Deterministic CNOT Gate on electron qubits using quantum-dot spins in double-sided optical microcavities

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We propose a scheme to construct a deterministic CNOT gate on static electron-spin qubits, allowing for deterministic scalable quantum computing in solid-state systems. The excess electron confined in a charged quantum dot inside a double-sided optical microcavity represents the qubit, and the single photons play a medium role. Moreover, our device can work in both the weak coupling and the strong coupling regimes, but high fidelities are achieved only when the ratio of the side leakage to the cavity loss is low. Finally, we assess the feasibility of this device and show it can be implemented with current technology.

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The controlled-not (CNOT) gate has numerous applications in the field of quantum information science and it is one of the elementary elements for a quantum computer [1–3]. In 1995, Barenco et al. [4] proved that any n-qubit quantum computation can be achieved by using a sequence of one-qubit gates and CNOT gates. The archetypal two-qubit CNOT gate, or its equivalents, have been demonstrated from various perspectives and for different physical systems, including trapped ions [2, 6], nuclear magnetic spins [7], superconducting circuits [8, 9], and linear optics [10, 11]. In fact, each of these systems has its bottleneck. For example, based on linear optical elements, the maximum probability for achieving a CNOT gate is 3/4 [12]. A Superconducting circuit is fragile to decoherence. In 2004, Beenakker et al. [13] proposed a theoretic protocol for CNOT gate on moving electrons. Nemoto and Munro [14] introduced a protocol for a CNOT gate on photons with cross-Kerr nonlinearity. The CNOT gate on static qubits is more useful for a scalable quantum computing.

Recent works show that the electron spin in a quantum dot (QD) [15] can be used to store and process quantum information due to the long electron-spin coherence time (∼μs) [16] using spin echo techniques, which is limited by the spin relaxation time (∼ms) [17], and it hold great promising in quantum communications, quantum information processing, and quantum networks. The spin-QD-cavity unit, e.g., an electron confined in a self-assembled In(Ga)As QD or a GaAs interface QD inside a single-sided or a double-sided optical resonant cavity was proposed by Hu et al. [18, 19]. In this unit, the spin represents the qubit and promises scalable quantum information computing. A single spin qubit can be read out by the information of a coupling photon, and spin manipulation is well developed using pulsed magnetic-resonance technique. This unit has been used for constructing a hybrid CNOT gate and a phase-shift gate, two-photon Bell-state analyzer (BSA), teleportation, entanglement swapping, entanglement purification, and creating photon-photon, photon-spin, and spin-spin entanglements [18–24].

In this paper, we investigate the construction of a CNOT gate on the two static electrons confined in two charged QDs inside two double-sided microcavities. We first propose a device which can convert the spin parity into the out-coming photon polarization information. Using two such parity measurements, we construct a CNOT gate on two static electron-spin qubits, resorting to an ancillary static electron-spin qubit, a single-qubit measurement, and the application of single-qubit operations. Moreover, a complete deterministic two-spin BSA was constructed. In our scheme, the CNOT gate promises a scalable quantum computing in solid-state systems, in which two single photons only are mediums. The device works in both the weak coupling and the strong coupling regimes, but high fidelities are achieved only when the side leakage and cavity loss is low.

The spin-QD-double-side-cavity unit, we consider here, is a singly electron charged self-assembled GaAs/InAs interface QD inside an optical resonant double-sided microcavity with two partially reflective mirrors. The potential of this system has also been recognized in Ref. [19]. An exciton (X−) that consists of two electrons and a hole can be created by optical excitation. Here, the dipole is resonant with cavity mode, probed with a resonant light. The four relevant electronic levels are shown in Fig. 1. An exciton (X−) that consists of two electrons and a hole can be created by optical excitation. Here, the dipole is resonant with cavity mode, probed with a resonant light. The four relevant electronic levels are shown in Ref. [19]. Due to Pauli’s exclusion principle, there are two dipole transitions, one involving a photon with the spin sz = +1 and the other involving a photon with sz = −1. Considering a photon with sz = ±1, if the injecting photon coupled to the dipole, the cavity is reflected, and both the polarization and the propagation direction of the photon will be flipped. Otherwise, the cavity is transmissive and the photon will acquire a π mod 2π phase shift relative to a reflected photon. The rules of the input states changed under the interaction of the photons with sz = ±1 and the cavity are described

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as follows:

\[
\begin{align*}
| R^\uparrow \rangle & \rightarrow | L^\downarrow \rangle, & | L^\uparrow \rangle & \rightarrow -| L^\downarrow \rangle, \\
| R^\downarrow \rangle & \rightarrow -| R^\uparrow \rangle, & | L^\downarrow \rangle & \rightarrow | R^\uparrow \rangle, \\
| R^\uparrow \rangle & \rightarrow -| R^\downarrow \rangle, & | L^\downarrow \rangle & \rightarrow -| L^\uparrow \rangle, \\
| R^\downarrow \rangle & \rightarrow | L^\uparrow \rangle, & | L^\downarrow \rangle & \rightarrow -| L^\downarrow \rangle.
\end{align*}
\]

(1)

Here, $| \uparrow \rangle$ and $| \downarrow \rangle$ represent the electron-spin states $| + \frac{1}{2} \rangle$ and $| - \frac{1}{2} \rangle$, respectively. The spin quantization axis for angular momentum is along the normal direction of cavity that is the z axis. $| R \rangle$ ($| L \rangle$) is the right (left) circular polarization of a photon, and the superscripts $\uparrow$ and $\downarrow$ indicate the propagation directions of a photon along the z axis. In Fig 1, $| \uparrow \rangle$ and $| \downarrow \rangle$ represent hole-spin states $| + \frac{1}{2} \rangle$ and $| - \frac{1}{2} \rangle$, respectively.

![FIG. 1: Relevant energy levels and spin selection rules for optical transition of negatively charged exciton $X^-$.

Now, let us describe the construction for the parity-check gate (PCG) and a CNOT gate for spin-QD-double-side-cavity units. Based on the rules discussed above, the principle of our PCG for two spin qubits (in the first two cavities) is shown in Fig 2. It is relied on the spin-to-polarization conversion. Two excess electron spins in the cavities are in two arbitrary states. A probe photon passes through the polarizing beam splitter in the circuit basis (C-PBS) and it injects into the first and the second cavities in succession. After it interacts with the cavities, the photon is detected. By detecting the output of the photon, one can distinguish the spin states of the two-electron system $\{| \uparrow \downarrow \rangle, \downarrow \downarrow \rangle \}$ from $\{| \uparrow \uparrow \rangle, \downarrow \uparrow \rangle \}$. If the two spins are parallel $| \uparrow \uparrow \rangle$ or $| \downarrow \downarrow \rangle$, the polarization of the probe photon in the state $| R \rangle$ ($| L \rangle$) will remain and the photon will trigger the detector $D_2$ ($D_1$); otherwise, the state of the probe photon will be flipped and the photon will be detected by the detector $D_1$ ($D_2$). The evolution of the photon-cavity state can be described as

\[
\begin{align*}
| R^\uparrow \rangle | \uparrow \downarrow \rangle & \rightarrow | R^\uparrow \rangle | \uparrow \downarrow \rangle, & | R^\downarrow \rangle | \uparrow \downarrow \rangle & \rightarrow -| R^\downarrow \rangle | \uparrow \downarrow \rangle, \\
| R^\uparrow \rangle | \downarrow \uparrow \rangle & \rightarrow | R^\uparrow \rangle | \downarrow \uparrow \rangle, & | R^\downarrow \rangle | \downarrow \uparrow \rangle & \rightarrow -| R^\downarrow \rangle | \downarrow \uparrow \rangle, \\
| L^\uparrow \rangle | \uparrow \downarrow \rangle & \rightarrow | L^\uparrow \rangle | \uparrow \downarrow \rangle, & | L^\downarrow \rangle | \uparrow \downarrow \rangle & \rightarrow -| L^\downarrow \rangle | \uparrow \downarrow \rangle, \\
| L^\uparrow \rangle | \downarrow \uparrow \rangle & \rightarrow | L^\uparrow \rangle | \downarrow \uparrow \rangle, & | L^\downarrow \rangle | \downarrow \uparrow \rangle & \rightarrow -| L^\downarrow \rangle | \downarrow \uparrow \rangle.
\end{align*}
\]

(2)

Based on spin-QD-double-side-cavity systems, the principle of our CNOT gate is shown in Fig 2. It is used to flips the spin of the target qubit if the spin of the control qubit is $| \downarrow \rangle$; otherwise, it does nothing. Suppose that the two excess electron spins in the first cavity and third cavity are considered as the control qubit and the target qubit, respectively. They are in two arbitrary states $| \psi_1 ^\alpha \rangle = \alpha_1 | \uparrow \rangle + \beta_1 | \downarrow \rangle$ and $| \psi_3 ^\beta \rangle = \alpha_3 | \uparrow \rangle + \beta_3 | \downarrow \rangle$, respectively. The ancilla qubit in the second cavity is prepared in the state $| \psi _{\text{anci}} ^\gamma \rangle = \frac{1}{\sqrt{2}} (| \uparrow \rangle + | \downarrow \rangle)$. Our scheme consists of three parts. (i) We take two PCGs on spin pairs 1-2 and 2-3 in series, with a Hadamard transformation (e.g., using a $\pi/2$ microwave pulse)

\[
| \uparrow \rangle \rightarrow \frac{1}{\sqrt{2}} (| \uparrow \rangle + | \downarrow \rangle), \quad | \downarrow \rangle \rightarrow \frac{1}{\sqrt{2}} (| \uparrow \rangle - | \downarrow \rangle),
\]

(3)

on the ancilla qubit and target qubit before and after the second PCG operation, respectively. The first probe photon (label $in_1$) is originally in state $| R^\uparrow \rangle$ and the second one (label $in_2$) is in $| L^\uparrow \rangle$. (ii) The ancilla qubit is measured. (iii) According to the result of two PCGs and the spin of the ancilla qubit, a proper classical feed-forward is performed on the control qubit and the target qubit to complete a CNOT gate with the success probability of 100%. The correspondences between the results of each measurements and specific feed-forwards are given in Table 1.

BSA is an important prerequisite for many quantum protocols, such as superdense coding, teleportation, entanglement swapping, and so on. Next, based on spin-QD-double-side-cavity units, we show the principle of our
TABLE I: The correspondences between the results of two PCG operations and the spin of the ancilla and the feedforward operators applied to the control and the target spins in the construction of a static two-spin-qubit CNOT gate.

| PCG1 | PCG2 | ancilla qubit | feedforward | control qubit | target qubit |
|------|------|---------------|-------------|--------------|--------------|
| R    | L    | ↑             | σ_x         | L            | σ_x          |
| R    | L    | ↑             | σ_x         | ↓            | σ_x          |
| L    | R    | ↑             | σ_x         | ↑            | σ_x          |

By far, we have shown the principles for PCG, CNOT gates, and BSA under the ideal condition. We consider imperfections due to side leakage of cavity field, the trion dephasing, and the heavy-light hole mixing.

The fidelity of the CNOT gate associates with the reflection and transmission operators of the system. The two operators, include the contributions both from the uncoupled and from the coupled cavities, can be described as

\[ F = 1 - P, \]

with

\[ r(\omega) = 1 + t(\omega), \]

\[ t(\omega) = \frac{-\kappa[\sqrt{2}(\omega_X - \omega) + \frac{\omega}{2}] + 2g^2}{[\sqrt{2}(\omega_X - \omega) + \frac{\omega}{2}][\sqrt{2}(\omega_X - \omega) + \kappa + \frac{\omega}{2}]} \]

Here, \( r(\omega) \) and \( t(\omega) \) are the reflection and the transmission coefficients of the coupled cavities with \( g \neq 0 \), respectively. \( r_0(\omega) \) and \( t_0(\omega) \) are the reflection and the transmission coefficients of the uncoupled cavities with \( g = 0 \) in Eq. (9). \( \omega, \omega_c, \) and \( \omega_X \) are the frequencies of the input photon, cavity mode, and \( X^- \) transition, respectively. \( g \) is cavity coupling strength. \( \gamma/2, \kappa, \) and \( \kappa_s/2 \) are the \( X^- \) dipole decay rate, the cavity field decay rate, and the cavity field leaky rate, respectively.

In our schemes, we consider the resonance with \( \omega_c = \omega_X^- = \omega \). Eq. (6) can be simplified as

\[ t_0(\omega) = -\frac{\kappa}{\kappa + \frac{\omega}{2}}, \quad r_0(\omega) = \frac{\omega}{\kappa + \frac{\omega}{2}}. \]

and

\[ r(\omega) = 1 + t(\omega), \quad t(\omega) = -\frac{2\kappa}{\kappa + \frac{\omega}{2}} + g^2. \]

For an ideal case, that is, the side leakage \( \kappa_s \) is much lower than the cavity decay rate \( \kappa \), and then \( |r_0(\omega)| \to 1, |r(\omega)| \to 0 \) for the cold cavity and \( |t(\omega)| \to 0 \), \( r(\omega) \to 1 \) for the hot cavity in the strong coupling regime \( g > \sqrt{\kappa \gamma} \) [19]. Our scheme for a CNOT gate can achieve a unity fidelity in the strong-coupling regime. However, this is a big challenge for QD-micropillar cavities although significant progress has been made [27].

For an unideal case, that is, the cavity side leakage \( \kappa_s \), which will cause bit-flip error, is taken into account, and then

\[ |R \downarrow \rangle \to |r||L \downarrow \rangle + |t||R \downarrow \rangle, \]

\[ |L \downarrow \rangle \to |r||R \downarrow \rangle + |t||L \downarrow \rangle, \]

\[ |R \uparrow \rangle \to -|t_0||R \uparrow \rangle - |r_0||L \uparrow \rangle, \]

\[ |L \uparrow \rangle \to -|t_0||L \uparrow \rangle - |r_0||R \uparrow \rangle. \]

The fidelity of the CNOT gate can be written as

\[ F = 1 - P, \]

\[ = \frac{(\frac{\omega_c}{\kappa})^4 + 16}{\kappa^4 + 16(\frac{\omega_c}{\kappa})^2 + 16} = \frac{200(\frac{\omega_c}{\kappa})^4 + 1}{200(\frac{\omega_c}{\kappa})^4 + 3} \]

by taking \( \gamma = 0.1 \) which is experimentally achieved, \( |t_0| = |r| \) (that is, \( \langle \frac{\omega_c}{\kappa} \rangle^2 = \frac{\kappa c}{\kappa c} \) which is required for our protocol, and \( \omega_c = \omega_X^- = \omega \). Here \( P \) is the error rate.

As shown in Fig. 3(a) and Fig. 3(b), a near-unity fidelity of the CNOT gate can be achieved with small \( \kappa_s/2\kappa \) in the strong-coupling regime with \( g/\kappa = 2.4 \), which can be achieved for the In(Ga)As QD-cavity system [27, 28]. The lower \( \kappa_s/2\kappa \), the higher \( F \). A higher fidelity could
optical dephasing can reduce the fidelity by only a few percents as the optical coherence time of exciton in self-assembled In(Ga)As QDs is ten times long as the cavity photon lifetime. The former can be reach several hundred picoseconds [29], however, the later is around tens of picoseconds in the strong coupling regime for a cavity with a Q-factor of $10^4 - 10^5$; the spin dephasing of the $X^{-}$ which mainly arises from the hole-spin dephasing, can be safely neglected, that is because spin coherence time is at least three order of the magnitude longer than the cavity photon lifetime [30, 31]. For the imperfect optical selection rule which is caused by the heavy-light hole mixing in realistic QD (due to the asymmetric in the QD shape and the strain field distribution), since the hole mixing could be reduced by engineering the shape and the size of QDs or choosing different types of QDs and in the valence band is in the order of a few percents [32] [e.g., for self-assembled In(Ga)As QDs], the imperfect optical selection rule can reduce the fidelity by only a few percents.

In summary, we have proposed a device which can convert the spin parity of two static electron-spin qubits confined in charged QDs inside double-sided microcavities into the out-coming photon polarization information. Using two such parity-check measurements, we construct a deterministic CNOT gate on electron-spin qubits, allowing for deterministic scalable quantum computing in solid-state systems. Subsequently, a possible application of the spin-QD-double-side-cavity, a spin Bell-state analyzer was discussed. Moreover, from the investigation on the fidelity of the CNOT gate, one can find that our proposal works in both the weak coupling and the strong coupling regimes, but high fidelities are achieved only when the ratio of the side leakage to the cavity loss is low.

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