Asymmetric solitons and domain walls supported by inhomogeneous defocusing nonlinearity

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We show that an inhomogeneous defocusing nonlinearity that grows toward the periphery in the positive and negative transverse directions at different rates can support strongly asymmetric fundamental and multipole bright solitons, which are stable in wide parameter regions. In the limiting case when nonlinearity is uniform in one direction, solitons transform into stable domain walls (fronts), with constant or oscillating intensity in the homogeneous region, attached to a tail rapidly decaying in the direction of growing nonlinearity.

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Interest in the evolution of light beams in materials with spatially inhomogeneous parameters, such as refractive index or nonlinearity, is motivated by the possibility to control diffraction broadening for beam shaping and steering. For example, the strength and sign of the effective diffraction can be managed in periodic refractive-index landscapes, with the aim to create nonlinear modes and propagation regimes that are not possible in uniform media [1-3]. Modulation of the local strength of the nonlinearity can be also used to control the beam dynamics via effective pseudo-potentials whose impact on light propagation crucially depends on its intensity. Various types of solitons were predicted in nonlinear pseudo-potentials [4], including one-dimensional solitons in nonlinear [5-9] and combined linear-nonlinear [10-12] lattices, exact modes supported by specially designed localized focusing nonlinearities [13], two-dimensional solitons supported by localized or periodic nonlinearities [14-17]. Formation of bright solitons in pseudo-potentials requires the presence of domains with focusing nonlinearity, assuming that the nonlinearity modulation depth is limited. However, it was recently shown that purely defocusing nonlinearities also support bright solitons with a finite total power, provided that the nonlinearity strength grows toward the periphery as \( \sigma(\eta) > 0 \) is the local strength of the defocusing nonlinearity. Here we consider a spatially inhomogeneous nonlinearity growing toward the periphery as \( \sigma(\eta) = \exp(\alpha_0 \eta^2) \) and \( \sigma(\eta > 0) = \exp(\alpha_1 \eta^2) \), where \( \alpha_0, \alpha_1 \) define the nonlinearity growth rates at \( \eta < 0 \) and \( \eta > 0 \), respectively. We set \( \alpha_1 = 1 \) by means of rescaling and vary \( \alpha_0 \), which results in asymmetric nonlinearity distributions. Such nonlinearity profiles may be realizable by inhomogeneous doping of suitable photorefractive materials, or by the inhomogeneous application of Feshbach resonances in the case of matter waves [4].

Soliton solutions of Eq. (1) with propagation constant \( b \) are looked for as \( q(\eta, \xi) = w(\eta) \exp(i k \xi) \). Their stability was investigated by adding small perturbations \( u(\eta), v(\eta) \), \( q = w + u \exp(i k \xi) + v \exp(i k \xi) \exp(\delta k \xi) \), and linearizing Eq. (1), which leads to the eigenvalue problem \( \delta u = -(1/2) d^2 v / d\eta^2 + b v + \sigma w^2 v \), \( \delta v = (1/2) d^2 u / d\eta^2 - b u - 3 \sigma w^2 u \) for the instability growth rate \( \delta \).

Despite the defocusing character of the nonlinearity, we have found a variety of asymmetric bright soliton solutions of Eq. (1), which are classified by the number of zeros (nodes) \( k \) in their shapes. Figure 1 depicts representative examples. The existence of such states is a consequence of the nonlinearizability of Eq. (1) for the decaying tails of the solitons, due to the growing nonlinearity strength [18,19]. When \( \alpha_0 \) decreases solitons develop a wide left lobe, as solutions adapt to the slower rate of the nonlinearity growth at \( \eta < 0 \).

Several noteworthy observations are suggested by Fig. 1. First, for a given value of \( \alpha_0 \), soliton solutions with different values of \( k \) feature identical asymptotic forms at \( \eta \to \pm \infty \) as follows from the comparison of fundamental and dipole solitons in Fig. 1(a), although their shapes...
differ considerably around $\eta=0$. Second, for different nonlinearity growth rates $\alpha_1$ at $\eta<0$, the soliton tails tend to coincide at $\eta>0$, as seen in the plot. Further, the soliton width rapidly grows when $\alpha_1$ decreases, due to the appearance of a wide left lobe, and the width diverges at $\alpha_1\to0$, with the nodes appearing in the profile of multiple solitons simultaneously shifting to $\eta<0$.

Solitons exist with negative values of the propagation constant, which is explained by the fact that the stationary version of Eq. (1) yields $b<0$ at the inflexion point, $\partial^2 w / \partial \eta^2 = 0$. The soliton amplitude increases with increasing $-b$ [Fig. 1(b)], consistent with the Thomas-Fermi approximation applied to this case [18]. While the asymptotic form of the solutions at $\eta\to\pm\infty$ is solely determined by $\alpha_1$ and does not depend on $b$, the soliton core changes considerably with $b$. In particular, increasing $|b|$ pushes all nodes to the right [Fig. 1(b)].

The soliton width, defined as $W=2U^{-1}\int_{\eta=0}^{\eta=\infty} |\eta| q^2 \, d\eta$, where $U=\int_{\eta=0}^{\eta=\infty} |\eta| q^2 \, d\eta$ is the total energy flow, rapidly decreases by increasing $|b|$, saturating at $b<-20$. Higher-order solutions, with a larger number of nodes, always carry a smaller energy flow [Fig. 2(a)]. On the other hand, the energy flow grows when $\alpha_1$ decreases and diverges at $\alpha_1\to0$, simultaneously being a monotonically increasing function of $|b|$ [Fig. 2(b)]. Dependencies similar to those shown in Figs. 2(a) and 2(b) were also obtained for modes with a larger number of nodes.

Linear stability analysis indicates that there is a broad range of parameters $(\alpha_1, \alpha_2)$, where solutions are stable for all values of the number of nodes $k$ up to $k=10$, at least. In particular, fundamental and dipole solitons are stable in the entire existence domain. Tripoles feature two instability domains, which are indicated by gray areas in Fig. 2(c). Remarkably, stability is possible at $\alpha_1\to0$ too, when the soliton strongly expands to $\eta<0$. Inside the gray areas in Fig. 2(c), the instability of tripoles is oscillatory, resulting in irregular shape oscillations, as seen in Fig. 3(a). In contrast to systems with homogeneous nonlinearity, where instability-induced emission of radiation may be considerable, in the setting analyzed here almost all light stays around the nonlinearity minimum, even if the unstable beam exhibits considerable oscillations. The structure of the stability and instability domains becomes more complex as $k$ increases, but stability domains are always found.

At $\alpha_1\to0$, the soliton solutions transform into DWs separating the decaying tail at $\eta>0$ and a cnoidal wave at $\eta\to\infty$, whose amplitude for a given propagation constant $b$ takes the values $w_0\leq w_{\text{max}}=|b|^{1/2}$. In fact, $w_0$ is the second free parameter of the DW solutions, with the maximal amplitude $w_{\text{max}}$ corresponding to an asymptotically flat state, as shown in Fig. 4. Note that solutions of this type can be found in an exact form for $b=-3/2$ and $w_0^2=(3-7^{1/2})/2$:

$$w(\eta)=\begin{cases} \left(2^{-1/2}\eta \exp(-\eta^2/2)\right), & \eta \geq 0, \\
0, & \eta < 0, \end{cases}$$

where $\beta^2=(3+7^{1/2})/2$ and $\kappa^2=(3-7^{1/2})/(3+7^{1/2})$ determines the elliptic modulus of the sn function.
The profile, in the form, is shown by the green line, also for, which is a monotonously, at for (the black line) corresponding to, may be characterized by the renormalized. This solution corresponds to, as shown in Fig. 4(a). A detailed, and, the elliptic. This conclusion is, and.

Summarizing, we have shown that defocusing nonlinear media with a nonlinearity strength increasing toward the periphery at different rates in two contiguous regions support strongly asymmetric stable fundamental and multipole solitons. Solitons are stable in large parameter regions, and they transform into stable domain walls. The results reported here may be generalized to the case of two transverse dimensions.

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