Do the SuperKamiokande atmospheric neutrino results explain electric charge quantisation?

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Abstract

We show that the SuperKamiokande atmospheric neutrino results explain electric charge quantisation, provided that the oscillation mode is $\nu_\mu \rightarrow \nu_\tau$ and that the neutrino mass is of the Majorana type.
The up-down asymmetry observed by SuperKamiokande [1] for the atmospheric muon-neutrino flux provides extremely clear evidence for $\nu_\mu$ disappearence, and hence for the violation of muon-type lepton number $L_\mu$. Furthermore, the detailed zenith angle and energy dependences observed for the atmospheric $\nu_\mu$ and $\nu_e$ fluxes are non-trivially consistent with a neutrino oscillation explanation based on either $\nu_\mu \rightarrow \nu_\tau$ or $\nu_\mu \rightarrow \nu_s$ [2] (where $\nu_s$ is a hypothetical sterile neutrino). In this paper we will show that these results lead to a theoretical explanation of the famous phenomenon of electric charge quantisation, provided that (i) the mode responsible is $\nu_\mu \rightarrow \nu_\tau$, and (ii) that the neutrino mass involved is of Majorana type.

The argument is extremely simple. We begin with the known result [3] that electric charge quantisation (ECQ) does not follow from the Minimal Standard Model (MSM) Lagrangian (where “minimal” means zero neutrino mass). Therefore, new physics is required to explain it [4]. It is important to recall, however, that the possible charge dequantisation allowed by the MSM is strongly constrained by the gauge invariance of the MSM Lagrangian and anomaly cancellation. Enforcing classical gauge invariance only, the allowed form for electric charge in the MSM is

$$Q_{\text{actual}} = Q_{\text{standard}} + \alpha L_e + \beta L_\mu + \gamma L_\tau + \delta B,$$

where $Q_{\text{actual}}$ and $Q_{\text{standard}}$ are the actual and standard electric charges, $L_{e,\mu,\tau}$ are the three types of lepton number, and $B$ is baryon number. The quantities $L_{e,\mu,\tau}$ and $B$ enter this formula precisely because they are conserved Abelian quantum numbers in the MSM. The continuous parameters $\alpha$, $\beta$, $\gamma$ and $\delta$ quantify electric charge dequantisation. The measured upper bounds on their magnitudes are, of course, tiny. Enforcing gauge anomaly cancellation [5] in addition to classical gauge invariance leads to the additional constraints

$$\gamma = (-\alpha^3 - \beta^3)^{1/3}, \quad \delta = -\frac{1}{3}[\alpha + \beta + (-\alpha^3 - \beta^3)^{1/3}],$$

where there are now only two continuous charge dequantisation parameters $\alpha$ and $\beta$. If one chooses to also enforce the cancellation of mixed gauge-gravitational anomalies [6], then the allowed electric charges are further constrained to be given by

$$Q_{\text{actual}} = Q_{\text{standard}} + \epsilon (L_e - L_\mu), \quad \text{or}$$
$$Q_{\text{actual}} = Q_{\text{standard}} + \epsilon (L_e - L_\tau), \quad \text{or}$$
$$Q_{\text{actual}} = Q_{\text{standard}} + \epsilon (L_\mu - L_\tau),$$

where $\epsilon$ now quantifies charge dequantisation, in addition to the threefold discrete choice one has between these forms. Note that baryon number $B$ plays no role if mixed gauge-gravitational anomalies are forced to cancel. The fact that the totality of classical and quantal constraints one may apply to the MSM do not uniquely specify electric charge constitutes the modern statement of the electric charge quantisation problem.

It is interesting that the charge quantisation problem depends crucially on the conservation of the family lepton numbers. Since neutrino oscillation experiments search for family lepton number violation, they also indirectly probe the charge quantisation problem.

There are of course many experimental indications in favour of neutrino oscillations and hence of family lepton number violation: the solar neutrino deficit [7] and the LSND
anomaly \[8\], in addition to the aforementioned atmospheric neutrino deficit observed by SuperKamiokande and other experiments \[9\]. At present, the strongest experimental evidence for neutrino oscillations comes from the SuperKamiokande atmospheric neutrino results. It is therefore interesting to ask what one may conclude about the charge quantisation problem on the basis of their results. Are the SuperKamiokande atmospheric neutrino results sufficient to explain electric charge quantisation? If not, what further experimental information is needed?

Conservatively speaking, SuperKamiokande has established the violation of \(L_\mu\). By itself, this is not enough to explain ECQ, because charge may, for example, be dequantised as per Eqn.(4) which does not involve \(L_\mu\). As a concrete illustration, one may explain the atmospheric neutrino results by \(\nu_\mu \rightarrow \nu_s\) oscillations only, leaving \(\nu_e\) and \(\nu_\tau\) as massless states unmixed with any others. Then, \(L_e - L_\tau\) is an anomaly-free conserved quantity that can dequantise electric charge. If one does not wish to impose gauge and/or mixed anomaly cancellation then there are of course more possibilities.

So, let us now suppose that the atmospheric neutrino results are explained by \(\nu_\mu \rightarrow \nu_\tau\) oscillations. Since we know that \(\nu_\tau\) almost certainly exists, this is perhaps a less speculative (though not necessarily more attractive) possibility. Conservation of \(L_\mu + L_\tau, L_e\) and \(B\) are consistent with this oscillation mode (but not required by it). Oscillations of \(\nu_\mu\) to \(\nu_\tau\) require that at least one of the mass eigenstates which superpose these flavour eigenstates to have a nonzero mass. (Of course, in general one would expect all participating neutrino masses to be nonzero.)

The conclusions depend on whether this nonzero mass is of Majorana or Dirac type \[10,11\]. If the mass is Dirac type, then the additional right-handed neutrino state required alters the anomaly cancellation calculations which lead to Eqns.(2) and (3-5).

Let us suppose first that it is of the more economical and theoretically more appealing Majorana type. In this case, \(L_\mu\) and \(L_\tau\) must be separately broken. (This conclusion is obviously unaltered if there are more nonzero neutrino eigenmasses.) One then immediately draws the interesting conclusion, either from Eqns.(3-5) or from the more general Eqn.(2), that electric charge must be quantised in the standard way. If both \(L_\mu\) and \(L_\tau\) are broken, then from the more general Eqn.(2) we must set

\[ \beta = 0, \quad \gamma = ( - \alpha^3 - \beta^3 )^{1/3} = 0 \quad \Rightarrow \quad \alpha = 0, \]

and hence \(Q_{\text{actual}} = Q_{\text{standard}}\). \textit{The SuperKamiokande atmospheric neutrino data explain electric charge quantisation provided that} \(\nu_\mu \rightarrow \nu_\tau\) \textit{oscillations induced by a Majorana mass} \textit{are their explanation.} This is our main result. The only ways to evade it are either to give up gauge anomaly cancellation, or to suppose that there are as yet unobserved heavy fermions which contribute to anomaly cancellation \[12\]. Note that both the choices – of \(\nu_\mu \rightarrow \nu_\tau\) over \(\nu_\mu \rightarrow \nu_s\) and of Majorana over Dirac mass – are ones of minimality with respect to (low energy) degrees of freedom.

For completeness, we now discuss the Dirac mass alternative even though it is much less appealing from a see-saw mechanism point of view. We first discuss the even less likely supposition that there is only one nonzero Dirac mass. We will also make the conservative assumption that the \(\nu_e\) does not mix the other neutrinos. Since \(\nu_\mu\) and \(\nu_\tau\) mix, the single Dirac mass term must be of the form
\[ m(\cos \theta \nu_{\mu L} + \sin \theta \nu_{\tau L})N_R + \text{H.c.} \]  

(7)

where \( m \) is the mass, \( \theta \) is the mixing angle\(^1\) and \( N_R \) is a right-handed neutrino field. This mass term does not preserve \( L_\mu \) or \( L_\tau \) separately but does preserve \( L_\mu + L_\tau \)\(^2\) (as well as, trivially, \( L_e \) and \( B \)). Thus, the classical gauge invariance of the Lagrangian implies that

\[
Q_{\text{actual}} = Q_{\text{standard}} + \alpha L_e + \beta (L_\mu + L_\tau) + \delta B.
\]

(8)

Furthermore, gauge anomaly cancellation only holds provided that

\[
\beta = -\alpha = -3\delta.
\]

(9)

Hence in this case electric charge is not quantised, since there is one theoretically unconstrained continuous parameter, \( \epsilon = \alpha \), such that

\[
Q_{\text{actual}} = Q_{\text{standard}} + \epsilon \left( L_e - L_\mu - L_\tau + \frac{B}{3} \right).
\]

(10)

Also note that in this case the mixed gauge gravitational anomaly provides no independent constraint.

Suppose now that both the Dirac eigenmasses involving \( \nu_\mu \) and \( \nu_\tau \) are nonzero. In this scenario there are two right-handed neutrino fields \( N_{1R} \) and \( N_{2R} \). The quantities \( L_\mu + L_\tau \), \( L_e \) and \( B \) are again classically conserved. Following similar reasoning to the one right-handed neutrino scenario discussed above, we find that electric charge is also dequantised in this case, again with one theoretically unconstrained continuous parameter, \( \epsilon = \beta \), such that

\[
Q_{\text{actual}} = Q_{\text{standard}} + \epsilon \left( L_\mu + L_\tau - \frac{2B}{3} \right).
\]

(11)

Again, the mixed gauge gravitational anomaly provides no independent constraint in this case.

Equations (10) and (11) were derived on the assumption that \( \nu_e \) does not mix with the other neutrinos. If we now alternatively suppose that it does mix (recall that the LSND anomaly might be due to \( \nu_e - \nu_\mu \) mixing), then \( L_e \) and \( L_\mu + L_\tau \) are no longer separate invariances, with only the linear combination \( L = L_e + L_\mu + L_\tau \) (total lepton number) being conserved. The coefficient of \( L_e \) in the formula for electric charge must now be put equal to the coefficient of \( L_\mu + L_\tau \), meaning that \( \epsilon = 0 \) in Equns. (10) and (11) and electric charge quantisation now follows in both of these scenarios.

Finally, suppose that the most general and perhaps most likely situation holds (given the Dirac mass assumption): there are three right-handed neutrino fields, and the neutrinos

\(^1\)It is measured to be close to \( \pi/4 \), though we only need to know that it is nonzero for the sake of our argument.

\(^2\) Provided of course that the \( L_\mu + L_\tau \) charge of \( N_R \) is the same as the \( L_\mu + L_\tau \) charge of the \( \nu_\mu \) and \( \nu_\tau \).
mix in an arbitrary manner. In this case, $B - L$ is anomaly-free and so charge may be dequantised as per $Q_{\text{actual}} = Q_{\text{standard}} + \epsilon (B - L)$ \[4,10].

To conclude, we have shown that the SuperKamiokande atmospheric neutrino results explain the charge quantisation mystery of the standard model provided that the oscillation mode is $\nu_\mu \rightarrow \nu_\tau$ and the neutrino mass is of the Majorana type. It is important in general to realise that neutrino oscillation and neutrinoless double $\beta$-decay experiments provide important information regarding the seemingly unrelated issue of electric charge quantisation.

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