A Coherent Photonic Crossbar for Scalable Universal Linear Optics

George Giamougiannis, Apostolos Tsakyridis, Yangjin Ma, Angelina Totović, Miltiadis Moralis-Pegios, David Lazovsky, and Nikos Pleros

Abstract—We demonstrate a novel interferometric coherent photonic crossbar architecture (Xbar) that can realize any tensor operator and allows for total loss-induced fidelity restoration, offering at the same time significant dimension scalability credentials compared to respective state-of-the-art solutions. The proposed Xbar layout demarcates from the prevalent photonic schemes that rely on the implementation of intended matrices factorized via the Singular Value Decomposition (SVD); instead, it harnesses the power of interference within a novel design where each matrix element can be mapped by one-to-one correspondence to a single, designated Xbar node, bringing down the number of programming steps to only one. In this paper, we present the theoretical foundations of the Xbar, proving that its insertion losses do not scale with the node losses as opposed to the exponential scaling witnessed by the SVD-based counterparts. This leads to a matrix design with significantly lower overall insertion losses and improved scalability potential compared to SVD-based schemes, allowing for the employment of alternative node technologies with lower energy consumption and higher operational speed credentials. Finally, we theoretically validate that the proposed Xbar architecture is the first linear operator that supports fidelity restoration, outperforming SVD schemes in loss- and phase-error fidelity performance.

Index Terms—Linear algebra, linear optics, linear transformations, multiport interferometer, optical signal processing.

I. INTRODUCTION

There has been an increasing interest in linear optics during the last years, aiming at deployment of programmable universal multiport interferometers capable of performing any linear transformation on any given set of input modes [1], [2], [3]. In this effort, the ultimate goal is the experimental realization of any real- or complex-valued matrix operator solely via optical elements, allowing for the exploitation of all speed-, size- and energy-related advantages of optics in the rapidly advancing fields of neuromorphic photonics [4], [5], [6], [7], microwave photonics [8], [9], machine learning [3], [10] and quantum photonics [2], [11] among other. Optical linear transformations have been so far the stronghold of matrix designs that rely, mainly, on the singular value decomposition (SVD) technique, as has been first proposed by Miller [1]. This scheme extends along the factorization of any matrix \( W \) in the form of \( W = U \Sigma V^\dagger \), where \( U \) and \( V \) are unitary matrices implemented in the optical domain and \( \Sigma \) is a diagonal matrix that can be optically realized via a column of variable attenuators interleaved between the two unitary matrix layouts. As such, the problem of enabling any matrix representation in the optical domain via the SVD approach turns into the constituent problem of optically realizing any unitary matrix. The field of optical unitary matrix operators was pioneered in the seminal work of Reck et. al. [12], where a unitary matrix decomposition scheme based on elementary \( 2 \times 2 \) unitary elements was adopted. With the \( 2 \times 2 \) unitary element inherently offered by a single lossless \( 2 \times 2 \) Mach-Zehnder interferometer (MZI) in the optical domain, along with two phase shifters (PSs), Reck et. al. proved that any \( N \times N \) unitary operator can be implemented via a triangular mesh of \( N(N-1)/2 \) MZI nodes. This was, recently, validated also experimentally in a silicon-integrated programmable nanophotonic processor (PNP) that was successfully employed in an all-optical neural network for vowel recognition [3] and a silicon optical neural chip for the implementation of complex valued neural networks [7], highlighting the advantages of linear optics in the rapidly emerging field of neuromorphic photonics. Moving towards an improved unitary matrix scheme, Clements et. al. [13] recently presented a more compact rectangular mesh architecture that requires the same number of \( 2 \times 2 \) MZI nodes but yields half the optical depth compared to the Reck design, supporting in this way a more loss-balanced and loss-error-tolerant design.

Both Reck and Clements architectures have been based on the assumption of lossless \( 2 \times 2 \) MZIs, as their elementary constituent nodes, towards realizing unitary matrix operators. Yet, the reality is that non-ideal optical elements will be utilized when these architectures will be transferred into experimental layouts, implying that lossy elementary MZI nodes and eventually also imperfect phase conditions will definitely affect the device and, consequently, the system performance. Given that both Reck and Clements designs rely on cascaded stages of \( 2 \times 2 \) MZI...
nodes, their overall insertion loss (IL) will scale exponentially with the MZI losses, eventually limiting the circuit size due to the loss build-up. Moreover, the employment of lossy optical MZI nodes yields a loss-imbalanced configuration that translates directly into reduced fidelity metrics. Although the Clements layout is inherently more loss-resistant than the Reck design, both schemes can neither sustain a 100% fidelity nor practically support any fidelity restoration mechanism. This indicates that both designs come with the demand of ultra-low-loss MZI node technology, in order to retain reasonable ILs and high fidelity performance, forming one of the main reasons for the use of low-loss thermo-optic MZI elements in the experimental deployments reported so far [3], [7].

These effects become even more pronounced when any real-and/or complex-valued linear transformation via the SVD is targeted, where two concatenated unitary matrix layouts are required. Fidelity degradation and high ILs will form an important drawback especially when higher matrix dimensions are targeted, restricting even more the employment of alternative MZI PS technologies that may eventually equip the matrix operator with additional energy, functionality or speed advantages. For example, both Clements and Reck designs can hardly cope with the losses of emerging, yet powerful, photonic technologies, like phase-change materials (PCMs) [14], [15] and electro-optic phase shifting structures [16], which typically have IL values beyond 1 dB per element but offer significant benefits in terms of energy consumption and speed, compared to their thermo-optic PS counterparts.

In this paper, we propose a novel interferometric coherent photonic crossbar architecture (Xbar) that can realize any real-and/or complex-valued matrix operator and outperforms SVD factorization-based schemes in several key aspects. First, it allows for natural one-to-one mapping of each matrix element into the designated Xbar node, avoiding cascaded nodes for a single matrix element representation and yielding a static dependence of the total ILs on the individual node losses. This significantly reduces ILs compared to SVD-based layouts, with the loss benefits becoming even more pronounced when a higher loss node technology or higher matrix dimensions are targeted. Second, it follows a modular design that can be tailored either for ensuring a loss-balanced performance among all columns or for associating every matrix column with a constant loss factor, leading in this way to a completely restorable fidelity performance. This outlines a highly loss-tolerant design that can always precisely implement the targeted linear transformation, yet being flexible in terms of MZI node technology, as it can use even higher-loss PS technologies for enriching the matrix deployment with energy, speed and/or functionality benefits. Third, its non-cascaded, one-to-one mapping architecture makes the design significantly more robust to fabrication errors caused by phase mismatches, restricting the error only to a single matrix element, as opposed to error being accumulated to the multiple matrix elements due to non-bijective correspondence between matrix element and photonic node element in cascaded architectures. Last but not least, it can be programmed in just a single-step compared to the N(N-1)/2, taking advantage of its modular design that exploits decoupled Xbar nodes for matrix element representation. The proposed Xbar architecture effectively extends our previous work on coherent dual-IQ modulator-based vector dot-product engine [17], [18] towards a full vector-by-matrix multiplication interferometric layout, employing multiple parallel columns within a split-and-recombine configuration.

II. BACKGROUND

The prevailing method for constructing any real and/or complex-valued matrix operator in the optical domain is based on the SVD factorization technique. This factorization relies on the decomposition of the matrix \( W \) into a product of the form \( U \Sigma V^\dagger \), where \( \Sigma \) is a diagonal matrix and \( U \) and \( V \) are two unitary matrices, with \( V^\dagger \) being the conjugate transpose of \( V \). To this end, the optical deployment of the SVD-based scheme can take advantage of the unitary matrix operator implementations that have been so far proposed in the optical domain [12], [13] in order to construct the \( U \) and \( V^\dagger \) matrices, incorporating also an additional column of optical attenuator elements in between the two photonic unitary matrix designs for realizing the \( \Sigma \) diagonal matrix. Given that the Clements unitary matrix design has been proven, so far, to be the optimal layout in terms of optical depth and loss-induced fidelity [13], we consider as the optimal SVD-based photonic linear operator the layout that adopts the Clements unitary matrix architecture for both its \( U \) and \( V^\dagger \) operators and we will refer to it as the SVD-Clements architecture. This implies that an \( N \times N \) optical SVD-Clements design will require \( N(N-1)/2 \) programming steps for its configuration [19], since the two constituent \( N \times N \) Clements unitary matrices can be programmed in parallel within \( N(N-1)/2 \) steps.

Fig. 1(a) illustrates an example of an \( N \times N \) SVD-Clements architecture for \( N = 6 \), when operating on an N-element input vector \( X \). The values of the input vector, \( x_r \), with \( r = 1, 2, \ldots, N \) denoting the row index, are imprinted onto the optical beams originating from the incoming optical continuous wave (CW) signal \( E_{in} \), split equally in terms of power by a 1:N optical splitter. Vector by matrix multiplication starts by modulated optical signals, \( x_r E_{in}/N \), entering the \( N \times N \) unitary operator \( V^\dagger \) that relies on the Clements rectangular mesh of \( 2 \times 2 \) MZI-based nodes, with every \( 2 \times 2 \) MZI node comprising two 3 dB couplers and two PSs \( \theta \) and \( \varphi \), as depicted in the yellow inset of Fig. 1(a). After exiting the optical layout of the \( V^\dagger \) unitary operator, the \( N \) parallel optical beams enter the optical diagonal matrix \( \Sigma \) actualized by a single column of \( N \) variable optical attenuators (VOAs). Every attenuator can be easily realized via an MZI element where only one of its inputs connects to the preceding \( V^\dagger \) matrix and only one of its two outputs connects to the succeeding \( U \) matrix, as shown in the dashed-line inset of Fig. 1(a). The \( N \times N \) matrix design, based again on the Clements architecture, employ the layout and basic building blocks that are used also for the \( V^\dagger \) matrix operator. In this way, the \( N \) optical beams emerging at the output waveguides of the \( U \) matrix design yield an \( N \)-element optical output vector \( Y \) with \( Y = U \Sigma V^\dagger X \).

Each of the \( N \times N \) and \( V^\dagger \) unitary operators requires a total number of \( N(N-1)/2 \) MZI nodes and has a maximum optical depth [13] of \( N \), with its shortest path equaling \( \lceil N/2 \rceil \), where \( \lfloor x \rfloor \)
denotes the lower integer bound of $x$. This implies that the entire $N \times N$ SVD-Clements layout will employ a total number of $N^2$ MZI nodes, out of which, $(N-1)$ come from $2 \times 2$ MZI nodes of both $U$ and $V$ matrices added to the $N$ MZI nodes used for the $\Sigma$ diagonal matrix. At the same time, the depth of the shortest optical path of the SVD-Clements design will be $(2\lceil N/2 \rceil + 1)$ and will be named hereinafter as SVD-best-case loss path, while the maximum optical depth will equal $(2N + 1)$ and will be hereinafter referred to as SVD-worst-case loss path, with both the best- and the worst-case loss paths illustrated schematically in Fig. 1(b).

When $2 \times 2$ MZI nodes are considered to be lossless, both $U$ and $V$ matrix implementations yield unitary transformations. Assuming that the input modulators $x_r$ are also lossless and operate at their transparency region, then the overall ILs of the SVD-Clements architecture are solely dictated by the attenuation levels enforced at its $\Sigma$ diagonal matrix, which, in turn, depend on the specific singular values of the targeted matrix. Decoupling the ILs of the overall SVD-Clements architecture from the matrix-specific attenuation values required in the $\Sigma$ column, the MZI nodes that form the $\Sigma$ diagonal matrix are also considered to perform in their transparency region, i.e., with no loss penalty. In this case, the entire SVD-Clements design can be indeed considered as lossless and the output power obtained at each individual output waveguide is exclusively defined by the targeted matrix dimensions through the $1/N$ splitting ratio.

The outcome becomes different when the constituent MZI nodes are not treated as lossless, as will, in fact, be the case in a practical implementation. Let us define as $\ell_{coup} \leq 1$ and $k \leq 1$ the electric field transmission coefficients of a 3 dB coupler and a PS, respectively, where $IL_{coup, dB} = -10 \log_{10}(T_{coup}^2)$ and $IL_{PS, dB} = -10 \log_{10}(k^2)$ denote the corresponding optical power ILs. In each MZI-node, where two 3 dB couplers and two PSs $\theta$, $\varphi$ are employed, the total electric field transmittivity factor is $T_{node} = T_{coup}^2 k^2 \leq 1$ and the total MZI-node insertion loss $IL_{node, dB} = -10 \log_{10}(T_{node}^2)$. Assuming transparent and lossless modulators $x_r$ and maximum transmission conditions that would occur in the special case of a unitary matrix implementation, we can define the overall electric field transmission coefficient between the matrix output and input as $f$, with $f^2$ denoting the power transmission coefficient. Taking into account that the electric field emerging at a single matrix output waveguide will be the coherent sum of multiple electrical fields that will have propagated through different paths within the matrix, $f^2$ will be bounded between the transmittivity of the longest and shortest optical paths within the SVD-Clements architecture respectively, $f^2 \in [\frac{1}{\sqrt{2}}(T_{node}^2)^{(2N+1)} \leq \frac{1}{\sqrt{2}}(T_{node}^2)^{(2}\lceil \frac{N}{2} \rceil + 1)] \iff f^2 \leq \frac{1}{\sqrt{2}}$. The upper bound of $f^2$ corresponds to the optical power transmittivity in the SVD-best-case path loss scenario, where the signal travels through $(2\lceil N/2 \rceil + 1)$ nodes, as depicted in Fig. 1(b)(top), while its lower bound stands for the optical power transmittivity in the SVD-worst-case path loss scenario, where the signal propagates through the maximum number of nodes $(2N + 1)$, as illustrated in Fig. 1(b)(bottom). This indicates that the overall ILs of the SVD-Clements architecture depend exponentially on the MZI-node IL $T_{node}^2$, regardless of the path taken, or, in terms of dB, statically on $IL_{node, dB}$, with only the slope being defined by the taken path. Expressing the total ILs in dB for the two extreme cases yields:

$$IL_{SVD-Clements, bc, dB} = 10 \log_{10}(N) + (2 \left\lceil \frac{N}{2} \right\rceil + 1) IL_{node, dB}$$

for the best-case scenario and

$$IL_{SVD-Clements, wc, dB} = 10 \log_{10}(N) + (2N + 1) IL_{node, dB}$$

for the worst-case scenario.

Besides this exponential scaling, the employment of MZI nodes with non-zero ILs is also expected to degrade the fidelity of the experimentally realized matrix layout, being the result of non-balanced losses through different paths. Each of the constituent unitary matrices relies on the Clements design that has been shown to yield a reduced fidelity performance when MZI nodes of nonzero ILs are used, with fidelity degradation becoming stronger with increasing MZI losses and circuit size [13]. To this end, the employment of the Clements design in the respective SVD-Clements architecture for realizing any linear and not only unitary transformations, as is the ultimate target in the field of linear optics, is expected to yield even stronger loss-induced fidelity degradation since two cascaded unitary layouts along with an additional diagonal matrix $\Sigma$ are required.

### III. Coherent Crossbar as a Linear Operator

Photonic crosstalks have been so far well-known from networking and switching sectors, with their main drawback being their differential path losses, enforcing the use of non-coherent...
multiwavelength input signals with unequal power levels when their utilization as a matrix operator is targeted [20], [21]. Here, we propose a coherent Xbar layout, able to operate using a single wavelength, allowing for wavelength multiplexing to become an additional degree of freedom to be used for other purposes, e.g., throughput boosting through operation parallelization [22], [23], [24]. Our Xbar architecture is an extension and a generalization of our recently reported coherent vector dot-product engine, operating as a coherent linear neuron, relying on dual-IQ modulation cell as its main building block [17].

An $N \times M$ Xbar layout when multiplying an $N$-element input vector with $N = 2^n$ and $n, M \in \mathbb{N}$ is depicted in Fig. 2. It comprises a 1:N front-end splitter that is followed by the input vector $X$ modulation stage and the main block of the Xbar. The latter is formed by $M$ columns, with every column comprising an array of the Xbar nodes followed by a back-end combination-and-forwarding stage (BCFS). An input CW signal $E_{in}$ is injected into the front-end splitter and is then equally split, in terms of power, in $N$ identical constituent beams via $\log_2(N-1)$ cascaded 3 dB Y-junction splitting couplers. These $N$ CW signals are then modulated by their corresponding input amplitude modulator in order to form the optical data signal vector $X = [x_1, x_2, \ldots, x_N]^T$, with its $r$-th element represented by $x_r$, which then enters the Xbar main block via its $r$-th row. Each of these signals is then split via a $\xi_{2:1}$ optical splitter, so that the part of the $r$-th row signal that gets coupled to the $\xi_{2:1}$ splitter output enters the Xbar node $X_r$, and the remaining part of the $r$-th row signal, that gets forwarded to its 3rd column (proportional to $t_{2:1}^2$). This process is repeated at every Xbar node $(r,c)$ via a respective $\xi_{c:2}^2 : t_{c:2}^2$ optical splitter.
until reaching the last column $M$, which includes no splitters, but rather relies on the ($M$-1)-th column splitter $\xi_{M-1}^2 : t_{M-1}^2$ to receive the optical signal from its $t_{M-1}^2$ output. In other words, column $M$ could be described by a virtual splitter with $\xi_M^2 = 1$ and $t_M^2 = 0$.

Each Xbar node at the $r$-th row and $c$-th column of the Xbar comprises an amplitude modulator, with its transfer function being proportional to $w_{r,c}$ and a PS $\varphi_{r,c}$, so that the $r$-th row signal part coupled through the $\xi_r^2$ branch and travelling vertically along the column $c$ gets multiplied by a factor $w_{r,c}e^{j\varphi_{r,c}}$. Taking into account that $w_{r,c}$ is non-negative, this factor is a complex value with a norm of $w_{r,c}$ and a phase $\varphi_{r,c}$, implying that every Xbar node $(r,c)$ can represent any real or complex number at its $(r,c)$-th matrix element. The experimental realization of an Xbar node can be accomplished through the employment of a tunable MZI that has only one of its 2 inputs and one of its 2 outputs connected to the circuit and acts as a VOA for defining the value $w_{r,c}$, followed by a tunable PS for defining the value $\varphi_{r,c}$, as shown in the legend of Fig. 2(a).

After exiting the Xbar node, the signal enters the BCFS of its corresponding column $c$ that is responsible for performing the following two functions: a) forward the signal part emerging at the $\xi_r^2$ coupling port of the $\xi_r^2 : \xi_c^2$ optical splitter to the next column via a passive optical waveguide section, b) allow the signals that exit the intra-column nodes from all Xbar rows to coherently recombine and produce the column output signal $E_{out,c}$. The BCFS is shown in detail in Fig. 2(b), with the coherent recombination being accomplished via a mirror version of the 1:N front-end splitter implemented through a cascade of 3 dB Y-junction combiner stages. In this way, the $N$ signals can recombine sequentially in clusters of two at every combination stage until reaching the single waveguide output. Waveguide crossings are required at every column $c \in [1, M-1]$ between successive combination stages until reaching the single column read-out in order to overcome the row waveguides that lead to the next columns acting as the forwarding section. It should be recognized that a different number of crossings is required at every row of each column, with the maximum being $\log_2(N) - 2$ at rows $(N/2) - 1$ and $(N/2) + 2$, for $N > 2^2$, which typically is the case. If $N = 2^2$, only a single crossing appears in rows $N/2 = 2$ and $N/2 + 1 = 3$, whereas for $N = 2$ no crossings are required. In summary, for $N \geq 2^2$, we have a maximum number of waveguide crossings that amounts to $\max\{1, \log_2(N) - 2\}$. In order to ensure a loss-balanced forwarding section, every row waveguide in the BCFS is equipped with dummy crossings up to a total number of $\max\{1, \log_2(N) - 2\}$ in each column $c$, for $N \geq 2^2$, as shown in Fig. 2(b). At the same time, the need to ensure balanced losses between the recombining signals within every column leads to the use of $2^{r-1}$ dummy waveguide crossings at the respective input port of every Y-junction combiner in the $s$-th combination stage, with $s = 2, 3, \ldots, \log_2(N) - 1$. Together with the final single crossing at the last $\log_2(N)$ stage, the total number of crossings along the recombination path that each row-signal encounters amounts to $1 + \sum_{s=2}^{\log_2(N)-1}2^{s-1} = N/2 - 1$, for $N \geq 2^2$. In the exception of $N = 2$, no crossings will exist, similar as along the forwarding waveguides. Active and dummy waveguide crossings are represented by black and grey crosses, respectively in Fig. 2(b).

As mentioned, the proposed Xbar requires a single programming step in order to map the vector- and transformation matrix-elements into the hardware. Fig. 2(c)–(e) illustrate an example of the programming of a $4 \times 5$ Xbar that calculates the linear transformations of a $4 \times 1$ input vector $X$ via a $4 \times 5$ transformation matrix $W$. As shown in Fig. 2(c), the #4 elements of the input vector are encoded into the respective modulators that emerge the 4 output rows of the 1:4 front-end splitting stage. At the same time, the #20 elements of the weight matrix are directly mapped into the nodes of the Xbar, where each row of $W$ is imprinted in each row of the Xbar, as shown in Fig. 2(d). Finally, the multiplication of the input vector with the weight matrix results into the $5 \times 1$ output vector $E_{out}$ after the signals’ recombination through the BCFS stages, as highlighted in Fig. 2(e).

It should be noted that a different front-end splitter layout has been assumed in the Xbar layout compared to the respective stage in the SVD-Clements scheme. Although a generic 1:N splitter has been described for the SVD-Clements setup, the front-end splitter in the Xbar-based linear optical circuit has been proposed to rely on a simple and fabrication-wise mature cascade of 3 dB splitters. This doesn’t affect the performance comparison between the SVD-Clements and the Xbar-based linear transformations as long as the number of rows is a power-of-2 but leads to a differentiated analysis when a matrix dimension of $M \times N$ is implemented with $N$ not being a power of 2. This scheme can be realized in the case of the 3 dB splitter-cascade front-end layout used in the Xbar analysis by simply deactivating or ignoring the number of rows that correspond to the difference between $N$ and $N_f$ within a $N_f \times M$ Xbar design, where $N_f = 2^n$ and $n = \lceil\log_2(N)\rceil$, with the brackets denoting the lower integer bound. For example, for an intended $N \times M$ layout where $N = 8$ and $M = 10$, then $N_f = N = 8$. In the case of $N = 5$, then $N_f$ will remain equal to 8 according to the relationship $N_f = 2^{\lceil\log_2(5)\rceil} = 8 > N$, but three of its rows will remain inactive. This indicates that part of the input signal power will be wasted resulting to additional IL; however, this loss penalty will never exceed 6 dB given that $N$ will be always between $2^{n-1}$ and $2^n$. Finally, it is important to note that, since every Xbar node represents an individual matrix element, all Xbar nodes can be configured independently and in parallel to each other requiring just a single-step programming operation for the entire matrix, offering a significant configuration time benefit compared to the respective N(N-1)/2 steps needed to program an SVD-Clements layout.

Focusing on a single, $c$-th column of the Xbar, it can be clearly seen how its operation represents a generalization of our coherent vector dot-product engine described in [17]. The modulated input signal vector $X$ is fed into every of the Xbar’s columns exploiting the $\xi_r^2 : \xi_c^2$ optical splitter stage, element-wise multiplied by the $c$-th column vector of the Xbar matrix, and coherently summed at the BCFS stage to yield the result of dot-product multiplication, $E_{out,c}$, at the output. Every individual Xbar column connects to the incoming optical signal vector $X$ as
described above, effectively implementing generalized version of the layout described in [17]. Considering again that the IL of each MZI-node equals $IL_{node,dB} = -10 \log(T_{node}^2)$, as in the case of SVD-Clements design, the magnitude of the electric field at every column output of the Xbar can be obtained through:

$$E_{out,c} = \alpha T_{node} L_c \frac{1}{N_f} \prod_{q=1}^{c-1} t_q \xi_c \left( \sum_{r=1}^{N} x_r w_r, e^{j\varphi_{r,c}} \right) E_{in}$$

(3)

where $\alpha \leq 1$ is the electric field transmission coefficient of the input amplitude modulator $x_r$, when operated in transparency, while $N_f$ and $N$ are the row-dimensions of the Xbar layout and the targeted matrix, respectively. The term $1/N_f$ equals to $(1/\sqrt{2})^{2\log_2(N_f)}$, with $2 \times \log_2(N_f)$ equal to the total amount of the Y-junction splitters and combiners the signal traverses, comprises the loss factor of the splitting and recombination stages. Additionally, in the case of the last column, no splitting stage exists and signal is only passed through the column itself, implying $\xi_M = 1$. The coefficient $L_c \leq 1$ is defined as the total passive circuitry transmission coefficient, denoting the transmission through all front-end and back-end 3 dB couplers (Y-splitters and Y-combiners, respectively), all $\xi^2_e : t^2_e$ couplers and all waveguide crossings within the signal’s path from Xbar input to c-th output column. For $N > 2$, it equals:

$$L_c = \begin{cases} 
\frac{2\log_2(N_f)}{l_{coupl}} & f_{\xi} \frac{N}{l_{in}} - 1 + (c-1) \max(1, \log_2(N_f) - 2), c \in [1, M - 1] \\
\frac{2\log_2(N_f)}{l_{M}} & f_{\xi} \frac{M-1}{l_{in}} \max(1, \log_2(N_f) - 2), c = M
\end{cases}$$

(4)

where $l_{in} \leq 1$ and $l_{coupl} \leq 1$ are the electric field transmission coefficients of a single $\xi^2_e : t^2_e$ directional coupler and a waveguide crossing, respectively. If $N = 2$, no waveguide crossings will exist, reducing the expression for $L_c$ to the case of $l_{in} = 1$, and raising the coefficient $L_c$ closer to the ideal value of 1. From this point onward, we focus on the case of $N > 2$, a condition under which we anticipate Xbar to typically operate. A detailed derivation of (3), together with the accompanying explanations, is given in section A of the Appendix.

Equation (3) verifies that the output of each column is proportional to the linear weighted sum of the inputs $x_r$, that is embraced in the parentheses, successfully yielding the vector dot-product between the Xbar column entries and the input column vector $X$. Based on the columns outputs given by (3), we define the Xbar associated $N \times M$ transfer matrix, i.e., a weight matrix, normalized to lossless case as:

$$W = \begin{bmatrix}
    w_{1,1}e^{j\varphi_{1,1}} & \cdots & w_{1,M}e^{j\varphi_{1,M}} \\
    \vdots & \ddots & \vdots \\
    w_{N,1}e^{j\varphi_{N,1}} & \cdots & w_{N,M}e^{j\varphi_{N,M}}
\end{bmatrix}$$

(5)

with each Xbar node being described by $w_{r,c}e^{j\varphi_{r,c}}$ where $r = 1, 2, \ldots, N$ and $c = 1, 2, \ldots, M$. The output electrical field vector $E_{out} = [E_{out,1}, E_{out,2}, \ldots, E_{out,M}]^T$ can now be expressed in a matrix-equation form as:

$$E_{out} = (WP)^T X E_{in} = P^T W^T X E_{in}$$

(6)

with $P$ being the diagonal $M \times M$ transmission matrix encompassing all the transmission factors corresponding to the ILs and splitting ratios $P = diag(p_1, p_2, \ldots, p_M)$, where the real elements along its diagonal equal:

$$p_c = \alpha T_{node} L_c \frac{1}{N_f} \prod_{q=1}^{c-1} t_q \xi_c$$

(7)

More details are given in section A of the Appendix.

IV. LOSS ANALYSIS

Both the Xbar and the SVD-Clements architectures are expected to carry out the same algebraic operation of vector ($x$) by matrix ($W$) multiplication in a photonic hardware. This implies that the total loss seen by the optical signal includes the IL (corresponding to the passive loss that a device introduces at the maximum of the transmission spectrum), but also an equivalent of the modulation loss, i.e., the loss penalty due to random values of the input vector and weight matrix. In this section we treat separately the two loss contributors, namely insertion loss and value dependent loss (VDL).

A. Insertion Loss Analysis

The IL analysis that follows has been performed at the condition where the device under test operates at its transparency point, i.e., all its incoming power is intentionally forwarded to its output port so that the IL calculation gets independent of the targeted operation and accounts only for passive loss factors like reflection, propagation, absorption, coupling etc. [25].

The overall IL at the c-th output column of the Xbar can be calculated using the respective transmission factor $p_c$. In order to do so, we assume transparent operation of all input modulators, $x_r = 1$, as well as all Xbar nodes with $w_{r,c}e^{j\varphi_{r,c}} = 1$, so as to exclude the matrix-value-enforced node losses, as was done in the case of the SVD-Clements architecture, where the specific case of a unitary matrix implementation was considered. A simple IL analysis can be carried out when considering a loss-balanced $N \times M$ matrix, by which we assume that all of Xbar’s column outputs have the same IL. Following the analysis described in detail in section B of the Appendix, we show that the $\xi^2_e$ coupling coefficients can be selected in such a manner to yield identical transmission factors at every column, i.e., $p_c = p_1$ for $c \in [2, M]$, effectively corresponding to the case where the input optical power is equally distributed among all Xbar columns. To this end, the total Xbar power ILs can be obtained for every column when excluding the losses of the input modulation stage by considering $a = 1$ and can be calculated as:

$$IL_{Xbar,dB} = IL_{node,dB} + 2 \log_2 (N_f) IL_{coupl,dB}$$

$$+ IL_{\xi,dB} + \frac{N_f}{2} - 10 \log_{10} (\frac{N}{N_f})$$

$$+ 20 \log_{10} \left( \frac{N_f - 1}{N} \right)$$

(8)

where $IL_{\xi,dB} = -10 \log_{10}(l_\xi)$ and $IL_{X,dB} = -10 \log_{10}(l_\xi^2)$ stand for the optical power loss per $\xi^2_e : t^2_e$ directional coupler.
and waveguide crossing, respectively. It should be underlined that $\xi$ can be determined based on the recursive procedure and depends only on $N_f$, $I_{Lx, dB}$ and $I_{Lx, dB}$, as described in section B of the Appendix. The base case and the formula for the recursive procedure are given by:

$$\xi_{M-1}^2 = \left[ 1 + 2x \left( \frac{N_x}{2} - 1 - \max(1, \log_2(N_f)-2) \right) \right]^{-1}$$  \hspace{1cm} (9a)

$$\xi_c^2 = \left[ 1 + \left( \xi_{c+1}^2 \xi_{x,dB}^2 \left[ \log_2(N_f)-2 \right] \right)^{-1}, \quad c \in [1, M-2]$$  \hspace{1cm} (9b)

The relationship given by (8) shows that the ILs of the Xbar depend only statically on the ILs of its node technology, $I_{Lx, dB}$, whereas the exponential scaling (proportional to the Xbar size, $N_f$) takes place only with respect to a) the Y-branch splitters and combiner losses $I_{Lcoup, dB}$ employed at the front-end splitting and BCFS stages, and b) the waveguide crossing insertion losses $I_{Lx, dB}$. Both of them, however, comprise well-established passive circuits with state-of-the-art ILs being as low as 0.06 dB [26] and 0.02 dB [27] for Y-branch MMI couplers and waveguide crossings, respectively. This forms a significant advantage compared to the SVD-Clements layout since the exponential scaling is retained only for low-loss passive circuitry instead of the entire MZI node, with the loss scaling with respect to the MZI node technology being transformed into a linear relationship. This allows the use of intra-node active PS technology that may have higher losses but can offer important energy consumption or speed benefits, like, for example, the BaTiO$_3$ electro-optic [16], PCM [14] or SiSCAP [28] technologies, without leading to prohibitive total ILs for the Xbar, as would be the case in the SVD-Clements design. Finally, the term $20 \log_{10}\left\{ N_f / N \right\}$ reveals the additional losses encountered when the matrix dimension $N$ is not a power of two, where an Xbar layout with $N_f = 2^{\left\lfloor \log_2(N) \right\rfloor} + 1$ is utilized with its $N_f - N$ rows being deactivated. Given that $N_f / N$ will never exceed the value of 2, the additional losses will be always lower than 6 dB, with the lowest-loss case being 0 dB for $N = N_f$.

As a consequence of its linear dependence on the MZI node ILs, the proposed N×M Xbar arrangement can yield significantly lower total ILs and better scaling performance compared to the respective SVD-Clements design. This is illustrated in Fig. 3 where a total loss comparison between Xbar architecture and the best- and worst-case paths of the SVD-Clements design is provided on an equal footing (i.e., with the Xbar number of outputs equaling its number of inputs, $M = N$), assuming existing silicon photonic technology with losses of $I_{Lcomp, dB} = 0.06$ dB for the MMI couplers used in the 3 dB Y-branch splitter and combiner stages [26], $I_{Lx, dB} = 0.1$ dB for the $\xi^2 : t^2$ optical directional couplers [29] and $I_{Lx, dB} = 0.02$ dB for the waveguide crossings [27]. More specifically, Fig. 3(a) depicts the total ILs provided by (1) and (2) for the best- and worst-case SVD-Clements paths (dash and dot lines, respectively) and by (8) for the Xbar design (dash-dot lines) for MZI-node loss values $I_{Lx, dB}$ ranging between 0–2 dB in the case of a 4×8 input (black lines) and an 8×8 (red short lines) matrix implementation. It can be clearly observed that the slope of the Xbar IL is significantly lower compared to the respective slopes of the SVD-Clements design, as a direct result of the linear, as opposed to the exponential, dependence of the IL on the MZI node losses. The linear dependence also leads to a constant IL slope of the Xbar, with the loss itself being only increased by a constant $\sim 3.5$ dB offset as the matrix dimension scales from 4×4 to 8×8. Moreover, it results to a more loss-tolerant configuration, since the SVD-Clements layout retains a slightly lower total IL only for ultra-low node losses of $I_{Lx, dB} < 0.15$ dB but leads to higher losses when $I_{Lx, dB}$ exceeds 0.15 dB, with the loss performance gap between the two architectures constantly increasing with $I_{Lx, dB}$. The Xbar losses remain as low as 8.5 dB and 12 dB in the case of a 4×4 and an 8×8 matrix design even when $I_{Lx, dB}$ equals 2 dB, while the corresponding loss values for the SVD-Clements design extend between 16–24 dB and 27–43 dB, respectively.

The scalability of the proposed Xbar layout in terms of the circuit size and its comparison with the respective SVD-Clements architecture are validated via Fig. 3(b) and (c), which illustrate the total losses with respect to the matrix dimension $N$, varying
between 4 and 64, for two different values of losses per node \( I_{lnode, dB} \). 1 dB in Fig. 3(b) and 2 dB in Fig. 3(c). These node-loss values have been selected to comply with various state-of-the-art silicon-based PS technologies, so as to highlight the potential of the Xbar to operate with powerful non-thermo-optic MZI node technology. Both Fig. 3(b) and (c) show that the losses in dB of the SVD-Clements layout (black dash and dot lines) for the best- and worst-loss path cases, respectively) increase with \( N \) linearly, suggesting an exponential dependence when losses are expressed in linear scale. The slope gets higher in the case of \( I_{lnode, dB} = 2 \) dB, as expected from (1) and (2). At the same time, the Xbar ILs (red straight lines) reveal a ramp-like behavior owing to the loss penalty for \( N < N_f \) originating from the factor \( 20 \log_{10} (N_f / N) \) of (8), experiencing a rapid jump from 0 dB to approximately 6 dB whenever \( N \) increases beyond \( 2^n \), with the loss-penalty slowly decaying until reaching 0 dB when \( N = 2^{n+1} \). The Xbar ILs increase with \( N \) at a slower rate compared to the SVD-Clements design, since they are primarily determined by the Y-junction coupler and the waveguide crossing losses and not by the MZI node losses as can be confirmed by (8). Moreover, the Xbar ILs are almost always lower than the respective SVD-Clements layout losses: Fig. 3(b) shows that the Xbar ILs extend slightly beyond the SVD-Clements best-case path losses only in the case of \( N = 5 \), while remaining lower for all other values of \( N \), with the loss difference between Xbar and SVD-Clements best-case increasing with \( N \) and reaching 28.2 dB for \( N = 32 \) and 55.6 dB for \( N = 64 \). In the case of higher node losses shown in Fig. 3(c), the IL advantages of the Xbar design become even higher since it offers constantly lower losses for all values of \( N \) and the loss difference gets even more pronounced reaching 60.2 dB for \( N = 32 \) and 119.6 dB for \( N = 64 \). It should be, also, noted that the overall ILs remain always within a rather feasible power budget of \( \sim 30 \) dB even for \( N = 64 \), while the respective SVD-Clements performance scales up to non-realistic power budget expectations of \( \sim 150 \) dB.

B. Value Dependent Loss for Arbitrary Matrix Implementation Analysis

The VDL originates from the loss that will be encountered when an arbitrary matrix is intended to be implemented in the photonic domain, so that finally the total loss of the experimentally realized matrix gets calculated by adding the VDL and the architecture IL. Since the proposed Xbar encodes the matrix elements in an element-to-element manner, its average VDL is expected to differ from the respective VDL of the SVD-based implementations based on the targeted matrix characteristics. This can be easily revealed by considering the following two cases: (a) a unitary matrix, and (b) a matrix with all its elements being identical. The SVD-Clements layout will obviously experience zero VDL when the targeted matrix \( W = UV \Sigma^T \) is unitary, since \( U \) and \( V \) are inherently lossless and \( \Sigma \) will comprise an identity matrix in this case. On the flip side, in this specific case, Xbar will encounter a non-zero VDL that will depend on the elements of the targeted matrix. However, in the specific case where all the matrix elements are equal, the Xbar architecture will have a zero VDL, while the SVD-based schemes will experience a non-zero VDL.

Towards calculating the total loss for the implementation of an arbitrary real-valued matrix, we performed a simulation analysis in order to estimate the VDL of both architectures and add this to the architectural IL. More specifically, a Monte Carlo analysis was carried out where a set of 1000 real-valued square matrices of random gaussian entries bounded within the \([-1,1]\) range were generated for every matrix dimension, with the matrix dimension ranging between \([4, 64]\). The mean VDL of both architectures was then calculated to be the average VDL of all columns in the Xbar layout and the average VDL of all rows in the SVD-Clements scheme, respectively. We discriminate our analysis into two cases based on the normalization factor utilized: (i) The targeted matrix is normalized to the square root of the product of its dimensions and (ii) the targeted matrix is normalized to its maximum value for the Xbar case and the \( \Sigma \) matrix to its maximum value for the SVD-Clements scheme (see Appendix section C). The first case corresponds to a universal matrix design approach that can differentiate between scalar transformations of different matrices and should be employed in applications where the absolute (and not relative) values of the targeted matrix is required. On the contrary, case (ii) corresponds to a VDL-optimization and subsequently to a total loss optimization approach that is a common criterion among experimental realization attempts and should be employed in applications where the scalar difference between matrices can be neglected. (10), (11) represent the respective expressions for the calculation of the VDL of a single column/row of case (i) and (ii):

\[
VDL_{Xbar, case (i)} = 20 \log_{10} N - 10 \log_{10} \left( \sum_{r=1}^{N} x_r u_{r,c} e^{j \varphi_{r,c}} \right)^2 \left( \sum_{r=1}^{N} x_r u_{r,c} e^{j \varphi_{r,c}} \right) \right)^* \tag{10a}
\]

\[
VDL_{SVD-Cl., case (i)} = 20 \log_{10} N - 10 \log_{10} \left( \sum_{r=1}^{N} x_r u_{r,c} e^{j \varphi_{r,c}} \right)^2 \left( \sum_{r=1}^{N} x_r u_{r,c} e^{j \varphi_{r,c}} \right) \right)^* \tag{10b}
\]

\[
VDL_{Xbar, case (ii)} = 20 \log_{10} \left( N \* \max (|W|) \right) - 10 \log_{10} \left( \sum_{r=1}^{N} x_r u_{r,c} e^{j \varphi_{r,c}} \right)^2 \left( \sum_{r=1}^{N} x_r u_{r,c} e^{j \varphi_{r,c}} \right) \right)^* \tag{11a}
\]

\[
VDL_{SVD-Cl., case (ii)} = 20 \log_{10} \left( \max (\Sigma) \right) - 10 \log_{10} \left( \sum_{r=1}^{N} x_r u_{r,c} e^{j \varphi_{r,c}} \right)^2 \left( \sum_{r=1}^{N} x_r u_{r,c} e^{j \varphi_{r,c}} \right) \right)^* \tag{11b}
\]
with $c \in [1, N]$. The calculated VDL corresponds to the mean VDL of all columns and rows in the Xbar and SVD-Clements, respectively. Fig. 4(a) illustrates the simulation results, that reveal a similar behavior for the comparing architectures in the normalization scheme of case (i) (dashed lines) and a VDL gap in the normalization scheme of case (ii) (solid lines), with SVD-Clements VDL plateauing, since its expression for its calculation is independent of the matrix dimensions and outperforms the VDL of the proposed Xbar. On the other side, Xbar’s VDL curves are almost identical for both normalization schemes, since the gaussian distributed elements of the targeted matrix will statistically contain a value close to $\pm 1$.

Consequently, the VDL of the comparing architectures when the normalization scheme of case (i) is employed, will add the same offset to their IL to formulate the total power budget required for the construction of an arbitrary matrix. On the other hand, the disparate behavior of the VDL of case (ii) will affect the total loss asymmetrically in the comparing architectures. Fig. 4(b) and (c) depict the average total loss of the Xbar (red curves) and SVD-Clements (black curves) best (dashed) and worst (dotted) path cases for an $\Pi_{\text{node, dB}} = 1 \text{ dB}$ and 2 dB, respectively. In the former, the total loss curve of the proposed Xbar ranges within the SVD-Clements best and worst path cases curves for matrix sizes <20 and starts to become lower than the SVD-Clements best-case loss value for matrix sizes $\geq 21$. The crossing point between the total loss curves of the two architectures depends, however, also strongly on the $\Pi_{\text{node, dB}}$ value and can shift significantly to the left or to the right when a different IL is experienced by its employed node technology. For example, in the case of $\Pi_{\text{node, dB}} = 2 \text{ dB}$, the total loss of the proposed Xbar is always lower than the SVD-Clements loss, as shown in Fig. 4(c), validating the credentials of the Xbar architecture to support higher IL node technology. Consequently, the lower IL experienced by the Xbar layout can compensate for its higher average VDL when reasonable node IL values are considered, with the Xbar total loss advantages becoming more pronounced with increasing node loss values and/or increasing matrix dimensions.

V. FIDELITY ANALYSIS: LOSS- AND PHASE ERROR TOLERANCE

Fidelity forms a highly critical factor for linear optics when lossy and/or imperfect elements are used, providing a measure of accuracy of the optical device when representing experimentally an algebraic matrix. In order to compare our Xbar design with the state-of-the-art architectures in terms of their fidelity performance, we calculate and compare the fidelity of our design with the fidelity accomplished by the SVD-Clements design for the cases of employing a) lossy optical elements, and b) phase errors in the MZI node phase shifting structures. The standard fidelity measure [13], [30] based on Frobenius inner product of two matrices, is given by the following equation:

$$ F(Y_{\text{exp}}, Y) = \frac{\left|\text{tr}(Y^\dagger Y_{\text{exp}})\right|^2}{\sqrt{\text{tr}(Y^\dagger Y) \cdot \text{tr}(Y_{\text{exp}}^\dagger Y_{\text{exp}})}} $$

(12)

where $Y$ and $Y_{\text{exp}}$ are the $N \times N$ target matrix and its experimental implementation, respectively, with $Y^\dagger$ and $Y_{\text{exp}}^\dagger$ denoting their conjugate transposes, respectively. This measurement indicates the accuracy/error of a device when it is implemented experimentally (incorporating loss). The matrix values are enforced over electrical fields, meaning that finally the resulting electrical field considered to form the trace of the matrix product has to account also for the phase (imaginary part) of the constituent electrical fields prior applying the absolute value. The normalization employed in this fidelity definition allows us to focus, solely, on unbalanced instead of balanced losses, so that we do not distinguish between matrices that differ by just a constant multiplicative factor.

The loss-induced fidelity analysis can be carried out via analytical expressions using the Xbar matrix formulation given by (6). We show that the fidelity is unity when designing a loss-balanced configuration where the factors $\xi_c$ of the directional couplers have been selected to ensure the same losses among all columns, i.e., $p_c = p_1$ for $c \in [2, M]$, transforming the transmittivity matrix into $P = p_1 I_M$, with $I_M$ standing for identity matrix of size $M$. With the target matrix being
\[ Y = W^T \]

The experimentally realized matrix through our Xbar architecture will equal to 
\[ Y_{exp} = P^TW^T = p_1 W^T \]

according to (6). Their conjugate transposes read 
\[ Y^\dagger = W^* \text{ and } Y_{exp}^\dagger = p_1 W^* \]

with asterisk denoting conjugate matrix and assuming 
\[ p_1 = 1 \]

since, according to (7) the coefficient \( p_1 \) is real. The matrix products become 
\[ Y^\dagger Y_{exp} = p_1 W^* W^T \text{ and } Y^\dagger Y = W^* W^T \]

and \( Y_{exp} \) differs only by a constant multiplicative factor given as a degree of \( p_1 \). Given that we are interested in the trace of the matrix product \( W^* W^T \), we determine the elements along its main diagonal, which read 
\[ (W^* W^T)_{k,k} = \sum_{i=1}^{N} |w_{ki}|^2 \]

implying, they are real, yielding also a real trace \( tr(W^* W^T) = \sum_{k=1}^{N} \sum_{i=1}^{N} |w_{ki}|^2 \), as can be also derived based on Frobenius inner product. Having all quantities of interest being real, we conclude to:

\[ F_{Xbar} = F(Y_{exp}, Y) = \frac{p_1 tr(W^* W^T)}{\sqrt{tr(W^* W^T) p_1^* tr(W^* W^T)}} = 1 \]  

The design of a loss-balanced layout, however, requires high accuracy and high resolution in the values of \( \xi \), which may not always be possible, especially for high number of Xbar columns, given the current technology and fabrication limitations. Given that \( P \) is a diagonal matrix with its diagonal elements in practice differing among themselves and from the unity value, the fidelity obtained through (12) will generally be lower than unity when an unbalanced loss design is employed. However, the Xbar layout allows - even in this case - for a completely restorable fidelity performance in direct contrast with the SVD-Clements design where fidelity restoration cannot be accomplished. The Xbar fidelity restoration can be achieved by defining a new matrix \( Y'_{exp} = (P^T)^{-1} Y_{exp} = (P^T)^{-1} P^T W^T = W^T = Y \) where fidelity will always equal unity, i.e., \( F(Y_{exp}, Y) = 1 \), requiring, in practice, a simple procedure of multiplying the Xbar’s output by the inverse of its \( P^T \) diagonal matrix. Multiplication with this new diagonal matrix from the left side effectively corresponds to the introduction of differentiated factor at every column output, i.e., by inserting a factor of \( 1/p_c \) at every column output \( c \). This yields, finally, a loss-balanced Xbar architecture where the power balancing is achieved by pondering the outputs and thus compensating for the power distribution inequality due to the non-optimal splitting ratios. Even if all \( \xi^2 \) \( t^2 \) optical couplers across columns are assumed to have the same coupling ratios \( \xi \) and \( t \), which would reduce design and fabrication complexity, the lack of an a-priori loss-balanced scheme is easily compensated at the Xbar’s output.

Assuming, for example, that \( \xi_c = \xi \) and \( t_c = t_1 \) for every \( c \in [1, N-1] \) in the case of an \( N \times N \) matrix arrangement, then 
\[ p_c / p_c - 1 = L_c / L_{c-1} / \xi_c / (1 - t_c / t_1) / \xi = t_1 / \xi \]

suggesting that the differential loss between two successive columns is equal to the losses of \( \max \{ 1, \log \xi (N_f - 2) \} \) wave-guide crossings employed at every row of the BCFS stage multiplied by the excess losses of a directional coupler \( \xi^2 \); \( t^2 \) and the coupling coefficient \( t_1 \). The exception is the last column \( N \), where the non-existent coupler is described by \( \xi_N = 1 \) and we have 
\[ p_{N-1} / p_{N-1} = L_{N-1} / L_{N-1} \xi_N / (1 - t_1) / \xi \]

Finally, the fidelity restoration matrix \( (P^T)^{-1} \), in this case, can be defined by its diagonal elements, where the \( c \)-th element \( p_c^{-1} \) can be calculated as:

\[ p_c^{-1} = \frac{1}{p_c} \left\{ \left( t_1^{-1} - \log \xi \right)^{c-1} \log \xi \right\} \left( c \in [1, N-1] \right) \]

This proves that our Xbar architecture supports a simple fidelity restoration mechanism requiring just the introduction of simple attenuation or gain elements at its column outputs.

This fidelity restoration mechanism is a significant advantage compared to SVD-based matrix designs, as it allows to accurately implement any target matrix through simple loss-balancing at the device output. On the contrary, SVD-based architectures can turn into loss-balanced configurations only when intervening in their inner-design and equipping every of their cascaded MZI stages with dummy MZIs [31], [32], which effectively means that all possible optical paths turn into equal-loss paths that experience the same loss with what has been considered as the worst-case optical loss path in our previous analysis. To this end, loss-balancing in the SVD-based designs can only be accomplished during device design and at the expense of much higher ILs, significantly increasing the gap between ILs of the SVD compared to the Xbar configuration, as has been already analyzed through Fig. [3].

The quantified benefits provided by the loss-induced fidelity restoration mechanism of the Xbar over the originally proposed SVD-Clements layout (i.e., without applying any restoration mechanism/ error correction techniques e.g., by employing dummy MZIs) are revealed in Fig. [5]. Having proven that the Xbar retains fidelity of 1 regardless of node losses, we perform fidelity calculations for the SVD-Clements design using Monte-Carlo method with 500 random target matrices \( Y \) generated for every given number of inputs \( N \), further decomposed to sets of \( (U, \Sigma, V^\dagger) \) (more details in section D of the Appendix). Fig. [5(a)] depicts the fidelity performance for two specific matrix dimensions of \( 4 \times 4 \) and \( 8 \times 8 \), when the losses of every MZI node vary between 0–2 dB. Fidelity degradation in the SVD-Clements layout becomes higher as the MZI node losses increase and the effect becomes even more pronounced when scaling the matrix dimension from \( 4 \times 4 \) towards \( 8 \times 8 \). This suggests that high fidelity performance in SVD-Clements-based schemes can, only, be obtained when ultra-low loss PS technologies with loss values <0.1 dB are used, as in the case of thermo-optic (T/O) MZIs [33] or Ring Resonators (RRs) [34], where, however, a power consumption of several mWs is required and only MHz-scale operational speeds can be sustained. Higher-loss phase shifting technologies would lead to significant fidelity degradation, implying that a whole range of highly promising
technologies should be probably excluded from their employment in SVD-based schemes, such as state-of-the-art PCM and silicon-based electro-optic PS technology \[14, 15, 16\], where losses can exceed 1 dB per node.

The fidelity performance gap between the SVD-Clements design and our architecture becomes dramatically wider when higher dimension layouts are implemented. This can be, also, confirmed by Fig. 5(b) and (c), where the fidelity performance with respect to the circuit size for two different MZI node loss values of 0.5 dB (Fig. 5(b)) and 1.5 dB (Fig. 5(c)) is illustrated. The SVD-Clements fidelity falls below 80% even for matrix dimensions as low as 18 × 18 and 7 × 7 when 0.5 dB and 1.5 dB losses per MZI node are employed, with the 20 × 20 matrix layouts yielding fidelity performance ~76% and ~45% for the two respective node loss cases.

In addition to the loss-induced fidelity degradation, θ and φ phase errors encountered at the Xbar node PSs can also critically affect the performance and lead to accuracy deviations in the experimental realization of a target matrix. This is also expected to be the case for the SVD-Clements architecture, given that phase-error-induced fidelity degradation has been already analyzed for unitary matrices relying on the Clements and Reck architectures \[30\]. In order to evaluate the phase-error tolerance of our Xbar architecture in comparison with the SVD-Clements layout, we employ again the standard fidelity measure of (12) and, using the Monte Carlo method, we calculate the respective fidelity metrics for the Xbar and the SVD-Clements designs by assuming lossless architectures for 500 arbitrary target matrices generated for every given matrix dimension N (more details in section D of the Appendix).

Fig. 6 illustrates the fidelity comparison of the two architectures. Fig. 6(a) and (b) depict the fidelity performance for a 4 × 4 and an 8 × 8 matrix implementation, respectively, when the phase-error standard deviation σ ranges between 0–0.2 rad. It can be observed that the Xbar architecture outperforms the SVD-Clements design in both cases, yielding a slightly reduced performance when increasing the matrix dimensions from 4 × 4 to 8 × 8, but retaining always a fidelity value that is greater than 97%, even in the extreme case where the standard deviation equals 0.2 rad. On the other hand, the fidelity of the SVD-Clements scheme degrades faster for higher standard deviation values, with the degradation becoming even more distinct when moving from the 4 × 4 to the 8 × 8 matrix implementation, yielding a fidelity value <87% when the standard deviation equals 0.2 rad. The effect of matrix dimensions on the fidelity performance can be more clearly witnessed in Fig. 6(c), where fidelity for both schemes has been plotted for a constant standard deviation value of 0.1 rad and for matrix dimension N ranging between 4 and 20. The Xbar architecture reveals an almost constant fidelity value as N increases, allowing fidelity to remain above 99% even when N = 20, whereas the respective SVD-Clements design shows a significant fidelity drop down to only 87%. This verifies
that the Xbar architecture shows a stronger phase-error-resistive behavior compared to the SVD-Clements scheme, with its fidelity experiencing a weaker dependence both on phase-error standard deviation and on matrix dimension scaling. This can be explained by the one-to-one mapping between the target matrix elements and Xbar nodes in our configuration, whereas, in the case of the SVD-Clements design, the mapping is not bijective, requiring a product of transfer matrices from multiple MZI nodes to achieve target matrix element. As such, assuming that a phase-error is experienced at a single $\left( \theta, \varphi \right)$ phase pair within a node, this will affect a single node and a single matrix element in the Xbar layout, as opposed to the multiple matrix entries, as will be the case in the SVD-Clements scheme. Moreover, given that the number of matrix entries affected by a phase error at a single $\left( \theta, \varphi \right)$ phase pair increases with matrix dimension as a result of the higher number of cascaded MZI nodes being employed, the impact of error becomes much stronger in the SVD-Clements scheme as the circuit size increases, as it can be seen also in Fig. 6(c). On the contrary, the independent nature of Xbar’s nodes and their one-to-one correspondence to target matrix elements is retained irrespective of the matrix size, obviously resulting to a more phase-error tolerant design for increasing circuit sizes.

It is worth noted that our fidelity analysis focuses only on active elements in both architectures, considering ideal performance for all necessary passive components. However, it is well-known that during the fabrication process, the passive elements might vary from their ideal values. For example, the coupling ratio of the 3-dB splitters and combiners in each node may slightly deviate from its ideal 50:50 value, which has been already studied in the case of the Reck and Clements architectures in view of the resulting fidelity degradation [32], [35], [36]. On top of this, the proposed Xbar architecture requires also the use of equal path lengths whenever coherent recombination of the signals is necessary, enforcing the need for waveguide length difference compensation during the photonic chip layout [37], [38], [39]. Finally, we would like to highlight that the fidelity metric is used to evaluate how accurately can the optical hardware encode the information of the targeted transformation matrix, which is independent of the front-end splitter and input modulation stage layout. Any imperfect front-end splitter performance would simply lead to inaccuracies in the experimental realization of the input signal vector and subsequently to the matrix-vector multiplication outcome but would not affect the optical matrix performance and the fidelity curves presented in our manuscript.

Table I summarizes the analysis reported in the above sections and provides an overview of the performance characteristics of the Xbar and the SVD-Clements architectures.

### Table I

| Performance parameter | Xbar | SVD-Clements |
|-----------------------|------|--------------|
| Insertion loss dependence on loss per node | Linear | Exponential |
| Loss-induced fidelity | Fully Restorable | Significant degradation |
| Phase-error-induced fidelity | Small degradation | Significant degradation |
| Scalability | High | Low |
| Versatility to node technology | High | Low |
| Programming complexity (for an N×N matrix) | O(1) | O(N³) |
| Insertion loss dependence on loss per node | Static | Exponential |

**A. Mathematical Analysis of Crossbar Operation**

In order to calculate the electric field at the output of each $N_f \times M$ Xbar’s column, let us first define $a \leq 1$ as the electric field transmission coefficient of the input amplitude modulator $x_r$ when operated in transparency. Additionally, let us define $l_{coup} \leq 1$ and $k \leq 1$ as the electric field transmission coefficients of a 3 dB coupler and a Phase Shifter (PS), respectively, where $IL_{coup, dB} = -10 \log_{10}(l_{coup}^2)$ and $IL_{PS, dB} = -10 \log_{10}(k^2)$ denote the corresponding optical power insertion losses. In each MZI-node, where two 3 dB couplers and two PSs $\theta$, $\varphi$ are employed, the total electric field transmittivity factor is $T_{node} = l_{coup}^2 k^2 \leq 1$. Moreover, we define, $l_q \leq 1$ as the electric field transmission coefficient of the $\xi^2$ coupler and $l_e \leq 1$ as the electric field transmission coefficient associated with $\Xi$.
with the losses per crossing, so that \( IL_{\xi, dB} = -10 \log_{10}(l^2) \) and \( IL_{x, dB} = -10 \log_{10}(l^2_x) \). After balancing the losses coming from different number of waveguide crossings within an Xbar column by adding dummy waveguide crossings in the BCFS circuit sections to amount to \( \max\{1, \log_2(N_f) - 2\} \) along the forward-propagating and \( N_f/2 - 1 \) along each path of the vertical coupling section for \( N_f \geq 2^2 \), or, equivalently, \( N > 2 \), as described in the main body of the manuscript, we define the total passive circuit transmission factor of the \( c \)-th column \( L_c \leq 1 \). Assuming \( N > 2 \), the total passive loss per column can be calculated as follows:

\[
L_1 = l_{coup}^{2 \log_2(N_f)} \left( \frac{2^f}{\xi^1} \right)
\]
\[
L_2 = l_{coup}^{2 \log_2(N_f)} \left( \frac{2^{f+1}}{\xi^2} \right) + \max\{1, \log_2(N_f) - 2\}
\]
\[
L_3 = l_{coup}^{2 \log_2(N_f)} \left( \frac{2^{f+2}}{\xi^3} \right) + 2 \max\{1, \log_2(N_f) - 2\}
\]
\[\vdots\]
\[
L_{M-1} = l_{coup}^{2 \log_2(N_f)} \left( \frac{2^{f+M-2}}{\xi^{M-1}} \right) + 1 + (M-2) \max\{1, \log_2(N_f) - 2\}
\]
\[
L_M = l_{coup}^{2 \log_2(N_f)} \left( \frac{2^{f+M}}{\xi^M} \right) + (M-1) \max\{1, \log_2(N_f) - 2\}
\]

or:

\[
L_c = \begin{cases} 
2 \log_2(N_f) & c = 1, M - 1, N > 1, \\
\frac{2 \log_2(N_f)}{\xi^c} & c = 1, M - 1, N = 2, \\
\frac{2 \log_2(N_f)}{\xi^c} & c = M, N > 2,
\end{cases}
\]

where \( N_f = 2^n, \) for \( n = \lfloor \log_2(N) \rfloor \), as analyzed in the main manuscript. In the special case of \( N = 2 \), no waveguide crossings exist, implying:

\[
L_c = \begin{cases} 
2 \log_2(N_f) & c = 1, M - 1, N > 1, \\
\frac{2 \log_2(N_f)}{\xi^c} & c = M, N > 2,
\end{cases}
\]

Consequently, the electric field at every column output can be obtained through the relation (A2):

\[
E_{out, c} = \alpha T_{node} L_c \frac{1}{N_f} \left( \prod_{q=1}^{c-1} t_q \right) \xi_c
\]

\[
\times \left( \sum_{r=1}^{M} x_r w_r e^{j \phi_r} \right) E_{in}
\]

The term \( 1/N_f \) equals to \((1/\sqrt{2})^{2 \log_2(N_f)}\), with \(2 \times \log_2(N_f)\) being equal to the amount of the Y-junction splitters and combiners the signal will traverse. More specifically, assuming that all the 2-input Y-junction splitters and combiners are formed by 3 dB directional couplers or MMI couplers neglecting one input or one output, respectively, so that a \( \pi/2 \) phase shift is always induced at the same (upper or lower) output arm, as indicated by their transfer function (A3), \((1/\sqrt{2})^{2 \log_2(N_f)}\) will be the transmittivity factor that corresponds to the loss that the split and recombining stages will introduce.

\[
\begin{bmatrix}
E_{out,1} \\
E_{out,2}
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & j \\
1 & j
\end{bmatrix} \begin{bmatrix}
E_{in,1} \\
E_{in,2}
\end{bmatrix}
\]

(A3)

Additionally, the product \( \prod_{q=1}^{c-1} t_q \) represents the total transmission coefficient of all \( \xi \) couplers preceding the column \( c \), and, by convention, in case of the empty set is equal to 1, i.e., for the 1-st column. \( \prod_{q=1}^{0} t_q = 1 \) in case of the last column, the splitting ratio can be defined by \( \xi_M = 1 \), as no coupler exists and the signal is only forwarded to the last Xbar column by the previous, (M-1)-th column’s splitting stage.

The output electric field given by (A2) can be written in matrix form as:

\[
E_{out} = (WP)^T E_{in}
\]

where \( E_{out} = [E_{out,1}, E_{out,2}, \ldots, E_{out,M}]^T \) stands for the output vector, \( E_{in} \) for the input optical signal, \( X = [x_1, x_2, \ldots, x_M]^T \) the vector of inputs,

\[
W = \begin{bmatrix}
\alpha_1 e^{-j \phi_{11}} & \cdots & \alpha_1 e^{-j \phi_{1M}} \\
\cdots & \cdots & \cdots \\
\alpha_M e^{-j \phi_{M1}} & \cdots & \alpha_M e^{-j \phi_{MN}}
\end{bmatrix}
\]

the targeted real/complex-valued matrix, implemented via the MZM-nodes of the Xbar and, finally, by \( P \) we denote an \( M \times M \) diagonal matrix \( P = \text{diag}[p_1, p_2, \ldots, p_M] \), encompassing all the transmission factors that correspond to the insertion losses and splitting ratios. The elements of the \( P \) matrix can be calculated by (A6):

\[
p_c = \alpha T_{node} L_c \frac{1}{N_f} \left( \prod_{q=1}^{c-1} t_q \right) \xi_c
\]

A. Insertion Loss Analysis

The optical power can be equally distributed among all columns of the Xbar when enforcing \( p_1 = p_2 = \cdots = p_M \), implying that the coupling coefficients \( \xi_c \), \( t_c \) have to satisfy the following relations (A7):

\[
L_c \left( \prod_{q=1}^{c-1} t_q \right) \xi_c = \text{const.}
\]

When \( N > 2 \), previous relation can be expanded to:

\[
\xi_1 t_1 = t_2 \xi_2 t_2^{2 \log_2(N_f) - 2} \\
\xi_2 t_2 = t_3 \xi_3 t_3^{2 \log_2(N_f) - 2} \\
\vdots
\]

\[
t_M - 2 \xi_M t_M - 2 t_M^{2 \log_2(N_f) - 2} \\
\xi_M t_M = t_1 t_2 \cdots t_{M-2} \xi_{M-1} t_{M-1}^{2 \log_2(N_f) - 2} - \left( \frac{N_f}{n} - 1 \right)
\]

(A8)

In addition to the previous set of equations, another set holds true, owing to the power conservation constraints. Namely, as
the loss of the couplers has been treated independently from their splitting/coupling ratio, through the parameter $l_x$, the sum of the two coefficients defining the power splitting ratio must, at all times, fulfill $t_x^2 + e_x^2 = 1$ for $c \in [1, M - 1]$. Focusing on the last two columns $(M - 1$ and $M)$, that share the $(M-1)$-th coupler, we have:

$$\xi_{M-1} = t_{M-1}l_x^{\max(1,\log_2(N_f)-2)} - \left(\frac{N_f}{2}\right)-1$$

$$\xi_{M-1}^2 = 1 + \left(\frac{1}{1 + l_x^{\max(1,\log_2(N_f)-2)}}\right)^2$$

(A9)

Same procedure can be extended to any two consecutive columns, e.g., $c$-th and $(c+1)$-th, for $c \in [1, M - 2]$, yielding:

$$\xi_c = t_c\xi_{c+1}l_x^{\max(1,\log_2(N_f)-2)}$$

$$\xi_c^2 = \frac{1}{1 + \left(\frac{1}{1 + l_x^{\max(1,\log_2(N_f)-2)}}\right)^2}$$

(A10)

Applying the recursive procedure based on the base case of $\xi_{M-1}^2$ and the recursive step defined above, all splitting coefficients can be determined. Moreover, enforcing identical coefficients $p_c$, allows us to express any column’s output via the splitting coefficient $\xi_1$, which itself is a function of $l_x$, $l_x$ and $N_f$, as:

$$E_{out,c} = p_c\left(\sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}}\right)$$

$$E_{in} = p_1\left(\sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}}\right)$$

$$E_{out,c} = \alpha T_{node} L_1 \frac{1}{N_f} \xi_1 \left(\sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}}\right)$$

(A11)

In order to calculate the overall loss of the Xbar, we consider lossless input modulators $(\alpha = 1)$ and assume the specific case where all input modulators and all weighting nodes operate at their transparency, i.e., $x_r = w_{r,c} = 1$ and $\varphi_{r,c} = 0$, $\forall r, c$, which implies:

$$\sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} = N$$

(A12)

and, hence, we get:

$$P_{out,c} = E_{out,c} E_{out,c}^* = p_c^2 N^2 P_{in}$$

(A13)

Considering $IL_{node,c} = -10 \log_{10}(T_{node})$ as the insertion loss of each MZI-node (2 couplers and 2 PSs), as in the case of SVD-Clements architecture, the total insertion loss of the Xbar can be expressed as:

$$IL_{Xbar,c} = -10 \log_{10}\left(\frac{P_{out,c}}{P_{in}}\right)$$

$$IL_{Xbar,c} = -10 \log_{10}\left(p_c^2 N^2\right)$$

$$= -10 \log_{10}\left(T_{node}^2 L_1 \frac{1}{N_f} \xi_1^2 N^2\right)$$

$$= -10 \log_{10}\left(T_{node}^2 L_1 \frac{1}{N_f} \frac{N^2}{\xi_1^2 N^2}\right)$$

(A14)

It should be underlined that $\xi_1$ can be determined based on the previously established recursive procedure and depends only on $N_f$, $IL_{\xi,c}$, and $IL_{\xi,dB}$.

C. Value-Dependent Loss Penalty

Using any of the two architectures outlined in the main manuscript, namely SVD-Clements or Xbar, vector-by-matrix multiplication leads to the value dependent loss (VDL) penalty, originating from the arbitrary values that the elements of the matrix $W$ or the input vector $X$ can take. Both architectures are designed such that their anticipated column-wise (Xbar) or row-wise (SVD-Clements) output equals the dot product between the input column-vector and a single row of the targeted matrix $W$, i.e., $y_c = \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}}$, with $N$ denoting the size of the architecture. Prior to defining the VDL, it is necessary to establish the domains on which elements of the targeted matrix $W$ and input vector $X$ are defined. Having in mind that both architectures are passive (no optical amplifiers employed) and that all employed optical components have amplitude transmission not larger than 1, we assume that the elements of the targeted matrix $W$ are defined in the complex domain with the maximum magnitude of 1, implying $w_{r,c} \in [0, 1]$ and $\varphi_{r,c} \in [0, 2\pi]$, and the elements of the input vector on the domain $x_r \in [0, 1]$. Further normalization of the targeted matrix and/or input vector may be necessary to guarantee that the total optical power at the output never surpasses the available optical power at the input, regardless of the choice of the targeted matrix and/or input vector, even under the assumption of zero IL. In this regard, we apply two different normalization approaches, depending on the application requirements.

The first, which we refer to the main manuscript as case (i), is dictated by the architecture itself and its validity can be verified by observing the loss of the architecture when all elements of input vector and targeted matrix are assumed to have a maximum magnitude of 1. If the total loss in this case equals the insertion loss, the power conservation is fulfilled.

Following the above outlined normalization principle, in case of the Xbar we can rewrite (A11) as:

$$E_{out,Xbar,c} = \alpha T_{node} L_1 \frac{N}{N_f} \xi_1 \left(\sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}}\right)$$

(A15)

highlighting the fact that, in the special case, when $x_r = 1$ ($\forall r$) and $w_{r,c} e^{j\varphi_{r,c}} = 1$ ($\forall r, c$), the pondered sum in the brackets
yields 1, as shown by (A12), and the remaining coefficients in front of the bracket define the insertion loss, as (A14) reveals for \( \alpha = 1 \). Based on (A15), it is possible to determine the VDL penalty, knowing that the total loss that the proposed Xbar experiences will be a combination of IL and VDL. The optical power at the output can be found as:

\[
P_{\text{out}, \text{Xbar}, c} = E \left[ E_{\text{out}, \text{Xbar}, c} E_{\text{out}, \text{Xbar}, c}^* \right]
\]

\[
P_{\text{out}, \text{Xbar}, c} = \left( \alpha T_{\text{node}} L_1 \frac{N}{N_f} \xi \right)^2 E
\]

\[
\left[ \left( \frac{1}{N} \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} \right) \left( \frac{1}{N} \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} \right)^* \right] P_{\text{in}}
\]

whereas the total loss equals:

\[
T L_{\text{Xbar}, dB} = IL_{\text{Xbar}, dB} + V LD_{\text{Xbar}} = -10 \log_{10} \left( \frac{P_{\text{out}, \text{Xbar}, c}}{P_{\text{in}}} \right)
\]

yielding the VDL penalty of:

\[
VDL_{\text{Xbar}, c, \text{case (i)}} = 20 \log_{10} N - 10 \log_{10} E
\]

\[
\times \left[ \left( \frac{1}{N} \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} \right) \left( \frac{1}{N} \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} \right)^* \right] \quad \text{(A16)}
\]

where \( E[\cdot] \) denotes the expected value of the quantity enclosed in the square brackets over a sufficiently large number of samples.

To derive the equations governing the electrical fields in the worst- and best-case scenario of the SVD-Clements architecture illustratively shown in Fig. 1 in the main manuscript, we start from the insertion losses given by (1), (2). Assuming all elements of the targeted matrix \( W \) and input vector \( X \) have a maximum magnitude of 1 (a condition under which the total loss equals the insertion loss), we have:

\[
E_{\text{out}, \text{SVD} - \text{Cl}, w,c} = f_{wc} E_{\text{in}} = \frac{1}{\sqrt{N}} T_{\text{node}}^{2N+1} E_{\text{in}} \quad \text{(A17a)}
\]

\[
E_{\text{out}, \text{SVD} - \text{Cl}, b,c} = f_{bc} E_{\text{in}} = \frac{1}{\sqrt{N}} T_{\text{node}}^{2[N/2]+1} E_{\text{in}} \quad \text{(A17b)}
\]

When employed in matrix-by-vector multiplication, the output field will obviously depend on the term \( \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} \), but will also be pondered by a normalization coefficient. To determine this normalization coefficient, which guarantees optical power conservation between the output and input in case of zero IL, we recognize that the targeted matrix \( W \) will be decomposed using SVD algorithm, into a product \( U \Sigma V^\dagger \), where \( U \) and \( V^\dagger \) are unitary matrices and \( \Sigma \) denotes the diagonal matrix with real elements, corresponding to the singular values of the matrix \( W \). As proven in relevant papers [12], [13], unitary matrices can be represented in photonic platform by Reck’s or Clements mesh of \( 2 \times 2 \)MZI interferometers without requiring any modification to their elements, as, by unitary matrix definition, no element of \( U \) or \( V^\dagger \) can have magnitude greater than 1. However, SVD algorithm does not restrict the values of the \( \Sigma \) diagonal matrix, and, in principle, allows them to be greater than one. Translating these values to a photonic platform using the attenuators to represent the elements of \( \Sigma \) requires a normalization constant which will be equal to the highest anticipated \( \sigma_r \) value for any given \( N \times N \) matrix, where \( \sigma_r \) stands for the \( r \)-th element of \( \Sigma \). Knowing that \( \sigma_r, r \in [1, N] \), represent singular values of targeted matrix \( W \), we recognize that the maximum \( \sigma_r \) value equals the spectral norm of the matrix (\( p \)-norm for \( p = 2 \)) and has the upper bound given by Frobenius norm for a rank-one matrix [40]. This implies that the maximum singular value of the \( N \times N \) targeted matrix \( W \) can be found starting from the \( N \times N \) matrix fully populated by ones, which has the spectral norm equal to:

\[
\sigma_{r,\text{max}} = \left( \sum_{c=1}^{N} \sum_{r=1}^{N} |w_{r,c} e^{j\varphi_{r,c}}|^2 \right)^{1/2} = N
\]

The electrical field from (A17) then becomes:

\[
E_{\text{out}, \text{SVD} - \text{Cl}, w,c} = \frac{1}{\sqrt{N}} T_{\text{node}}^{2N+1} \left( \frac{1}{N} \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} \right) E_{\text{in}}
\]

\[
E_{\text{out}, \text{SVD} - \text{Cl}, b,c} = \frac{1}{\sqrt{N}} T_{\text{node}}^{2[N/2]+1} \left( \frac{1}{N} \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} \right) E_{\text{in}}
\]

resulting in the VDL penalty:

\[
VDL_{\text{SVD} - \text{Cl}, c, \text{case (i)}} = 20 \log_{10} N
\]

\[
-10 \log_{10} E \left[ \left( \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} \right) \left( \sum_{r=1}^{N} x_r w_{r,c} e^{j\varphi_{r,c}} \right)^* \right] \quad \text{(A18)}
\]

which is statistically equal to the VDL calculated for Xbar, given by (A16). This is to be expected since both architectures have the same algebraic task and should suffer the same penalty originating from random distributions of weights and inputs defined on the above stated domains. Yet, despite the universality of this normalization scheme, optimization normalizing factors may be applied. For instance, the VDL of the SVD-based approaches when a unitary matrix is targeted, can and should be equal to zero, since \( U \) and \( V \) are unitary matrices and \( \Sigma \) comprises an identity matrix. However, following the above normalization scheme the VDL becomes non-zero. An equivalent example in the Xbar’s VDL is the special case where all the elements of the targeted matrix are equal.

In order to optimize the VDL we could adopt a matrix specific normalization scheme, that in the main manuscript is defined as case (ii). In this case, instead of normalizing the targeted matrix based on its dimensions, the maximum value of its elements comprised the normalization factor in the case of the Xbar VDL, while in the SVD-based approach this factor was defined by the maximum value of the elements of the \( \Sigma \) diagonal matrix. For the Xbar case, since the gaussian distributed elements of the targeted
matrix will statistically contain a value close to ±1, the VDL of the two discrete normalization cases will be approximately equal. The respective electric field expression at the cth column of the Xbar will be modified as:

\[
E_{out,Xbar,c} = \alpha T_{node} L_1 \frac{N_{f}}{N_f} \sum_{r=1}^{N} x_r w_r e^{j \varphi r,c} E_{in} \tag{A19}
\]

\[
VDL_{Xbar,case (ii)} = 20 \log_{10} \left( N \ast \left( \max \left( \left| W \right| \right) \right) \right)
\]

\[
- 10 \log_{10} E \left[ \left( \sum_{r=1}^{N} x_r w_r e^{j \varphi r,c} \right) \left( \sum_{r=1}^{N} x_r w_r e^{j \varphi r,c} \right) \right] \tag{A20}
\]

On the other hand, the VDL of the SVD-Clements approach with the normalization of case (ii), will be, apparently, reduced, with the related expressions being:

\[
E_{out,SV D Cl,we,c} = \frac{1}{\sqrt{N}} T^{2 \left( N+1 \right)}_{node} \left( \sum_{r=1}^{N} x_r w_r e^{j \varphi r,c} \right) E_{in} \tag{A21a}
\]

\[
E_{out,SV D Cl,be,c} = \frac{1}{\sqrt{N}} T^{2 \left( N/2 \right)+1}_{node} \left( \sum_{r=1}^{N} x_r w_r e^{j \varphi r,c} \right) E_{in} \tag{A21b}
\]

\[
VDL_{SV D Cl.,case (ii)} = 20 \log_{10} \left( \max \left( \Sigma \right) \right)
\]

\[
- 10 \log_{10} E \left[ \left( \sum_{r=1}^{N} x_r w_r e^{j \varphi r,c} \right) \left( \sum_{r=1}^{N} x_r w_r e^{j \varphi r,c} \right) \right] \tag{A22}
\]

We can observe that the normalization factor is no longer dependent on the matrix dimensions N, but solely on the maximum value of the \( \Sigma \) or equivalently the maximum eigenvalue of the targeted matrix W.

### D. Crossbar and SVD-Clements Fidelity Calculation Method

We compare the fidelity ((10) of main manuscript) of our Xbar with the fidelity accomplished by the SVD-Clements design for the cases of employing a) lossy optical elements, and b) phase errors in the MZI node phase shifting structures. Fidelity calculations have been performed using the Monte-Carlo method, where, for a given number of inputs N, we initially generate 500 random target matrices Y. In case of Xbar, the elements of the matrix should be mapped to the designated node modulator and phase shifter as given, whereas in the SVD-Clements case, the matrices are subsequently decomposed using SVD, producing the U, \( \Sigma \) and V\(^\dagger \) matrices. All elements of each matrix (U, \( \Sigma \), V\(^\dagger \)) are then normalized by being divided by the maximum norm (magnitude) among its elements. We then calculate the corresponding \( \theta \) and \( \varphi \) phases of each lossless MZI node for the decomposition of U and V\(^\dagger \) via the Clements method and of the MZI nodes of \( \Sigma \).

In case of the lossy-elements SVD-Clements fidelity evaluation case, these phase values are enforced onto lossy MZI nodes constructing the \( Y_{exp} \) matrix, required for the fidelity calculation. We use a rather simple loss model for the SVD-Clements architecture where all MZI nodes are assumed to have the same insertion losses, considering a fixed IL\(_{coup,dB} = 0.06 \) dB loss for each 3 dB MMI coupler and \(-10 \log_{10}(k^2)\) loss for every PS, determined such that in total we reach the targeted value of the total node insertion losses IL\(_{node,dB}\). In the case of Xbar’s loss-induced fidelity, we prove via analytical expressions that it can always 1.

Regarding the phase-tolerance fidelity, for each arbitrary matrix Y, we generate 100 two-element sets of random phase deviation values corresponding to \( (\theta, \varphi) \) that follow a Gaussian distribution with a mean value of zero and a standard deviation \( \sigma \in [0, 0.2] \) rad. Fidelity is then calculated for the SVD-Clements architecture by applying the generated phase deviations onto the ideal \( \theta \) and \( \varphi \) values, enforcing these new phase values onto the respective MZI nodes within the SVD-Clements design and concluding to the experimental matrix representation \( Y_{exp} \), which is subsequently used in the fidelity calculation (10) of the main manuscript. Thereafter, following the same procedure, matrices and sets of phase deviations, we obtained the respective fidelity values for the Xbar architecture.

### E. Crossbar and SVD-Clements Footprint Comparison

The proposed Xbar and the SVD-based approaches employ an equivalent front-end splitter that will occupy the same space and the same number of nodes i.e., \#N\(^2\). Although the amplitude of the weight matrix elements can be encoded via any modulator technology in the proposed Xbar and not only via 2 × 2 MMIs which is a strict requirement in the SVD-based approaches, for the sake of an apples-to-apples comparison we consider that the nodes’ architecture and technology in the comparing layouts is the same. Therefore, the BCFS employed in the Xbar comprises the only differentiator in terms of the overall architectures’ footprint. The latter can be constructed either via a 3 dB MMI tree, as shown in Fig. 2(b) of the main manuscript, or via 50:50 directional couplers tree, that exhibit significantly lower footprint compared to MMIs. In general, we can conclude that the Xbar’s footprint would be expected to be greater than the SVD based layouts, however the extra chip area required by the BCFS stages could be compensated through a) the deployment of lower footprint node technologies in the Xbar architecture, like for example electro-absorption modulators [41], ring modulators [42], which can’t or is not easy to accomplish in the SVD-based setups due to their requirement for #2 I/O nodes and ultra-low loss matrix nodes, respectively, or b) the employment of out-of-plane BCFS fabrication, where the BCFS stages can be fabricated on an additional overlying photonic layer without consuming additional 2D chip real-estate, as has been already reported in [43].
George Giamougiannis received the Diploma in electrical and computer engineering from the Aristotle University of Thessaloniki, Thessaloniki, Greece, in 2017, and the M.Sc. degree in communication network and systems security from the same department in the beginning of 2020. Since September 2019, he has been a Member of the WinPhos Research Group, working as a Research Assistant. His research interests include optical communication and neuromorphic photonics.

Apostolos Tsayridis received the university degree from the Department of Electrical and Computer Engineering, University of Thessaly, Volos, Greece, in 2016, and the M.S. degree in computer system and networking in the beginning of 2019 from the Department of Informatics, Aristotle University of Thessaloniki, Thessaloniki, Greece, where he is currently working toward the Ph.D. degree. His research interests include optical communications and neuromorphic photonics.

Yangjin Ma’s biography is not available at the time of publication.

Angelina R. Totović was born in Kragujevac, Serbia, in 1988. She received the B.Sc., M.Sc., and Ph.D. degrees in electrical engineering from the School of Electrical Engineering, University of Belgrade, Belgrade, Serbia, in 2011, 2013, and 2018, respectively. From 2012 to 2018, she was a Teaching and Research Assistant at the School of Electrical Engineering, University of Belgrade. In 2019, she joined the William-Hedberg Research Group, Aristotle University of Thessaloniki, Thessaloniki, Greece, where she is currently a Senior Researcher. Her research interests include neuromorphic photonics and exploitation of SOAs in various levels of photonic systems. Dr. Totović is the recipient of several scholarships and awards for her scientific work.

REFERENCES

[1] D. A. B. Miller, “Self-configuring universal linear optical component [Invited],” Photon. Res., vol. 1, pp. 1–15, 2013.
[2] J. Carolan et al., “Universal linear optics,” Science, vol. 349, pp. 711–716, 2015.
[3] Y. Shen et al., “Deep learning with coherent nanophotonic circuits,” Nature Photon., vol. 11, pp. 441–446, 2017.
[4] A. N. Tait, M. A. Nahmias, B. J. Shastri, and P. R. Prucnal, “Broadcast and weight: An integrated network for scalable photonic spike processing,” J. Lightw. Technol., vol. 32, no. 21, pp. 4029–4041, Nov. 2014, doi: 10.1109/JLT.2014.2345652.
[5] J. Capmany, I. Gasulla, and D. Pérez, “The programmable processor,” Nature Photon., vol. 10, pp. 6–8, 2016.
[6] L. Yang, R. Ji, L. Zhang, J. Ding, and Q. Xu, “On-chip CMOS-compatible optical signal processor,” Opt. Exp., vol. 20, pp. 13560–13565, 2012.
[7] H. Zhang et al., “An optical neural chip for implementing complex-valued neural network,” Nature Commun., vol. 12, 2021, Art. no. 457, doi: 10.1038/s41467-020-20719-7.
[8] L. Zhang, C. G. H. Roelfzemen, M. Hoekman, K. J. Boller, and A. J. Lowery, “Programmable photonic signal processor chip for radiofrequency applications,” Optica, vol. 2, pp. 854–859, 2015.
[9] D. Pérez, J. Gasulla, J. Capmany, and R. A. Soref, “Reconfigurable lattice mesh designs for programmable photonic processors,” Opt. Exp., vol. 24, pp. 12093–12106, 2016.
[10] M. Miscouglio and V. J. Sorger, “Photonic tensor cores for machine learning,” Appl. Phys. Rev., vol. 7, no. 3, 2020, Art. no. 031404.
[11] N. C. Harris et al., “Quantum transport simulations in a programmable nanophotonic processor,” Nature Photon., vol. 11, pp. 447–452, 2017.
[12] M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani, “Experimental realization of any discrete unitary operator,” Phys. Rev. Lett., vol. 73, pp. 58–61, 1994.
[13] W. R. Clements, P. C. Humphreys, B. J. Metcalf, W. Steven Kolthammer, and I. A. Walmsley, “Optimal design for universal multiport interferometers,” Optica, vol. 3, pp. 1460–1465, 2016.
[14] W. Jiang, “Nonvolatile and ultra-low-loss reconfigurable mode (De)multiplexer/switch using triple-waveguide coupler with Ge2Sb2Se4Te1 phase change material,” Sci. Rep., vol. 8, no. 1, pp. 1–12, 2018.
[15] A. Manolits et al., “Non-volatile integrated photonics memory using GST phase change material on a fully etched Si3N4/SiO2 waveguide,” in Proc. Conf. Lasers Electro-Opt., OSA Tech. Dig., 2020, Paper STh3R4.
[16] F. Eltes et al., “A BiTaO3-based electro-optic pockels modulator monolithically integrated on an advanced silicon photonics platform,” J. Lightw. Technol., vol. 37, no. 5, pp. 1456–1462, Mar. 2019.
[17] G. Mourigas-Alexandris et al., “Neuromorphic photonics with coherent linear neurons using dual-IO modulation cells,” J. Lightw. Technol., vol. 38, no. 4, pp. 811–819, Feb. 2020.
[18] N. Pleros et al., “Neuromorphic photonics with coherent linear neurons,” U.S. Patent 17305486, Jan. 13, 2022.
[19] S. Y. Siew et al., “Review of silicon photonics technology and platform development,” J. Lightw. Technol., vol. 39, no. 13, pp. 4374–4389, Jul. 2021, doi: 10.1109/JLT.2021.3066203.
Miltiadis Moralis-Pegios received the B.Sc. and M.Sc. degrees in electrical engineering from the Democritus University of Thrace, Komotini, Greece, in 2011 and 2013, respectively, and the Ph.D. degree in silicon-based photonic integrated circuits and high-capacity switching systems for datacenters interconnects from the Department of Informatics, Aristotle University of Thessaloniki, Thessaloniki, Greece. He has been involved on several tasks of the EU-funded projects PhoxTroT, L3MATRIX, ICT-STREAMS, and MOICANA, PLASMONIA, while his research interests include large-scale switching architectures for datacenter applications, silicon photonic interconnects for datacenter and high-performance computing systems, and neuromorphic photonics. Since 2020, he has been a Postdoctoral Researcher with the Photonic Systems and Networks Group, Aristotle University of Thessaloniki.

David Lazovsky received the Bachelor of Science degree in mechanical engineering from Ohio University, Athens, OH, USA. He founded Intermolecular in 2004, a semiconductor and clean energy research, development and Intellectual Property licensing company. From September 2004 to October 2014, he was the company’s Chief Executive Officer, President and as a Member of the board of directors. As the President and CEO, he led all aspects of the business through its lifecycle from early-stage start-up to public company. Intermolecular (IMI) went public on the NASDAQ in 2011. He has an in-depth knowledge of semiconductor, data/telecommunications, photonics and clean energy industries, and extensive international business experience. He was the Chairman, Energy Storage Systems, industry leader in low-cost, long-duration energy storage during 2017–2018. Prior to Intermolecular, he held several senior management positions at Applied Materials. He was responsible for managing more than $1 billion in Applied Materials’ semiconductor manufacturing equipment business in the Metal Deposition and Thin Films Product Business Groups. He was Ernst and Young Entrepreneur of the Year 2011, Northern California finalist. He currently has more than 50 issued and five pending U.S. patents.

Nikos Pleros obtained the Diploma and the Ph.D. degree in electrical & computer engineering from the National Technical University of Athens (NTUA) in 2000 and 2004, respectively. He joined the Faculty of the Department of Informatics, Aristotle University of Thessaloniki, Thessaloniki, Greece, in September 2007, where he is currently an Associate Professor. His research interests include optical interconnect technologies and architectures, neuromorphic photonics, photonic and plasmonic integrated circuits, and optical signal processing and optical switching. He has more than 400 archival journal publications and conference presentations including several invited contributions, while his work has been cited more than 5000 times (h-index = 35, GS source). Dr. Pleros has coordinated >7 FP7 and H2020 European projects having participated as partner in more than 10 additional projects. He has received the 2003 IEEE Photonics Society Graduate Student Fellowship granted to 12 Ph.D. candidates world-wide in the field of photonics, while he was proud to (co-) supervise three more Fellowship winners during their Ph.D. Dr. Pleros was the recipient of the AUTH Excellence Award for his research project funding ID between 2016 and 2018.