma_{\mu\nu} \otimes \xi_\rho)(\gamma_{\mu\nu} \otimes \xi_\sigma)\) |
|---|---|---|---|
| 26 | 0.052 | -0.528 | 0.067 |
| 27 | 0.022 | 0.176 | -0.022 |
| 28 | 0.031 | 0.095 |
| 29 | -0.900 | -0.771 | 0.918 |
| 30 | -0.385 | 0.257 | -0.675 |
| 31 | -0.052 | -0.044 | -0.003 |
| 32 | -0.022 | 0.014 | -0.168 |
| 33 | 0.624 | 0.485 | 1.944 |
| 34 | -0.208 | 0.208 | -0.648 |
| 35 | 4.145 | -19.434/ - 6.274 | -5.960 |
| 36 | -1.381 | -3.000 | 1.986 |
| 37 | -0.222 | 0.764 |
| 38 | 0.668 | -0.254 |
| 39 | -0.222 | 0.764 |
| 40 | 0.668 | -0.254 |
| 41 | -1.465 | -0.485 | -0.416 |
| 42 | 2.733 | -0.208 | 0.138 |
| 43 | 21.486/8.327 | -3.224 | -2.763 |
| 44 | -3.157 | -1.381 | 0.921 |
Table 6: Renormalization factors for the gauge non-invariant operators $VV = (\gamma_\mu \otimes \xi_5)(\gamma_\mu \otimes \xi_5)$ and $AA = (\gamma_\mu \otimes \xi_3)(\gamma_\mu \otimes \xi_5)$ in the Landau gauge. Matrix elements are arranged in the same way as in Table. 

| mixed operator | $\gamma$ | $VV_1$ finite | $\gamma$ | $VV_2$ finite | $\gamma$ | $AA_1$ finite | $\gamma$ | $AA_2$ finite |
|----------------|---------|--------------|---------|--------------|---------|--------------|---------|--------------|
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 4.725 | -3.374 | -4.238 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -1.575 | -1.617 | 1.078 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.624 | -0.485 | -0.416 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.208 | -0.208 | 0.138 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.052 | -0.044 | -0.067 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.022 | 0.014 | 0.022 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -1.466 | -1.257 | 0.565 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.628 | 0.419 | -0.188 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 4.853 | -3.675 | -3.150 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -1.617 | -1.575 | 1.050 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.624 | 0.485 | 0.416 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.208 | 0.208 | -0.138 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.052 | 0.044 | 0.067 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.022 | -0.014 | -0.022 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.439 | -0.376 | 1.885 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.188 | 0.125 | -0.628 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.428 | -0.538 | -0.461 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.142 | -0.230 | -0.153 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -3.744/11.127 | 9 | -7 | -5.253 | -6 | -4.502 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -3 | -2.913 | 28.706/2.387 | -3 | -2.251 | 2 | 1.501 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -3 | -1.289 | 1.104 | 3.016 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.552 | 0.368 | -1.005 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.174 | 0.149 | 0.224 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.074 | -0.049 | -0.074 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -1.813 | -1.554 | 2.342 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.777 | 0.518 | -0.780 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.174 | 0.149 | 0.224 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.074 | -0.049 | -0.074 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.538 | -0.461 | 0.428 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.230 | 0.153 | -0.142 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -7 | -5.253 | -6 | -4.502 | 9 | 37.976/11.657 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -3 | -2.251 | 2 | 1.501 | -3 | -4.504 | 24.464/ - 1.854 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -3 | 2.832 | -1.813 | -1.554 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.780 | -0.777 | 0.518 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.224 | 0.174 | 0.149 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.074 | -0.049 | -0.074 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 2.016 | -1.289 | -1.104 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -1.005 | -0.552 | 0.368 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.224 | 0.174 | 0.149 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.074 | -0.049 | -0.074 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -2.242 | -2.763 | 5.433 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -1.838 | 0.921 | -1.811 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.485 | -0.416 | -0.624 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.208 | 0.138 | 0.208 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.485 | 0.416 | 0.624 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.208 | -0.138 | -0.208 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.067 | -0.052 | -0.044 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.022 | -0.022 | 0.014 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 1.293 | -0.900 | -0.771 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.431 | -0.385 | 0.257 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 0.067 | 0.052 | 0.044 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.022 | 0.022 | -0.014 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | 1.157 | -1.005 | -0.862 |
| $I (I \otimes \xi_5)(I \otimes \xi_5)$ | -0.385 | -0.431 | 0.287 |
Table 7: Renormalization factors for the gauge non-invariant operators $V A = (\gamma_\mu \otimes \xi_5)(\gamma_\mu_5 \otimes \xi_5)$ in the Landau gauge. Matrix elements are arranged in the same way as in Table. 2.

| mixed operator | $V A_1$ finite | $V A_2$ finite | $V A_3$ finite | $V A_4$ finite |
|---------------|----------------|----------------|----------------|----------------|
| 1 ($\gamma_\mu \otimes I)(\gamma_\mu_5 \otimes I)$ | 0.428 | | |
| 2 ($\gamma_\mu \otimes \xi_5)(\gamma_\mu_5 \otimes \xi_5$) | 9 \(37.711/11.392\) | | |
| 3 ($\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_\mu_5 \otimes \xi_{\mu\nu}$) | -1.289 | -1.104 | |
| 4 ($\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_\mu_5 \otimes \xi_{\mu\nu}$) | 0.174 | 0.149 | |
| 5 ($\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_\mu_5 \otimes \xi_{\mu\nu}$) | 0.074 | -0.049 | |
| 6 ($\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_\mu_5 \otimes \xi_{\mu\nu}$) | -0.777 | 0.518 | |
| 7 ($\gamma_{\sigma_5} \otimes I)(\gamma_\sigma \otimes I)$ | | | |
| 8 ($\gamma_{\sigma_5} \otimes \xi_5)(\gamma_\sigma \otimes \xi_5$) | -7 \(-5.253\) | -6 \(-4.502\) | |
| 9 ($\gamma_{\sigma_5} \otimes \xi_{\mu\nu})(\gamma_\sigma \otimes \xi_{\mu\nu}$) | -2.342 | | |
| 10 ($\gamma_{\sigma_5} \otimes \xi_{\mu\nu})(\gamma_\sigma \otimes \xi_{\mu\nu}$) | -0.780 | | |
| 11 ($\gamma_{\sigma_5} \otimes \xi_{\mu\nu})(\gamma_\sigma \otimes \xi_{\mu\nu}$) | | | |
| 12 ($\gamma_{\sigma_5} \otimes \xi_{\mu\nu})(\gamma_\sigma \otimes \xi_{\mu\nu}$) | | | |
| 13 ($\gamma_{\mu\nu} \otimes \xi_5)(\gamma_\mu_5 \otimes \xi_5$) | 0.174 | 0.149 | |
| 14 ($\gamma_{\mu\nu} \otimes \xi_5)(\gamma_\mu_5 \otimes \xi_5$) | 0.074 | -0.049 | |
| 15 ($\gamma_{\mu\nu} \otimes \xi_{\mu\nu})(\gamma_\mu_5 \otimes \xi_5$) | -0.74 | 0.049 | |
| 16 ($\gamma_{\mu\nu} \otimes \xi_{\mu\nu})(\gamma_\mu_5 \otimes \xi_5$) | -0.28 | 0.074 | |
Table 8: Renormalization factors for the gauge non-invariant operators $VV = (\gamma_\mu \otimes I)(\gamma_\mu \otimes \xi_5)$ in the Landau gauge. Matrix elements are arranged in the same way as in Table 2.

| mixed operator | $\gamma_1$ finite | $\gamma_2$ finite |
|---------------|------------------|------------------|
| 1 $(\gamma_\mu \otimes I)(\gamma_\mu \otimes \xi_5)$ | 37.711/11.392 | -3.708 |
| 2 $(\gamma_\mu \otimes \xi_5)(\gamma_\mu \otimes I)$ | 0.428 | -0.142 |
| 3 $(\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_\mu \otimes \xi_{\mu\sigma\nu})$ | -0.174 | -0.149 |
| 4 $(\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_\mu \otimes \xi_{\mu\sigma\nu})$ | -1.813 | -1.554 |
| 5 $(\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_\mu \otimes \xi_{\mu\sigma\nu})$ | 0.174 | 0.149 |
| 6 $(\gamma_\mu \otimes \xi_{\mu\nu})(\gamma_\mu \otimes \xi_{\mu\nu})$ | 0.074 | -0.049 |
| 7 $(\gamma_{\sigma\delta} \otimes I)(\gamma_{\sigma\delta} \otimes \xi_5)$ | -5.253 | -6.502 |
| 8 $(\gamma_{\sigma\delta} \otimes \xi_5)(\gamma_{\sigma\delta} \otimes I)$ | -2.253 | 2.501 |
| 9 $(\gamma_{\sigma\delta} \otimes \xi_{\mu\nu})(\gamma_{\sigma\delta} \otimes \xi_{\mu\nu})$ | 0.224 | 0.153 |
| 10 $(\gamma_{\sigma\delta} \otimes \xi_{\mu\nu})(\gamma_{\sigma\delta} \otimes \xi_{\mu\nu})$ | 3.016 | -1.005 |
| 11 $(\gamma_{\sigma\delta} \otimes \xi_{\mu\nu})(\gamma_{\sigma\delta} \otimes \xi_{\mu\nu})$ | -0.224 | 0.074 |
| 12 $(\gamma_{\sigma\delta} \otimes \xi_{\mu\nu})(\gamma_{\sigma\delta} \otimes \xi_{\mu\nu})$ | 2.342 | -0.780 |
| 13 $(\gamma_{\mu\nu} \otimes \xi_\rho)(\gamma_{\mu\sigma\nu} \otimes \xi_\rho)$ | 0.174 | 0.149 |
| 14 $(\gamma_{\mu\nu} \otimes \xi_\rho)(\gamma_{\mu\sigma\nu} \otimes \xi_\rho)$ | 0.074 | -0.049 |
| 15 $(\gamma_{\mu\nu} \otimes \xi_\rho)(\gamma_{\mu\sigma\nu} \otimes \xi_\rho)$ | -0.174 | -0.149 |
| 16 $(\gamma_{\mu\nu} \otimes \xi_\rho)(\gamma_{\mu\sigma\nu} \otimes \xi_\rho)$ | -0.074 | 0.049 |
| 17 $(\gamma_{\mu\nu} \otimes \xi_\rho)(\gamma_{\mu\sigma\nu} \otimes \xi_\rho)$ | 0.224 | -0.074 |
| 18 $(\gamma_{\mu\nu} \otimes \xi_\rho)(\gamma_{\mu\sigma\nu} \otimes \xi_\rho)$ | -0.224 | 0.074 |

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Table 9: Renormalization factors for the gauge non-invariant operators $VA = (\gamma_\mu \otimes I)(\gamma_\mu \otimes \xi_5)$ and $AV = (\gamma_5 \otimes I)(\gamma_\mu \otimes \xi_5)$ in the Landau gauge. Matrix elements are arranged in the same way as in Table 2.

| mixed operator | $VA_1$ finite | $VA_2$ finite | $AV_1$ finite | $AV_2$ finite |
|----------------|---------------|---------------|---------------|---------------|
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 1.885 | -0.439 | -0.376 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.628 | -0.188 | 0.125 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.067 | 0.052 | 0.044 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.022 | 0.022 | 0.014 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.485 | 0.416 | 0.624 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.208 | -0.138 | -0.208 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -3.675 | -3.150 | 4.853 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -1.575 | 1.050 | -1.617 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.565 | -1.466 | -1.257 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.188 | -0.628 | 0.419 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.067 | -0.052 | -0.044 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.022 | -0.022 | 0.014 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.485 | -0.416 | -0.624 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.208 | 0.138 | 0.208 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -3.774 | -3.235 | 4.725 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -1.617 | 1.078 | -1.575 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 37.976/11.657 | -7 | -5.253 | -6 | -4.502 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -3 | -4.503 | 24.464/ -1.854 | -3 | -2.251 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.428 | -0.538 | -0.461 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.142 | -0.230 | 0.153 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.174 | -0.149 | -0.224 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.074 | 0.049 | 0.074 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -1.813 | -1.554 | 2.342 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.777 | 0.518 | -0.780 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.174 | 0.149 | 0.224 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.074 | -0.049 | -0.074 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -1.289 | -1.104 | 3.016 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.552 | 0.368 | -1.005 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -7 | -5.253 | -6 | -4.502 | 9 | 37.446/11.127 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -3 | -2.251 | 2 | 1.501 | -3 | -2.913 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.538 | -0.461 | 0.428 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.230 | 0.153 | -0.142 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.224 | 0.174 | 0.149 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.074 | 0.074 | -0.049 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 3.016 | -1.289 | -1.104 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -1.005 | -0.552 | 0.368 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.224 | -0.174 | -0.149 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.074 | 0.074 | 0.049 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 2.342 | -1.813 | -1.554 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.780 | -0.777 | 0.518 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -1.005 | -0.862 | 1.157 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.431 | 0.287 | -0.385 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.52 | 0.044 | 0.067 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.222 | -0.014 | -0.022 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.900 | -0.771 | 1.293 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.385 | 0.257 | -0.431 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.052 | -0.044 | -0.067 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.022 | 0.014 | 0.022 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.624 | 0.485 | 0.416 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.208 | 0.208 | -0.138 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 4.145 | -4.226 | -3.622 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -1.381 | 1.811 | 1.207 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -0.624 | -0.485 | -0.416 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 0.208 | -0.208 | 0.138 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | 5.433 | -3.224 | -2.763 |
| $I \otimes (\xi_5 \otimes \xi_{\mu 5})$ | -1.811 | -1.381 | 0.921 |
Figure 1: One-loop diagrams for bilinear operators. Diagrams (b) and (e) are absent for gauge non-invariant operators.
Figure 2: One-loop diagrams for color two-loop four-quark operators $\bar{Q}^a(\gamma_S \otimes \xi_F)Q^a \cdot \bar{Q}^b(\gamma_{S'} \otimes \xi_{F'})Q^b$. Thick horizontal bars at the four-quark vertices signify contraction of spin-flavor quantum numbers, while dotted lines represent link factors and flow of color indices.
Figure 3: One-loop diagrams for color one-loop four-quark operators $\bar{Q}^a(\gamma_S \otimes \xi_F)Q^b \cdot \bar{Q}^b(\gamma_S' \otimes \xi_{F'})Q^a$. The meaning of the symbols is the same in Fig. 2.