Thermomechanical performance analysis and experiment of electrothermal shape memory alloy helical spring actuator

Yang Xiong, Jin Huang and Ruizhi Shu

Abstract
In this paper, an electrothermal shape memory alloy helical spring actuator constructed from shape memory alloy with copper-cored enameled wire is presented and fabricated. Based on the shear constitutive model of a shape memory alloy, the Thermo equilibrium equation and the geometrical equation of helical spring establish the thermomechanical theoretical model of helical spring actuator with electrothermal shape memory alloys under different scenarios. The thermomechanical behaviors of the actuator were verified by numerical simulation with experimental tests, and the actuator thermomechanical properties were derived from the analysis with current, temperature, response time, restoring force, and axial displacement as parameters. The experimental results show that the actuator produces a maximum recovery force of 70.2 N and a maximum output displacement of 7.7 mm at 100°C. The actuator response time is 26 s at a current of 3 A. It is also demonstrated that the theoretical model can effectively characterize the complex thermo-mechanical properties of the actuator due to the strong nonlinearity of the shape memory alloy. The experimental temperature-force response and temperature-displacement response, as well as the force-displacement response at different temperatures, provide references for the design and fabrication of electrothermal shape-memory alloy coil spring actuators.

Keywords
Shape memory alloys, shape-memory effect, actuator, helical springs, experimental test

Date received: 14 May 2021; accepted: 18 August 2021

Handling Editor: James Baldwin

Introduction
Shape memory alloys (SMA) are a class of smart alloy materials with shape memory effect (SME) and superelasticity. Smart actuators constructed from SMA can accomplish complex functions driven by stress or temperature, and because of the unique and excellent characteristics of SMA, such as compact structure, high power density, and high fatigue resistance to cyclic motion, SMA actuators have unique technical advantages in lightweight robots and miniaturized systems.1–3 The process by which SMA produces shape memory effect is that loading SMA transforms the austenite phase to stable non-twinned martensite phase when SMA is below the austenite phase transition temperature, heating SMA after unloading brings it to the austenite phase finish transition temperature, SMA reverses from non-

College of Mechanical Engineering, Chongqing University of Technology, Chongqing, China

Corresponding author:
Jin Huang, College of Mechanical Engineering, Chongqing University of Technology, 69 Hongguang Avenue, Banan District, Chongqing 400054, China.
Email: jhuangcq@cqut.edu.cn
twinned martensite to austenite phase and macroscopically shows that deformation gradually recovers after SMA heating.4–6 Smart components made of SMA-based shape memory effect are widely used in medical,7,8 mechanical,9,10 and Aerospace.11,12

SMA helical spring is a device that combines an actuator, a temperature sensor, and a displacement amplifier. Although the structure of the SMA spring is simple, the performance of the SMA spring is complex due to its strong non-linear thermomechanical properties.15–18 Ma et al.19 analyzed a biased bidirectional SMA actuator composed of SMA spring and steel spring, and established the relationship of output displacement and force of SMA spring actuator with temperature, stress-strain, material parameters as well as dimensional parameters. Gédouin et al.20 present a theoretical method for the analytical study of SMA helical spring actuators, which can effectively predict the thermomechanical behavior of linear biased spring actuators. Stachowiak and Kurzawa21 proposed an SMA micro spring-driven rotary motor achievable to operate in a continuous and bi-directional rotation mode where the speed and torque are determined by the driving sequence. Wang et al.22 perform finite element simulation on the training behaviors of SMA wave spring actuators, which provides an effective method for the design and optimization of SMA wave spring actuators.

Most of the above SMA actuators transfer heat with fluid as medium or load current to produce Joule heat to cause shape memory effect, but these two ways have certain defects. Limited by the heat transfer performance of fluids, SMA actuators that transfer heat from the fluid to the medium respond slowly, and the temperature rise of the fluid medium during heat transfer causes the loss of heat energy. Because SMA resistance is small, it needs to load larger current on SMA to generate enough heat to cause shape memory effect, higher performance requirements on power supply, and higher energy consumption, meanwhile, SMA exhibits many alterations in metallographic organization during temperature increase, which leads to the SMA resistance becoming highly non-linear, so it is very difficult to precisely control this type of SMA actuators. To address the deficiencies of the above SMA actuators, a novel helical spring actuator with electrothermal SMA is proposed, which is compact in structure and high energy density and can significantly reduce actuator response time.

**Theoretical model**

The electrothermal SMA helical spring specimen is shown in Figure 1(a). Polyurethane copper-cored enameled wires are spirally wrapped around the SMA wires to prevent short circuits in electrical appliances.

![Figure 1](image)

Meanwhile, to generate significant Joule heat and to exchange heat rapidly with SMA wire, the enameled wire and SMA wire are tightly bonded. The upper and lower ends of the electrothermal SMA helical spring are tightened and ground for load-bearing stability, so no enameled wire was wrapped around the SMA wires at the upper and lower ends.

The simplified model established from the electrothermal SMA helical spring specimen is shown in Figure 1(b). In the simplified model of electrothermal SMA spring, the diameter of SMA wire is $d = 1.95$ mm, the helical angle of the spring is $\alpha = 6^\circ$, the diameter of SMA spring is $D = 12.5$ mm ($D = 2R$), the free height of SMA spring is $L = 19$ mm, and the number of active coils is $n_e = 6$; Diameter of polyurethane enameled wires on SMA wire is $d' = 0.4$ mm, and the turns of painted wire is $N = 300$.

**Working principle**

The critical temperatures of SMA in the unstressed state are $M_s$, $M_f$, $A_s$, and $A_f$, respectively (Martensitic transformation start temperature and finish temperature, Austenite transformation start temperature and finish temperature). In the initial state, the temperature of the SMA helical spring is $T = 25^\circ C$ ($T < M_f$), and the parent phase of SMA is twin martensite; During the loading of SMA, the stress gradually increased to the beginning transformation stress of martensite, and the SMA transformed from the twinned martensitic phase to the non-twinned martensitic phase. The volume fraction and shape memory factor of martensite in SMA increase gradually at this time; After finishing loading, the temperature of the electrothermal SMA helical spring is $T = 25^\circ C$ ($T < A_s$). No phase transformation occurs in the SMA non-twinned martensite during unloading, and after the end of
unloading, the recoverable non-linear strain of the SMA transforms to the residual strain; In the heating stage, the current is applied to make the temperature of SMA spring rise to $T = 100^\circ C (T > T_A)$. In this process, the twinned martensite gradually transformed into austenite, the shape memory factor and residual strain gradually decrease, and the electrothermal SMA helical spring returns to its initial shape. The loading and unloading stage of the SMA has been completed during manufacture. It is only necessary to energized lacquered wire in electrothermal SMA helical spring in practical use. The desired displacement or restoring force can be generated by applying a displacement constraint to electrothermal SMA helical spring actuators, while continuously precise control of the restoring force or displacement generated by electrothermal SMA helical spring actuators can be achieved by controlling the magnitude and time of the enameled wire current.

**Pure shear constitutive model of SMA**

The thermodynamic process of shape memory effect of SMA can be accurately described by shape memory factor $\eta$ ($\eta \in (0, 1)$). During the SMA loading process, SMA began to produce recoverable non-linear strain, and the shape memory factor $\eta$ increased from 0 to 1 as the load gradually increased, at which point the recoverable non-linear strain produced by SMA reached a maximum; During SMA unloading process, the shape memory factor and recoverable non-linear strain do not change; Heated to SMA after unloading, the SMA transforms from non-twinned martensite to austenite when it reaches the phase transformation critical temperature, the shape memory factor gradually decreases with temperature, and the nonlinear strain can be recovered gradually by SMA. Combined with the microscopic mechanism of SMA, the relationship between the shape memory factor $\eta$ and the volume fraction $\xi$ of martensite can be expressed as

$$\xi = \xi_0 + (1 - \xi_0)\eta$$

(1)

Assuming a linear relationship between shape memory factor $\eta$ and shear stress as well as temperature, according to the phase transition critical stress temperature relationship, it can be concluded that the relationship between shape memory factor $\eta$ and shear stress $\tau$ as well as temperature during unloading of SMA in pure shear state is

$$\eta = \eta_0 \frac{\tau - \tau_{A_x}}{\tau_{A_x} - \tau_{A_y}}$$

(2)

Where $\tau_{A_x}$ and $\tau_{A_y}$ are the shear stress at the beginning of austenite and at the finish of austenite respectively, and $\tau_{A_x}$ and $\tau_{A_y}$ can be described as

$$\left\{ \begin{array}{l} \tau_{A_x} = \frac{\sqrt{3}}{2} C_A (T - A_x), T > T_A, \\ \tau_{A_y} = \frac{\sqrt{3}}{2} C_A (T - A_y), T > T_A \end{array} \right.$$ 

(3)

The strain of the SMA has three components: elastic strain, shape memory strain as well as thermal expansion strain, which puts the SMA strain $\varepsilon_{ij}$ expressed as a function of stress $\sigma_{ij}$ and a function of temperature $T$.

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl} + \frac{3}{2} \varepsilon_t \frac{\sigma_{ij}}{\sigma_e} \eta + \alpha_0 (T - T_0)$$

(4)

Where $\varepsilon_{ij}$ is the strain tensor, $\sigma_{ij}$ is the stress tensor, $S_{ijkl}$ is the flexibility coefficient ($i, j, k$, and $l$ represent the different directions of the axis), $\varepsilon_t$ is the maximum residual strain, $T_0$ is the initial value of temperature $T$, the equivalent stress $\sigma_e$, and deviatoric stress tensor $\sigma_{ij}$ can be described as

$$\begin{align*}
\sigma_e &= \sqrt{\frac{3}{2} \varepsilon_t^2 \frac{\sigma_{ij}}{\sigma_e} \eta} \\
\sigma_{ij} &= \sigma_{ij} - \frac{1}{3} \sigma_{kk}
\end{align*}$$

(5)

Assuming SMA to be an isotropic material, $\gamma_{12} = 2\varepsilon_{ij}$ is the shear strain, $\tau_{12} = 2\sigma_{ij}$ is shear stress, when the SMA is in pure shear condition $\tau_{12} = \tau$. Therefore the three-dimensional micromechanical constitutive equation for SMA in pure shear mode is

$$\gamma = \frac{\tau}{G(\xi)} + \frac{\sqrt{6}}{2} \varepsilon_t \eta$$

(6)

the SMA shear modulus $G(\xi)$ at different martensitic volume fractions is

$$G(\xi) = \frac{E}{2(1 + \nu)} = G_A + (G_M - G_A)\xi$$

(7)

Where $E$ and $\nu$ are the elastic modulus and Poisson’s ratio of SMA, respectively.

From equation (6), it can be concluded that corresponding to a certain initial state, the three-dimensional fine constitutive equation of SMA under pure shear mode can be expressed as

$$\gamma = \frac{\tau}{G(\xi)} + \frac{\tau_0}{G(\xi_0)} + \frac{\sqrt{6}}{2} \varepsilon_t (\eta - \eta_0) + \gamma_0$$

(8)

**Thermodynamic model of electrothermal SMA spring**

Enameded wire as a heat source to activate the shape memory effect of SMA, with one part of its Joule heat being transmitted to the SMA and the other part to the air around the electrothermal SMA spring, which is thermally exchanged as shown in Figure 2. In order to avoid the heat transfer from the enameded wire into the external environment causing a decrease in electrothermal performance, a polyurethane thermal insulation is
The SMA wire can be expressed as

\[ \rho_{s} \frac{dV_{s}}{dt} = H \frac{dV_{s}}{dt} \]

\[ m_{s}c_{s} \frac{dT_{s}}{dt} + \frac{S_{s}}{R_{s}}(T_{s} - T_{c}) = H \frac{dT_{s}}{dt} \]

\[ m_{c}c_{c} \frac{dT_{c}}{dt} + \frac{S_{c}}{R_{c}}(T_{c} - T_{s}) + S_{c}h_{ca}(T_{c} - T_{a}) = I^2 R_{c} \]

Where \( m_{s} \) and \( m_{c} \) is the quality of SMA and enameled wire, \( c_{s} \) and \( c_{c} \) Specific heat capacity of SMA and enameled wire, \( T_{s} \) and \( T_{c} \) is the temperature of SMA and enameled wire, \( S_{s} \) and \( S_{c} \) is the heat transfer surface area of SMA and enameled wire, \( H \) is the phase transition latent heat of SMA, \( h_{ca} \) is the convective heat transfer coefficient between the enameled wire and air, \( I \) is the current leading into the enameled wire, \( R_{c} \) is the resistance of the enameled wire and \( \xi \) represents the volume fraction of martensite in SMA.

The rate of change of the martensitic volume fraction \( \xi \) during the inverse martensitic transformation is

\[ \frac{d\xi}{dt} = -\frac{\pi dT_{s}}{2(A_{f} - A_{s})} \sin \left( \frac{\pi}{A_{f} - A_{s}} \right) \]

The temperature of enameled wire increases gradually after current is applied, and the relationship between its resistance \( R_{c} \) and temperature \( T_{c} \) can be approximately expressed as follows

\[ R_{c} = \rho_{0}(1 + \alpha T_{c}) l_{c} \]

Where \( \rho \) is the electrical resistivity of the enameled wire at 0°C, \( \alpha \) is the temperature coefficient, \( l_{c} \) is the total length of the enameled wire, and \( S \) is the cross-sectional area of the lacquered wire.

**Thermomechanical properties of electrothermal SMA spring**

During electrothermal SMA spring loading, the direction of force follows the axis of the spring, so the SMA wire is always subjected to pure shear load. When \( x \) is the distance from the center to the outer edge of the section, the linear shear strain of the SMA wire section is

\[ \gamma = \theta xx \in (0, r) \]

Where \( \theta \) is the twist angle of the SMA wire, \( r \) is the radius of the SMA wire.

The axial deformation of the SMA spring can be expressed as

\[ f = l - l_{0} = 2\pi n \theta r^{2} = \frac{\pi n \theta D^{2}}{2} \]

The spring geometry equation shown in equation (13) and the SMA pure shear constitutive model shown in equation (8) indicate that the relationship between the torsion angle of the SMA wire and the shear stress in the section is

\[ \gamma_{s} = \frac{\theta d}{2} = \frac{\tau}{G(\xi)} + \frac{\tau_{0}}{G(\xi_{0})} + \frac{\sqrt{6}}{2} \epsilon_{2}(\eta - \eta_{0}) + \gamma_{0} \]

Figure 2. Schematic diagram of electrothermal SMA spring thermal exchange.

placed on the outer ring, and to simplify the complexity of the analytical model, the thermal insulation is assumed to be completely adiabatic. Meanwhile, in order to maintain the structural stability of the electrothermal SMA spring, the inner ring of the spring is set on a cylinder, which is also insulated to avoid heat loss.

The process of Joule heat transfer from enameled wire to SMA includes three forms: heat radiation, heat conduction and heat convection. But electrothermal SMA spring during thermal transfer, the ultimate operating temperature of enameled wire is below 150°C and at the same time the austenite finish transition temperature \( A_{f} \) of SMA is also much lower than this temperature, so less heat is delivered by thermal radiation at this temperature, and the heat exchange between enameled wire and SMA mainly relies on the heat conduction from the contact surface between enameled wire and SMA; convective heat transfer from the gap between enameled wire and SMA. The contact area between enameled wire and SMA wire is line-to-surface contact, only a small amount of heat is transferred through the contact between enameled wire and SMA, so the heat generated by enameled wire is mainly exchanged by heat convection. The air gap thermal resistance \( R_{k} \) formed between the enameled wire and the SMA wire can be expressed as

\[ R_{k} = \int_{0}^{r} \frac{r - \sqrt{r^{2} - l^{2}}}{\lambda r^{2}} dl \]
Subsequently, equation (14) is brought into equation (15), and the axial deformation \( f \) of electrothermal SMA helical spring is

\[
f = \frac{\pi n D^2}{d} \left( \frac{\tau}{G(\xi)} + \frac{\tau_0}{G(\xi_0)} + \frac{\sqrt{6}}{2} \varepsilon_L (\eta - \eta_0) + \gamma_0 \right)
\]

(16)

Where \( \tau \) is the shear stress of the cross section of SMA wire, and the shear stress of spring can be expressed as

\[
\tau = k \frac{8FD}{\pi d^3}
\]

(17)

Where \( F \) is the axial load on the spring and the stress modification factor \( K \) can be written as follows

\[
k = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}
\]

(18)

Where \( C \) is the helical spring winding ratio (\( C = D/d \)).

Combined with equations (15) and (16), it is concluded that the axial displacement \( f \) of electrothermal SMA spring under axial load \( F \) is

\[
f = k \frac{8nFD^3}{d^4G(\xi)} - \frac{\pi n D^2}{d} \beta
\]

(19)

The intermediate variable \( \beta \) can be written as

\[
\beta = \frac{\tau_0}{G(\xi_0)} + \frac{\sqrt{6}}{2} \varepsilon_L (\eta - \eta_0) + \gamma_0
\]

Based on equation (19), it can be concluded that the recovery force \( F_r \), generated by the electrothermal SMA spring with axial displacement \( f \) is

\[
F_r = \frac{d^4G(\xi)}{8kD^3} f + \frac{\pi d^3 G(\xi)}{8kD} \beta
\]

(20)

The stiffness coefficient \( k_s \) of electrothermal SMA spring derived by differentiation in equation (20) can be written as follows

\[
k_s = \frac{dF_r}{df} = \frac{d^4G(\xi)}{8kD^3}
\]

(21)

Shear modulus \( G(\xi) \) is a function of temperature, so the electrothermal SMA spring have different stiffness coefficients \( k_s \) at different temperatures.

**Numerical simulation of electrothermal SMA spring**

**Material properties and analysis setup**

When the enameled wire is energized to produce Joule heat to raise the SMA temperature to \( \Delta_s \), the SMA produces a shape memory effect, but the SMA spring has different actuation characteristics under different restraint conditions. As shown in Figure 3(a), only fixed end restraint is applied to the bottom surface of the spring, and no restraint is applied to the top surface. In this case, the SMA spring will extend upwards when heated to produce displacement, which can be used as a displacement actuator. As shown in Figure 3(b), a fixed end restraint is applied at the bottom of the spring and a force is applied at the top, in which case the SMA spring can act as a force actuator.

The spatial structure of SMA spring is too complex to solve by finite element method (FEM), and the surface heat flux \( \phi_q \) of SMA wire can be simplified as

\[
\phi_q = \frac{I^2 R_0 (1 + \alpha T_c) l_c}{S_{sma}}
\]

(22)

Where \( S_{sma} \) is the effective convective heat transfer surface area of SMA silk, and the effective convective heat transfer surface area of SMA wire can be approximately expressed as the surface area after the spirals are spread out into a straight line, so the effective convective heat transfer surface area \( S_{sma} \) can be expressed as

\[
S_{sma} = \pi d - \theta D \frac{1}{2 \cos \alpha}
\]

(23)

Where \( \theta \) is the polar angle of the helical spring.

Electrothermal SMA spring can be used as different types of actuators, but the process by which SMA obtains shape memory characteristics is consistent. Therefore, the driving characteristics of electrothermal SMA spring as different types of actuators can be analyzed by applying different constraints or loads when SMA recovers non-linear strain. For the stability of the SMA spring during loading and unloading, the bottom of the spring is a fixed constraint. Meanwhile, the
Table 1. SMA (Ni51Ti49 (at. %)) material parameters.

| Parameter | Value       | Parameter | Value       |
|-----------|-------------|-----------|-------------|
| $E_a$     | 200 (GPa)   | $C_A$     | 14 (MPa · K) |
| $E_m$     | 70 (GPa)    | $C_M$     | 8 (MPa · K)  |
| $\nu_A = \nu_M$ | 0.33 | $A_s$     | 313 (K)     |
| $\alpha_A = \alpha_M$ | 10^{-4}(K^{-1}) | $A_p$     | 334 (K)     |
| $\delta_l$ | 6.8 (%)    | $A_f$     | 354 (K)     |
| $\sigma_A^0$ | 100 (MPa) | $M_s$     | 305 (K)     |
| $\sigma_M^0$ | 170 (MPa) | $M_p$     | 300 (K)     |
| $\rho$    | 6500 (kg/mm³) | $M_f$     | 296 (K)     |

The mechanical properties of SMA are defined by subroutine. The material parameters of SMA (Ni51Ti49 (at. %)) are shown in Table 1.

**FEM analysis results**

The stress distribution of an electrothermal SMA spring as a displacement actuator is shown in Figure 4. The spring in the initial state is shown in Figure 4(a). In the loading phase, the spring is compressed, and the stress at the ends of the spring is small, and the stress in the rest of the spring is 66.3 MPa at its maximum value, and the stress distribution is shown in Figure 4(b). In the unloading stage, the spring compression is reduced by a part, and the SMA spring stress is reduced to 0 MPa, and the stress distribution is shown in Figure 4(c). In the heating stage, the spring shape returns to the initial state shown in Figure 4(a), and the stress distribution is as shown in Figure 4(d).

The complete process of SMA generating shape memory effect is composed of Figure 4(a) to (d). In this process, the electrothermal SMA helical spring has a fixed constraint at one end only, and the compressed spring is restored to its initial form by heating, so the process mainly simulates the driving characteristics of the electrothermal SMA coil spring when it as a displacement actuator.

The stress distribution of the electrothermal SMA spring as a pressure actuator is shown in Figure 5. The initial state, loading state, and unloading stage are consistent with the stress distribution of the displacement actuator. In the heating stage, the temperature increases SMA produces shape memory effect, but the two ends of the electrothermal SMA spring are fixed constraint and force constraint respectively, the electrothermal SMA helical spring can only produce a reaction force of 71 N on the constrained surface and the stress increases to 57.5 MPa, as shown in Figure 5(d). The thermomechanical characteristics of the electrothermal SMA spring as a pressure actuator are completely simulated by Figure 5(a) to (d).

The stress-strain curve during the shape memory effect generated by the electrothermal SMA spring displacement actuator simulated by the FEM is shown in Figure 6. The nonlinear stress caused by the external force load ($F = 80$ N) during the loading phase increases to 66.3 MPa, at which point the SMA generates a nonlinear strain of 7.6%. After completing the loading, the external load is withdrawn and the SMA spring is cooled ($T = 300$ K), and the SMA enters the unloading phase, where the recoverable nonlinear strain is transformed into residual strain during the stress reduction to 0 MPa, at which time the residual strain is 6.8% ($\varepsilon = \varepsilon_L$). Finally, by heating the SMA temperature to 370 K, the SMA transforms from non-twin martensite to austenite, the shape memory factor...
The stress-strain curve during the shape memory effect generated by the electrothermal SMA spring pressure actuator simulated by the FEM is shown in Figure 7. The stress-strain curve of the pressure actuator and the displacement actuator during the loading and unloading phases of SMA are the same. However, there is a force constraint at one end of the SMA spring of the pressure actuator, and the residual strain cannot be recovered to recover during the heating phase, so the SMA spring generates a reaction force of 71 N on the constrained surface.

Experiments and analysis

**Experimental setup**

Thermal actuation performance test device of electrothermal SMA helical spring as shown in Figure 8, the device can test the driving characteristics when electrothermal SMA spring is a pressure actuator. The main components involved in the test include: digital display force dynamometer (SH-200N), digital display thermometer (TES-1310), manual base, digital display scale (HLB), rotating resistance box, digital DC power supply (RXN-3010D), and digital multimeter (DT-9927).

The beaker sets up a layer of thermal insulation (polyurethane foam) and the top of the beaker is fitted with a removable thermal insulation cover (polyurethane foam). To keep the SMA spring stable when tested, the bottom of the beaker was bonded with a guide bar that could install the SMA spring. Upon powering up heating on an electrothermal SMA spring, which will cause high temperature and large strain on the SMA surface, the thermocouples bonded on the SMA surface were prone to fall off resulting in inaccurate test data. Therefore, to accurately measure the temperature of SMA, the electrothermal SMA spring was filled with hydraulic oil (specific heat: 1.7 KJ/(kg°C), heat transfer rate: 0.2W/(m°C)) between the inner wall of the beaker. Compared with the air hydraulic oil has better heat transfer, the hydraulic oil effectively transfers heat from SMA to thermocouple through heat conduction, thus effectively improving the precision of temperature measurement on SMA.

When the electrothermal SMA spring as a pressure actuator, the testing device mainly measured the relationship between the electrothermal SMA spring recovery force $F_r$, current as well as the SMA temperature. The beaker equipped with an electrothermal SMA helix spring was injected with hydraulic oil at normal temperature (27°C) before the test started, then the top of the beaker was closed with an insulated cover. Then, the height of the force dynamometer was adjusted by rotating the handwheel so that the probe contacts the top surface of the spring without pressure, in which case the spring can generate the maximum recovery force $F_r$. Currents ($I = 3A$) was applied to the electrothermal SMA spring by power supply, and the experimental data were recorded.

When an electrothermal SMA spring as a force-displacement actuator, the test setup measures the relationship between the electrothermal SMA spring recovery force $F_r$, axial deformation $f$, current, and SMA temperature. The preparation before the test starts is the same as when testing the electrothermal SMA coil spring as a pressure actuator. During the test, the probe height of the force gauge was adjusted by rotating the handwheel on the base to produce different distances ($f = 1–11$ mm) between the probe and the top surface of the electrothermal SMA spiral spring. 3A current was applied to the electrothermal SMA spiral spring using a DC power supply, the temperature of the SMA surface was recorded by a digital thermometer, and the
recovery force $F_r$ generated by the electrothermal SMA spiral spring was recorded by a force gauge. Finally, the displacement data generated by the digital display scale, the response force data generated by the force gauge, and the temperature data generated by the digital display thermometer are read and analyzed, and stored by the computer.

The test device for thermal-displacement performance of electrothermal SMA spring is shown in Figure 8, which can test the driving characteristics of electrothermal SMA spring as displacement actuator. The main components involved in the test device include: TES-1310 digital thermometer, SK-199 digital dial indicator, magnetic base, rotary resistance box, DC power supply and DT-9927 digital multimeter. To accurately measure the surface temperature of SMA (similar to the test device shown in Figure 9), the electric SMA spring was submerged in an insulated beaker filled with hydraulic oil. At the same time, to reduce heat loss, a removable thermal cover was installed on the top of the heat insulation beaker, and polyurethane foam was installed between the bottom of the beaker and the metal base to insulate the heat.

This device mainly measures the relationship between axial spring deformation $f$, current and SMA temperature. A beaker equipped with an electrothermal SMA spring was injected with hydraulic oil at a normal temperature ($27^\circ C$) before the test started. During the test, adjust the magnetic base so that the digital dial indicator contacts the top surface of the electrothermal SMA spring without pressure. Current ($I = 3A$) was applied to the electrothermal SMA helical spring, and the experimental data were recorded.
Force driving properties

The temperature dependence of the recovery force derived from the electrothermal SMA coil spring thermal drive performance test setup is shown in Figure 10. The force-temperature curve of the electrothermal SMA spring is highly nonlinear at 3A current, and there are two obvious inflection points in the curve, which are closer to $A_s$ and $A_r$, respectively.

The experimental results of the finite unit method with the temperature curve and the recovery force when the loading current is 3A are the same. When the temperature of SMA reaches $40^\circ C$, the spring recovery force $F_r$ is 1.6 N when the temperature reaches $A_s$, so the recovery force is mainly generated by thermal expansion. When the SMA temperature reaches $81^\circ C$, the temperature reaches $A_r$, and the spring recovery force $F_r$ is 67.3 N, so the recovery force is mainly generated by the shape memory effect. When the SMA temperature exceeds $81^\circ C$, the recovery force is mainly generated by thermal expansion, and the maximum recovery force $F_r$ generated by the electrothermal SMA spring at $100^\circ C$ is 70.2 N.

By estimating the gradient of the electrothermal SMA helical spring force-temperature curve, the SMA recovery rate at 3A current is obtained as shown in Figure 11. As the temperature of the SMA spring increases, there is a large wave peak in the recovery rate curve, and the temperatures at the beginning and end of the wave peak are close to the SMA $A_s$ and $A_r$. The SMA recovery rate is at its maximum when the SMA temperature reaches $49^\circ C$. The SMA recovery rate curve obtained from the FEM simulation is basically consistent with the experimental data.

The surface temperature of the electrically heated SMA spring under a current loading of 3A versus the current loading time is shown in Figure 12. At a loading current of 3A, the Joule heat generated by the enameled wire can bring the SMA to the $A_r$, and the time required for the electrothermal SMA spring to reach $100^\circ C$ in this case is 168 s.

The thermodynamic equilibrium equation for the electrothermal SMA spring shown in equation (10) shows that the temperature is linearly related to the time when a constant current is loaded, so the SMA temperature at different currents simulated by the FEM is highly linearly related to the current loading time. Due to the simplification of the ambient heat exchange conditions set by the FEM and the existence of errors in the test environment, the temperature derived from the test is close to a linear relationship with the loading time. At SMA temperatures in the range of $40^\circ C$–$80^\circ C$, the test data are overall smaller than the simulated results, but the simulation results of the FEM are in general agreement with the test data.
Displacement driving characteristics

The relationship between displacement and temperature at 3A current obtained from the electrothermal SMA spring thermal-displacement test device is shown in Figure 13. When current \( I \) was 3A, the displacement-temperature curve is basically the same as the change pattern of the curve of the force-temperature curve, and there are two more obvious inflection points of the displacement-temperature curve, and the temperatures corresponding to the two inflection points are closer to \( A_s \) and \( A_f \), respectively. When the SMA temperature reaches 40°C, at which temperature reaches \( A_s \), the displacement of the spring is 0.1 mm, so the displacement is mainly generated by thermal expansion. When the SMA temperature reaches 81°C, which is the temperature reaches \( A_f \), the spring displacement is 7.4 mm, and the return force is mainly generated by the shape memory effect at this time. When the SMA temperature exceeds 81°C, at which point the recovery force is mainly generated by thermal expansion, and the maximum displacement generated by the electrothermal SMA spring at 100°C is 7.7 mm.

Due to the measurement errors in the experiments and the simplification of the theoretical model, the theoretical values of the displacement-temperature curves shown in Figure 13 are greater than the experimental values in the range of 25°C–55°C, and the theoretical values of the displacement-temperature curves are less than the experimental values in the range of 55°C–100°C. However, the axial displacement-temperature curves of the electrothermal SMA coil spring from the finite unit method simulation are consistent with the experimental values.

By estimating the gradient of the displacement versus temperature curve, the displacement recovery rate of the electrothermal SMA spring is obtained as shown in Figure 14. When the SMA temperature does not reach \( A_s \), the displacement recovery rate of the SMA approaches 0. As the temperature of the SMA spring increases, there is a large wave crest in the recovery rate curve, which starts and ends at temperatures closer to \( A_s \) and \( A_f \) of the SMA, and the SMA recovery rate is at its maximum when the SMA temperature reaches 57°C.

Force-displacement driving characteristics

When the electrothermal SMA spring is used in smart actuators, it is necessary not only to output pressure as a force actuator or displacement as a displacement actuator, but also to output displacement followed by a recovery force in specific application scenarios. The force-displacement response curve of an electrothermal SMA spring actuator at 100°C, as obtained by the force-displacement test device, is shown in Figure 15.
As shown in Figure 15, the height $H$ of the force dynamometer is the height between the bottom surface of the force dynamometer and the reference surface of the test platform, the free height of the electrothermal SMA spring $L = 19$ mm, and the displacement $D$ is the difference between the height of the force dynamometer $H$ and the free height of the spring $L$. In this test, the electrothermal SMA spring needs to output displacement so that the top surface of the spring is in contact with the force dynamometer and then output the recovery force, and the two endpoint values of the force-displacement response curve belong to two special states. This state is consistent with the characteristics of the electric SMA spring as a force actuator. The data at the right endpoint shows that the force dynamometer is not in contact with the top surface of the spring at all times, and that only displacement is produced during the process of temperature reaching 100°C. This state is consistent with the characteristics of the electrothermal SMA spring as a displacement actuator. As the output displacement increases, the recovery force generated by the electrothermal SMA spring actuator gradually decreases from the maximum value to 0 when the output displacement reaches the maximum value. Therefore, when the electrothermal SMA spring is used as a force-displacement actuator, the recovery force $F_r$ is linearly and negatively correlated with the height $H$ of the force dynamometer, and the linear fitting curve can be expressed by the function ($F_r = -5.9H + 180$).

From equation (20), it can be seen that when the SMA temperature is constant, the return force $F_r$ of the electrothermal SMA spring is linearly related to the displacement $f$. Combining the data in Figures 10 and 13, the force-displacement response curves of the electrothermal SMA spring at different temperatures are shown in Figure 16. The slope of the tested force-displacement response curve decreases with increasing temperature, which is expressed as the stiffness coefficient $k_s$, shown in equation (21), and the tested stiffness coefficients $k_s$ are 51.8, 15.5, 10.2, 9.9, 9.4, and 9.3 for temperatures of 50°C, 60°C, 70°C, 80°C, 90°C, and 100°C, respectively. The dotted lines in Figure 16 represent the force-displacement response curves of electrothermal SMA springs at different temperatures obtained by the FEM. Compared with the measured data, the force-displacement response curves obtained by the FEM show a directional deviation with a reduced slope in each temperature range. When the temperatures are 50, 60, 70, 80, 90, and 100, respectively, the stiffness coefficient $k_s$ obtained by the FEM are 40.1, 14.1, 10.67, 9.7, 9.3, and 9.1 respectively. The stiffness coefficient $k_s$ obtained by the FEM is in general agreement with the experimental data.

### Conclusions

In this paper, a new type of electrothermal SMA helical spring actuator is proposed, which generates Joule thermal excitation SMA by enameled wire to generate shape memory effect, continuously and stably controls the phase transformation process of SMA by controlling current. The actuator has a compact structure, high energy density, and fast response. The thermomechanical characteristics of electrothermal SMA spring actuator were derived by theoretical and FEM analysis. The numerical simulation of the electrothermal SMA coil spring actuator was realized by the FEM, and the thermo-mechanical characteristics of the electrothermal SMA coil spring actuator obtained by the FEM are consistent with the experimental results.

The results show that the electrothermal SMA coil spring produces a nonlinear response force or nonlinear displacement with increasing temperature when used as an actuator. The proposed theoretical model can be used for the design and experimental analysis of electrothermal SMA helical spring actuators. The experimental temperature-force response and temperature-displacement response, as well as the force-displacement response at different temperatures, provide references for the design and fabrication of electrothermal shape memory alloy coil spring actuators.

### Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.
Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The authors would like to gratefully acknowledge the National Natural Science Foundation of China (NO. 51875068, NO. 51905060).

ORCID iD
Yang Xiong https://orcid.org/0000-0003-3002-0646

References
1. Spaggiari A, Castagnetti D, Golinelli N, et al. Smart materials: properties, design and mechatronic applications. Proc Inst Mech Eng, Part L: J Mater Des Appl 2019; 233: 734–762.
2. Mohd Jani J, Leary M, Subic A, et al. A review of shape memory alloy research, applications and opportunities. Mater Des 2014; 56: 1078–1113.
3. Yuan H, Fauroux J, Chapelle F, et al. A review of rotary actuators based on shape memory alloys. J Intell Mater Syst Struct 2017; 28: 1863–1885.
4. Paiva A and Savi MA. An overview of constitutive models for shape memory alloys. Math Probl Eng 2006; 2006: 1–30.
5. Franco V, Blázquez JS, Ipus JJ, et al. Magnetocaloric effect: from materials research to refrigeration devices. Prog Mater Sci 2018; 93: 112–232.
6. Xu L, Baxevanis T and Lagoudas DC. A three-dimensional constitutive model for the martensitic transformation in polycrystalline shape memory alloys under large deformation. Smart Mater Struct 2019; 28: 074004.
7. Praveen Kumar G, Louis Commillus A and Cui F. A finite element simulation method to evaluate the crimpability of curved stents. Med Eng Phys 2019; 74: 162–165.
8. Morgan NB. Medical shape memory alloy applications—the market and its products. Mater Sci Eng A 2004; 378: 16–23.
9. Lu Y, Xie Z, Wang J, et al. A novel design of a parallel gripper actuated by a large-stroke shape memory alloy actuator. Int J Mech Sci 2019; 159: 74–80.
10. Henke M and Gerlach G. Mono- and bi-stable planar actuators for stiffness control driven by shape memory alloys. Sens Actuators A Phys 2016; 238: 95–103.
11. Seok S, Onal CD, Cho K-J, et al. Meshworm: a peristaltic soft robot with antagonistic nickel titanium coil actuators. IEEE/ASME Trans Mechatron 2013; 18: 1485–1497.
12. Santiago JJDM, Simões JDB and de Araújo CJ. Thermo-mechanical characterization of superelastic Ni-Ti SMA helical extension springs manufactured by investment casting. Mater Res 2019; 22: 22.
13. Weirich A and Kuhlenkötter B. Applicability of shape memory alloys in aircraft interiors. Actuators 2019; 8: 61.
14. de Sousa VC and De Marqui Junior C. Airfoil-based piezoelectric energy harvesting by exploiting the pseudelastic hysteresis of shape memory alloy springs. Smart Mater Struct 2015; 24: 125014.
15. Mohd Jani J, Leary M and Subic A. Designing shape memory alloy linear actuators: a review. J Intell Mater Syst Struct 2017; 28: 1699–1718.
16. Sofla AY, Meguid SA, Tan KT, et al. Shape morphing of aircraft wing: status and challenges. Mater Des 2010; 31: 1284–1292.
17. Andronov IN, Demina MY and Polugrudova LS. Method for designing springs using materials with shape memory as the actuators of power units. J Machinery Manuf Reliab 2018; 47: 196–204.
18. Yi H. Simulation of shape memory alloy (SMA)-bias spring actuation for self-shaping architecture: investigation of parametric sensitivity. Materials 2020; 13: 2485.
19. Ma J, Huang H and Huang J. Characteristics analysis and testing of SMA spring actuator. Adv Mater Sci Eng 2013; 2013: 1–7.
20. Gédouin P-A, Pino L, Arbab Chirani S, et al. R-phase shape memory alloy helical spring based actuators: modeling and experiments. Sens Actuators A Phys 2019; 289: 65–76.
21. Stachowiak D and Kurzawa M. A computational and experimental study of shape memory alloy spring actuator. Prz Elektrotechn 2019; 1: 31–34.
22. Wang J, Zhang W, Zhu J, et al. Finite element simulation of thermomechanical training on functional stability of shape memory alloy wave spring actuator. J Intell Mater Syst Struct 2019; 30: 1239–1251.
23. Zhou B, Liu Y, Leng J, et al. A macro-mechanical constitutive model of shape memory alloys. Sci China Phys Mech Astron 2009; 52: 1382–1391.
24. Jafarzadeh S and Kadkhodaei M. Finite element simulation of ferromagnetic shape memory alloys using a revised constitutive model. J Intell Mater Syst Struct 2017; 28: 2853–2871.
25. Xu L, Solomou A, Baxevanis T, et al. Finite strain constitutive modeling for shape memory alloys considering transformation-induced plasticity and two-way shape memory effect. Int J Solids Struct 2021; 221: 42–59.
26. Erbao D, Min X, Xin LY, et al. Modeling and experimental study on a new electric-thermal heating method for shape memory alloy wire actuators. China Mech Eng 2010; 21: 2857–2861.