Measurement of the angular momentum of a rotating Bose-Einstein condensate

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We study the quadrupole oscillation of a Bose-Einstein condensate of $^{87}$Rb atoms confined in an axisymmetric magnetic trap, after it has been stirred by an auxiliary laser beam. The stirring may lead to the nucleation of one or more vortices, whose presence is revealed unambiguously by the precession of the axes of the quadrupolar mode. For a stirring frequency $\Omega$ below the single vortex nucleation threshold $\Omega_c$, no measurable precession occurs. Just above $\Omega_c$, the angular momentum deduced from the precession is $\sim \hbar$. For stirring frequencies above $\Omega_c$ the angular momentum is a smooth and increasing function of $\Omega$, until an angular frequency $\Omega_c'$ is reached at which the vortex lattice disappears and the precession stops.

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The achievement of Bose-Einstein condensation in atomic gases has led to a new impulse in the study of quantum gases. Among the several questions which can be investigated in these systems the properties of quantum vortices are some of the most intriguing and debated. A vortex is a singularity line around which the circulation of the velocity field is non zero. For a superfluid this circulation is quantized and equal to $nh/M$, where $n$ is an integer and $M$ the atomic mass. These quantized vortices play an essential role in macroscopic quantum phenomena, such as the superfluidity of liquid helium or the response of a superconductor to an external magnetic field. In this paper we determine the angular momentum of a gaseous Bose-Einstein condensate, as it is stirred near the vortex nucleation threshold. We show that the angular momentum $L_z$ per atom along the stirring axis jumps from zero to $\hbar$ (within experimental uncertainty), as expected from the quantization of the velocity field. We also measure the increase of $L_z$ as more vortices are nucleated in the system, for a stirring frequency higher than the nucleation threshold.

We consider here a condensate harmonically trapped in a quasi-cylindrically symmetric potential. The angular momentum of the condensate along the symmetry axis $z$ of the trap is measured using the frequencies of its collective excitations. More precisely, as suggested in [4], we study the two transverse quadrupole modes carrying angular momentum along the $z$ axis of $m = \pm 2$ (see also [1,11]). In the absence of vortices, the frequencies $\omega_{\pm}/2\pi$ of these two modes are equal as a consequence of the reflection symmetry about the $xy$ plane. By contrast, for $L_z \neq 0$, this degeneracy is lifted by an amount:

$$\omega_+ - \omega_- = \frac{2L_z}{Mr_z^2},$$

where $r_z^2$ stands for the average value of $x^2 + y^2$ for the condensate. Consequently the measurements of $\omega_+ - \omega_-\ldots$ and of the transverse size of the condensate provide the angular momentum of the gas.

The result [1] is valid if the system is properly described by the hydrodynamic theory for superfluids [8]. This requires that the number of atoms $N$ is much larger than the ratio $a_{ho}/a$, where $a_{ho}$ is the size of the ground state of the transverse motion in the trap and $a$ the scattering length describing the interactions between atoms at low temperature. For our experimental conditions the quantity $Na/a_{ho}$ is larger than 1000 which ensures the validity of [1]. The prediction [1] can be interpreted in terms of the Sagnac effect: in presence of a vortex the rotation of the condensate, in which the two excitations $m = \pm 2$ propagate, lifts their degeneracy.

Two experiments have led so far to the observation of a vortex line in a gaseous condensate [2,3]. The method used in [2] uses a combination of a laser and a microwave to print the desired velocity field onto the atomic wave function. One generates in this way a condensate with atoms in a given internal state rotating around a second, stationary condensate in another internal state. The interference of these two condensates allows for a measurement of the $2\pi$ phase shift of the condensate wave function around the vortex which proves the quantization of the circulation. The second method [3], which is used in the present paper, is a transposition of the rotating bucket experiment performed on $^4$He. We superimpose onto the cylindrically symmetric magnetic potential a non-axisymmetric, dipole potential created by a stirring laser beam. The combined potential leads to a cigar-shaped harmonic trap with a slight anisotropic transverse profile. The transverse anisotropy is rotated at an angular frequency $\Omega$ and can nucleate a vortex if above a critical frequency $\Omega_c$. The details of the experimental setup have been described in [3]. For the preparation of the condensate we start with $10^5$ $^{87}$Rb atoms in an Ioffe–Pritchard magnetic trap at a temperature $\sim 200 \mu K$. The oscillation frequency of the atoms along the longitudinal axis of the trap is $\omega_z/2\pi = 10.3$ Hz. For the results presented here, the transverse frequencies $\omega_{\perp}/2\pi$ have been varied between 170 Hz and 210 Hz by adjusting the bias field at the center of the trap [3].
An experimental sequence consists in four steps: (i) condensation via evaporative cooling, (ii) vortex nucleation, (iii) excitation and evolution of the quadrupolar modes, and (iv) characterization using absorption imaging after a time-of-flight expansion of $T_{\text{tof}} = 25$ ms. The probe laser for the imaging propagates along the $z$-axis, and the image gives the transverse $xy$ distribution of atomic positions after the expansion. From each image we extract the temperature of the cloud, the size of the condensate in the $xy$ plane, and the number of vortices which have been nucleated.

We evaporatively cool the atoms with a radiofrequency sweep. The condensation threshold is reached at a temperature 550 nK. We continue the evaporative cooling to a temperature below 80 nK at which point approximately $3 \times 10^5$ atoms are left in the condensate. This number is evaluated from the size of the condensate after expansion, assuming an initial Thomas-Fermi distribution [18].

After the end of the cooling phase we switch on the stirring laser beam, which is parallel with the long axis of the condensate. The central position of this beam is varied in time with respect to the $x = y = 0$ axis by two acousto-optic modulators. This temporal variation and the stirring light intensity are chosen such that the stirring laser creates on the trapped atoms a dipole potential which is well approximated by $M \omega^2 (\epsilon_x X^2 + \epsilon_y Y^2)/2$ with $\epsilon_x = 0.05$ and $\epsilon_y = 0.15$. The $X, Y$ basis rotates at constant angular frequency $\Omega$ with respect to the fixed $x, y$ basis. The stirring phase lasts 900 ms which is well beyond the typical vortex nucleation time found experimentally to be about 450 ms [17]. During this phase the evaporation frequency is raised to a relatively large value (magnetic well depth equal to 2 $\mu$K) in order not to perturb the nucleation process, and we observe a slight heating of the cloud with a final temperature of $130 \pm 50$ nK.

At the end of the vortex nucleation phase we excite a quadrupolar oscillation using the dipole potential created by the stirring laser now on a fixed basis ($X, Y = x, y$) and with a 10-times larger intensity. This potential acts on the atoms for a 0.3 ms duration, which is short compared to the transverse oscillation period. The potential created can be decomposed into (i) a part proportional to $x^2 - y^2$, which excites the transverse quadrupole motion of the condensate, and (ii) a part proportional to $x^2 + y^2$ which excites the transverse $m = 0$ breathing mode which is not relevant for the present study [18].

The transverse quadrupolar mode excited is a linear superposition of the $m = \pm 2$ modes. The lift of degeneracy between the frequencies of these two modes causes a precession of the eigenaxes of the quadrupole mode at an angular frequency given by $\dot{\theta} = (\omega_+ - \omega_-)/2|m| = (\omega_+ - \omega_-)/4$. Therefore the measurement of $\dot{\theta}$ together with the size of the condensate gives access to $L_z$.

To determine $\dot{\theta}$ we let the atomic cloud oscillate freely in the magnetic trap for an adjustable period $\tau$ (between 0 and 8 ms) after the quadrupole excitation. We then perform the time-of-flight + absorption imaging sequence, and we analyze the images of the condensate using a Thomas-Fermi type distribution [19] as a fit function, with an adjustable ellipticity and adjustable axes in the $xy$ plane.

![Fig. 1. Transverse oscillations of a stirred condensate with $N = 3.7 \times 10^5$ atoms and $\omega_+/2\pi = 171$ Hz. For a,b,c,d the stirring frequency is $\Omega/2\pi = 114$ Hz, below the vortex nucleation threshold $\Omega_c/2\pi = 115$ Hz. For e,f,g,h, $\Omega/2\pi = 120$ Hz. For a,e: $\tau = 1$ ms; b,f: $\tau = 3$ ms; c,g: $\tau = 5$ ms. The fixed axes indicate the excitation basis and the rotating ones indicate the condensate axes. A single vortex is visible at the center of the condensate in e,f.g. The figures d and h give the variations with $\tau$ of the direction $\theta$ of the large axis of the elliptical atomic cloud in the $xy$ plane. Each circle represents a single realization of the experiment.](image)
in absence of the vortex nucleation phase. As shown in fig. 1h the angle of the large axis of the elliptical condensate in the $xy$ plane oscillates between 0 and $\pi/2$ with the frequency 250 Hz ($\pm 3$ Hz), which is in good agreement with the value $\sqrt{2}(\omega_{\perp}/2\pi)$ expected for a zero temperature condensate in the Thomas-Fermi limit [18].

For a stirring frequency $\Omega/2\pi = 120$ Hz for which a vortex is systematically nucleated at the center of the condensate, the behaviour of the system is dramatically different. A precession of the long axis of the condensate occurs, as can be seen in the sequence of pictures fig. 1efg. A measurement of the angle of the large axis of the condensate in the Thomas-Fermi limit [18].

We have performed similar experiments for several values of the stirring frequency $\Omega/2\pi$ between 170 Hz and 220 Hz we have found that $\Omega_c \simeq 0.65 \bar{\omega}_{\perp}$, where $\bar{\omega}_{\perp} = \omega_{\perp}(1 + (\epsilon_x + \epsilon_y)/2)^{1/2}$ is the average transverse oscillation frequency in presence of the stirring laser. The sensitivity of $\Omega_c$ to the atom number is very small: a change of $N$ by a factor of 2 changes $\Omega_c$ by less than 5%. This is the reason for which a transition as sharp as that of fig. 2 is observed for $\Omega = \Omega_c$, although the relative dispersion of the atom number is 40%. The measured value for $\Omega_c$ is notably larger (by $\sim 50\%$) than the predicted threshold above which the vortex state is energetically favored with respect to the non-vortex state [10,23,24]. A better account for the experimental result may be obtained by estimating the stirring frequency at which the energetic barrier between these two states disappears [23,24].

An important feature of fig. 2 is the linear variation of the angular momentum as a function of $\theta/\tau$, from which we deduce $\dot{\theta} \simeq \theta/\tau$, and the angular momentum.

![FIG. 2. Variation of the angular momentum deduced from (2) as a function of the stirring frequency $\Omega$ for $\omega_{\perp}/2\pi = 175$ Hz and 2.5 ($\pm 0.6$) $\times 10^5$ atoms.](image)

The results are shown in fig. 3 for the transverse frequency $\omega_{\perp}/2\pi = 175$ Hz. These data allow for a quantitative definition of the critical frequency $\Omega_c$. This quantity was previously defined using a qualitative criterion: the presence of a density dip at the center of the condensate. It can be defined now as the value for which $L_z$ jumps from 0 to a value close to $\hbar$. The precision of this determination is of the order of 1 Hz, and it may be limited by the characteristic nucleation time (450 ms). For $\Omega \sim \Omega_c$ the large error bar in fig. 3 reflects the large dispersion in the results for $\theta$. This is illustrated more clearly in the histograms of fig. 4 which give the value of $\theta$ for $\Omega/2\pi = 115 \ldots 119$ Hz.

![FIG. 3. Distribution of the values for the direction $L_z/\hbar$ of the large axis of the condensate, after a 5 ms quadrupole oscillation for a stirring frequency $\Omega/2\pi = 115 \ldots 119$ Hz (same parameters as for fig. 2).](image)
frequencies were 256 (±1) Hz and 350 (±1) Hz respectively, which is very close to what is expected of a zero angular momentum. The nature of the atomic sample in this region of Ω is still an open question: is it condensed or uncondensed? To address this question, we have performed for ω⊥/2π = 175 Hz and Ω/2π = 145 Hz a series of measurements analogous to the ones of fig. 1 fast and found that the quadrupole and monopole oscillation frequencies were 256 (±1) Hz and 350 (±1) Hz respectively, which is very close to what is expected of a zero temperature condensate at rest i.e. $\sqrt{2} \omega_1$ and $2 \omega_1$.

To summarize we have presented in this paper a direct measurement of the angular momentum of a Bose-Einstein condensate after it has been stirred at a given frequency Ω. Using only “macroscopic” quantities such as the precession angle and the spatial extension of the condensate we have access to a “microscopic” value of angular momentum $\sim \hbar$ per particle. This measurement can be viewed as the transposition to gaseous condensates of the experiment performed with superfluid liquid helium by Vinen, which detected single quanta of circulatory angular momentum $\hbar$ per particle. This measurement is found to provide a good fit for the condensate image, except in the vicinity of the vortex core.

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