Experimental and Numerical Investigation of Vibration Power Flow of Thin Plate Using Cross Power Spectral Density Based Technique

O K Nyein¹*, B A Aminudin¹, S Asnizah²
¹Department of Mechanical Precision Engineering, Malaysia-Japan International Institute of Technology, Universiti Teknologi Malaysia, Malaysia
²Mechanical Engineering Department, Politeknik Sultan Salahuddin Abdul Aziz Shah, Shah Alam, Malaysia

*Corresponding author: kokyawnyein@gmail.com

Abstract. The application of thin plates as elements of machines and vehicles component working under various operational condition causes degradation and occurrence of damage. Hence, it is extremely important to identify the dynamic characteristic and the vibration power flow of the thin structure that indicates magnitude and direction of vibration power distribution, and transmission path. This study aims to examine the potential changes in the dynamic characteristics and the vibration power flow of the thin plate at its first four natural frequencies. The numerical modal analysis and the experimental modal analysis of the thin plate was conducted in the first step. The vibration power flow was measured utilizing the cross power spectral density-based technique which was validated with the results from calculation. The power flow maps of the plate at the first four natural frequencies are presented. The visualization utilizing vector plot and streamline shows the vibration power flow direction, magnitude, and transmissiion paths in the plate. The results show that the higher frequency, the path of the vibration power flow become more complicated. This research proposed a measuring method, through simple calculation and visualization of vibration power flow of the plate that may give a contribution to the design phase in manufacturing sectors, the manufacturer, and structural health monitoring.

1. Introduction

Nowadays, plates of various thickness are widely being used in as components of different structures in civil work, mechanical, aerospace, naval architecture, and marine engineering applications with the advancement of technology. These structures are facing vibration problems in different frequency ranges which occurs due to the external dynamic loading which leads the results in fatigue failure of structures, NVH performance deficiency and destruction of mechanical equipment. On the other hand, in the designing process, the structural system must be satisfied certain criteria for a safe application and maintenance. This is important that the designed structure can be utilized with their intended environment and their design basis service life. Therefore, the designed structure or the existing structures which are being used and operating usually, are necessary to be examined for the vibration, noise, and harshness NVH performance or preventive actions to avoid failure and its aftereffects.
For structural reliability of designing processes and manufacturing processes, and minimizing vibration level, not only the dynamic characteristics of the structures play an important role but also the vibration power flow which indicates vibration energy transmission path, positions of sources and sinks of vibration in the structure. The vibration power flow has been recognized as a very interesting method in reporting the vibrational response of structure since the past few decades [1-4]. Hambric, S. A [5] has developed a method for computing power flow variables in finite element model. In addition, the mobility power flow approach, the wave transmission approach, and the spectral element method were reported for predicting the power transmission of structure [6-8]. Zhao Y, et al. [9] studied power flow transmission in complex space truss structures by travelling wave method. Wu W, et al. [10] proposed a method to investigate the characteristic of power flow and transmission path within built-up structures by forming dynamic stiffness matrix of the plate. Zhang X, et al. [11] also discussed the vibration power flow characteristics of the simply supported composite plate considering effect of elastic modulus ratio. Moreover, Ullmann R, et al. [12] evaluated the energy exchange of vehicle-related framework-like structures by means of active mean power. While many researchers focused on the methods and power flow of structure, Liu C.-C, et al. [13] investigated the influence of the plate thickness on the vibration power flow by comparing Mindlin plate theory and classical plate theory.

The visualization of power flow offers valuable information such as location of vibration source, and transmission path of energy by plotting vector flow. Therefore, some researchers have generated the vector flow of structural intensity to characterize the joint behavior of the plates which is connected by loosened bolts [14], to predict the surface mobility of a finite plate utilizing structural intensity [15], and to analyze welded connection of the plate which has depicted the influence of the connection stiffness on structural intensity vector field [16]. Bochniak W and Cieślik J [17] discussed the relations between the structural intensity distribution and structural mode shapes, and the effects of excitation frequency on the changes in energy flow in plate. Sparavigna A and Montrucchio B [18] developed an algorithm for vector field visualization with oriented streamlines. The streamlines representation method has been used to visualize the energy transmission path at plate surface [19, 20]. Wang B, et al. [21] developed a judging principle for analysing and applying the vibrational transmission path. However, the streamlines are not able to give information on the strength of vector field or vibration energy. Chen YH, et al. studied the energy flow behaviour of a coupled cylindrical shell-plate structure through parametric power flow analysis [22]. Most studies explained above were conducted by numerical and analytical methods. Petrone G, et al. [23] proposed a mixed approach, based on numerical and experimental data to investigate structural intensity in plates. In their study, the experimental displacements were used as imposed condition to generate the used forces from the numerical model which were applied for the calculation of structural intensity. Cieślik J investigated the total energy flow in the plate [24]. The method used in this investigation has disadvantage which is poor convergence and greater number of mode shapes are needed. For measuring power flow, Verheij J.W formulated cross power spectral density methods for bending, longitudinal and torsional waves [25]. Up to author’s knowledge, the experiment measurement of power flow is very limited.

The knowledge of the dynamic behaviour of the structure is important because the structural vibration problems present a major hazard and design limitation. To minimize this issue the idea with creative ways to identify the vibration transmission path and vibration power flow is necessary. In this study, the vibration power flow of a thin plate was investigated experimentally and numerically utilizing cross power spectral density-based technique. As first step, the model validation process was performed by the experimental and numerical modal analysis. Then, the measured power flow was compared with the calculated power flow, and the results show good agreement. The calculation of power flow at the first four natural frequencies was conducted using the data from the simulation to generate the power flow maps of the plate.
2. Methodology

The numerical and experimental techniques evaluate the dynamics characteristics detecting vibration and acoustical issues are one of the design processes that required to be done. This is because the modal frequency response analysis can generate the parameters which are required to calculate the dynamics characteristics and the vibration power flow.

In this study, the experimental work was performed by using a shaker, triaxial accelerometers, and integrated Simcenter test lab which is functioning as data acquisition for a complete experiment setup. For the laboratory testing of structure (aluminium plate), free boundary condition is the most frequently employed. Since, the interested boundary condition of the plate was free-free condition, a small diameter of rubber string from bungee cord was used to simulate the free-free model of the plate. This cord has very small stiffness and a much lower frequency than the lowest frequency of the plate. So, the effect of these suspension cords can be negligible. A small diameter and seven centimetres length of stinger rod that should be rigid in the axial direction and flexible in the lateral direction was used in this experiment to decouple the shaker from the testing object and transmit force to the testing object. To measure and transmit force to the structure, an impedance sensor was fitted at the top of the stinger. The excitation point was selected at position (27.5 cm, 5.2 cm) of the plate. The measuring point should be taken as same as the point of the finite element model to specify modal properties. A tri-axial accelerometer that was used in this testing, can measure vibration acceleration in X, Y and Z directions of every measuring point. The accelerometer was mounted on the measuring point by using beeswax. The experimental setup is shown in Figure 1.

![Simcenter test lab](image)

**Figure 1.** Experimental setup

To generate the results from the experiment, post processing task was performed using Simcenter Testlab. The summed frequency response function of the plate which is a summation of FRF of all measured points is shown in Figure 2. In this figure, it shows the series of “s” in red colour which means that the mode is stable. In addition, mode indicator function (MIF) in green colour is also depicted in the figure. This function indicates how many of poles to select at each resonance. In this way, all possible modes were selected which are depicted in the left side of Simcenter Testlab window. Although there are three peaks, MIF indicates four poles showing stable mode symbol.
The auto modal assurance criterion (MAC) was used to verify the validity of the generated modes within the same mode set. The modal assurance criterion of this experiment is shown in Figure 3.

In the figure, along the diagonal, every mode is identical to itself which is indicating a red colour bar (100%). Off the diagonal, the MAC values are very low indicating blue colour (less than 6%) and yellow colour (less than 27%). Therefore, it can be concluded that the results from testing are reliable. Then, the vibration power flow was measured as the cross-power spectral density.

On the other hand, the simulation for the modal analysis was conducted using MSC Patran/Nastran software. A shell element was used to create finite element model of the plate because it is dealing with bending effect and membrane effect. The dimensions of the plate are 0.328m width, 0.232m height, and 0.003m thickness. After that, the input of material property was entered in the material form that provides the option to create various MSC Nastran materials. The material properties of the aluminium plate are modulus of elasticity 61 GPa, density 2700 kg/m³, and Poisson ratio 0.3. To clarify the modelling of the plate structure, the dynamics characteristics which are results from experimental and simulation work are validated. Then, the frequency response analysis was performed to generate the parameters which are necessary to calculate the vibration power and to visualize the vibration power flow map. The data were extracted from Nastran output and the calculation of power flow was conducted in MATLAB software. The results were post-processed by utilizing two MATLAB functions: quiver and streamslice. The quiver function allows to draw a vector map from two vibration power components which show the magnitude and direction of the vibration power. The streamline provides the information of the power flow path across the plate.
3. Results and Discussion

In this study, experimental modal analysis was conducted to validate the results from numerical modal analysis and to improve the reliability of the finite element model because the agreement of the natural frequency of the plate is very important for model reliability. The comparison of natural frequencies of the plate is shown in Table 1.

| Modal order | Natural frequency from experiment (Hz) | Natural frequency from simulation (Hz) | % Error |
|-------------|----------------------------------------|---------------------------------------|---------|
| 1           | 119.45                                 | 119.68                                | 0.19    |
| 2           | 137.83                                 | 137.29                                | 0.39    |
| 3           | 280.84                                 | 281.46                                | 0.22    |
| 4           | 292.17                                 | 289.75                                | 0.82    |

This study focuses on the first four natural frequencies. The relative errors between numerical and experimental natural frequencies are reported. The natural frequencies have good agreement of experimental and simulation results with the range of percentage error; 0.19% to 0.82%. Therefore, it can be said that the finite element model of the plate has been validated in term of natural frequencies for further study.

The advancement of digital equipment has inspired the use of signal processing in frequency domain. Therefore, cross power spectral density technique is utilized for measuring vibration power flow of the plate. For the numerical result of power flow, the required data; transverse shear forces, bending moments, twisting moment, membrane force, membrane shear, translational velocity, and rotational velocity are generated from numerical frequency response analysis in MSC Nastran. The quadrilateral element that is used in the modeling is two dimensional. Therefore, the power flow in the x and y directions were calculated. The comparison of vibration power flow from measurement and simulation is shown in Figure 4. The natures of magnitude of both vibration power flow curves are similar. These power flow curves indicate power along with the frequency of any selected point of the plate. In this figure, there are four peaks indicating the level of highest vibration power at certain frequencies. These frequencies are very near to natural frequencies of the plate. Therefore, the power flow map of the plate should be investigated at these frequencies.

![Figure 4. comparison of vibration power flow](image-url)
The real part of the power flow is called active power flow. In practical conditions, the active power flow which is responsible for vibration energy transfer is analyzed. For energy conservation of the system, the imaginary power flow which is called reactive power flow is analyzed.

Figure 5. vibration power flow maps on the plate with free-free boundary condition

In this study, active power flow at every node of finite element model is calculated at each natural frequency to visualize vibration power flow on the plate as vector flow map and power flow path as streamline. The vibration power flow map on the plate with free-free boundary condition is shown in Figure 5.

In this figure, the black arrow shows the magnitude and direction of the active power flow, and the blue streamline indicates the vibration transmission path. As shown in Figure 5.a and 5.c, power sinks are observed at its first and third natural frequencies. At first natural frequency, all vibration power flows toward the center of the plate except the power at four corners. In another two modes (second and fourth mode), vibration power is spreading over the plate. The source of vibration and high vibration appear at the corners of the plate.

4. Conclusion

This study validated the measured vibration power using cross power spectral density with the calculated power. The visualization utilizing vector plot and streamline shows the vibration power
flow direction, magnitude, and transmission paths in the plate. These pictures play an important role in analysis and understanding of vibration power flow showing power dissipation by structural damping at each natural frequency. The vibration power flow distribution of the plate can be clearly seen through the description of vibration power flow map. The changes of vibration power flow provide information for the effective control of the vibration of the structure such as where the structure should be strengthened, and where the high vibration power occurs at certain frequency. It can also be observed as the identification of the locations for application of additional damping to reduce the vibration level. In addition, the power flow offers transmission paths, and positions of sources and sinks of vibration energy that can be useful for the engineer concerning with noise, vibration, and harshness NVH problems.

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