Phonon-Induced Resistance Oscillations of Two-Dimensional Electron Systems Drifting with Supersonic Velocities

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We present a theory of the phonon-assisted nonlinear dc transport of 2D electrons in high Landau levels. The nonlinear dissipative resistivity displays quantum magneto-oscillations governed by two parameters which are proportional to the Hall drift velocity \( v_H \) of electrons in electric field and the speed of sound \( s \). In the subsonic regime, \( v_H < s \), the theory quantitatively reproduces the oscillation pattern observed in recent experiments.

We also find the \( \pi/2 \) phase change of oscillations across the sound barrier \( v_H = s \). In the supersonic regime, \( v_H > s \), the amplitude of oscillations saturates with lowering temperature, while the subsonic region displays exponential suppression of the phonon-assisted oscillations with temperature.

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A high-mobility two-dimensional electron gas (2DEG) in a weakly quantizing perpendicular magnetic field \( B \) displays a rich variety of magneto-oscillation phenomena. These phenomena are defined by the ratio of energy parameters and the Landau level spacing \( \hbar \omega_c \), where \( \omega_c = eB/m^*c \), or by ratio of the spatial length scales and the cyclotron radius \( R_c = v_F/\omega_c \), where \( v_F (p_F = mv_F) \) is the Fermi velocity (momentum).

Examples of magneto-oscillations include: (i) the microwave-induced resistance oscillations (MIRO) \[1-3]\] and associated zero-resistance states (ZRS) \[4,5\] controlled by the ratio of the microwave frequency \( \omega \) and cyclotron frequency \( \omega_c \), \( \epsilon_{dc} \equiv \omega/\omega_c \); (ii) the Hall field-induced resistance oscillations (HIRO) \[6,7\] and associated zero-differential resistance states (ZdRS) \[8\] in the dc electric field \( E \), governed by \( \epsilon_{dc} \equiv eE(2R_c)/\hbar \omega_c \); (iii) the phonon-induced resistance oscillations (PIRO) \[9-12\], characterized by the ratio \( \epsilon_{ph} \equiv \omega/\omega_c \), where \( \hbar \omega_c = 2sp_F \) is the energy of an acoustic phonon with momentum \( 2p_F \) and \( s \) is the speed of sound. The above oscillations stem from commensurability of energy change in photon- (MIRO); disorder- (HIRO), and phonon-assisted (PIRO) scattering events and the \( \omega_c \)-periodic oscillations of the density of states (DoS). MIRO and HIRO \[1-3,4-8\], as well as their interplay \[13-15\], have been studied in detail over the past decade.

In this paper, we present a theory of the phonon-assisted transport in strong dc electric fields. In the nonlinear dc-response, our results are consistent with the observed evolution of PIRO in the regime \( \epsilon_{dc} < \epsilon_{ph} \) in recent experiment \[10\]. We predict that the phase change of oscillations across the sound barrier \( \epsilon_{dc} = \epsilon_{ph} \), where the Hall drift velocity \( v_H = eE/B \) equals to the speed of sound. We also analyze the temperature dependence of the phonon-assisted transport in different regimes.

Model. For a homogeneous 2DEG in a region of classically strong \( B \), when \( \omega_c \) exceeds the transport scattering rate \( 1/\tau_q \), the dissipative current \( j_x \) in response to an electric field \( E \) (pointing along \( x \) direction, \( \varphi = 0 \)) is

\[
j_x E = 2\nu_0 \int W_{\varphi,\varphi'} P_{\varphi,\varphi'}(\varepsilon) d\varepsilon \frac{d\varphi d\varphi'}{4\pi^2}.
\]

Here \( \nu_0 = m/2\pi \) is the DoS at zero magnetic field (hereafter we put \( \hbar = 1 \)) and \( P_{\varphi,\varphi'}(\varepsilon) \) denotes the semiclassical scattering rate of an electron with energy \( \varepsilon \) with the change in its momentum direction from \( \varphi \) to \( \varphi' \):

\[
P_{\varphi,\varphi'}(\varepsilon) = \frac{\tau_{\varphi,\varphi'}}{\tau_{\varphi,\varphi}} M(\varepsilon, \varepsilon + W_{\varphi,\varphi'}) + \frac{g^2 |\omega_{\varphi,\varphi'}|}{2} N_{\omega_{\varphi,\varphi'}} M(\varepsilon, \varepsilon + W_{\varphi,\varphi'}) + \frac{g^2 |\omega_{\varphi,\varphi'}|}{2} (N_{\omega_{\varphi,\varphi'}} + 1) M(\varepsilon, \varepsilon + W_{\varphi,\varphi'} - |\omega_{\varphi,\varphi'}|).
\]

In scattering of static disorder, represented by the first line in Eq. (2), electron energy \( \varepsilon \) changes by the electrostatic work \( W_{\varphi,\varphi'} = eER_0 \sin(\varphi - \varphi') \) for displacement \( \Delta R \) of electron cyclotron trajectory in electric field, see Fig. [1]. The second (third) line in Eq. (2) describes processes with change in electron electrostatic energy \( W_{\varphi,\varphi'} \) and absorption (emission) of a phonon with energy \( \omega_{\varphi,\varphi'} \), where \( \omega_{\varphi,\varphi'} = 2p_F s \sin(\varphi - \varphi')/2 \) for \( s \ll v_F \). The Planck’s function \( N_{\omega_{\varphi,\varphi'}} = 1/[\exp(|\omega_{\varphi,\varphi'}|/T) - 1] \) accounts for the thermal occupation of phonon modes, and \( g \) is the dimensionless electron-phonon coupling constant \[10\].

Factors \( M(\varepsilon, \varepsilon') = \nu(\varepsilon) \nu(\varepsilon') f(\varepsilon)[1 - f(\varepsilon')] \) in Eq. (2) contain the Fermi function \( f(\varepsilon) = 1/[\exp(\varepsilon - \mu)/T + 1] \) and the product of the DoS of initial and final states. The DoS in the experimentally relevant region of weak magnetic fields, \( \tau_q \omega_c \lesssim 1 \), has the form

\[
\nu(\varepsilon) \equiv \nu(\varepsilon)/\nu_0 = 1 - 2\lambda \cos(2\pi \varepsilon/\omega_c),
\]

where \( \lambda = \exp(-\pi/\omega_c \tau_q) \) and \( \tau_q \) is the quantum scattering time off disorder.

Using Eqs. (1)–(3) and assuming \( T \tau_q \lesssim 1 \), we obtain

\[
j_x E = \frac{\rho_D E^2 \tau_q}{\rho_H} \left( \Gamma_{\text{dis}} + \Gamma_{\text{ph}}^{(1)} + \Gamma_{\text{ph}}^{(osc)} \right),
\]

where \( \Gamma_{\text{dis}} \) is the disorder scattering rate, and \( \Gamma_{\text{ph}}^{(1)} \) and \( \Gamma_{\text{ph}}^{(osc)} \) are the elastic and inelastic scattering rates of electrons on acoustic phonons, respectively.
The position of four regions Fig. 1a-d in the yellow stripes). Dashed line marks the position of the Fermi level.

Fig. 1 illustrates the real-space shift $\Delta R$ of the cyclotron guiding center and the change in the electron momentum for scattering from $\varphi$ to $\varphi'$.

FIG. 1: Illustration of electron-phonon scattering processes in different regimes: (a) for $\epsilon_{dc} = 0$; (b) for $\epsilon_{dc} < \epsilon_{ph}$; (c) for $\epsilon_{dc} = \epsilon_{ph}$; (d) for $\epsilon_{dc} > \epsilon_{ph}$. Arrows show the change of the total energy of electron due to emission or absorption of a 2$\omega_p$-phonon accompanied by the shift of the guiding center of the cyclotron orbit by $2R_C$ along or against the dc field. The arrows are directed along the sound cone (solid lines), while the strength of dc field determines the tilt of Landau levels (maxima of oscillatory DoS Eq. (3) are marked by yellow stripes). Dashed line marks the position of the Fermi level.

The position of four regions Fig. 1a-d in the $\epsilon_{ph} - \epsilon_{dc}$ plane is shown in Fig. 1e. Fig. 1f illustrates the real-space shift $\Delta R$ of the cyclotron guiding center and the change in the electron momentum for scattering from $\varphi$ to $\varphi'$.

where $\rho_D = \frac{\pi v_0}{(e^2 N_e \tau_0)}$ and $\rho_H = B/(eN_e)$ are the Drude and Hall resistivities, $N_e = \frac{me^2 F(\pi \epsilon_{dc})}{\rho_H}$ is the electron surface density. The effective scattering rate of disorder was obtained in [11]: $\Gamma_{\text{dis}}(\epsilon_{dc}) = \tau_\pi^{-1} - 2\lambda^2 F''(\pi \epsilon_{dc})$, with $F(x) = \sum n_i^2(x)\tau_\pi$ and $J_i(x)$ standing for the Bessel functions; the scattering rate $\tau_{\varphi'-\varphi'}^{-1} = \sum \tau_\pi^{-1} e^{i\varphi}(\varphi'-\varphi)$ is presented by its angular harmonics $1/\tau_\pi (\tau_0 \equiv \tau_\pi)$.

Here we focus on the phonon-assisted scattering rates

$$
\begin{align*}
\Gamma_{\text{ph}}^{(\text{sm})} = g^2 T \int \left\{ \frac{1}{2 \lambda^2 \cos \frac{\varphi - \varphi'}{\omega_c/2\pi}} \right. & \\
& \times \left. (\sin \varphi - \sin \varphi')^2 \Lambda \left( \frac{\omega_c - \varphi'}{2\pi} \right) \right\} \\
& \left( W_{\varphi - \varphi'} \right) d\varphi d\varphi',
\end{align*}
$$

where $\Gamma_{\text{ph}}^{(\text{sm})}(\epsilon_{dc}, \epsilon_{ph})$ and $\Gamma_{\text{ph}}^{(\text{osc})}(\epsilon_{dc}, \epsilon_{ph})$ denote the smooth and oscillatory components of the electron-phonon scattering rate, and $\Lambda(x, y) = S(x)S(x - y)/S(y)$ is written in terms of $S(x) = x/\sinh x$. We first study the behavior of $\Gamma_{\text{ph}}^{(\text{sm})}$ and $\Gamma_{\text{ph}}^{(\text{osc})}$ in the experimentally relevant limit $T \gg \omega_c$ and then analyze how these rates evolve with lowering $T$.

**Main results.** At high $T \gg W, \omega$, we approximate $\Lambda(\omega/2T, W/2T) \approx 1$ in Eq. (5) and obtain constant $\Gamma_{\text{ph}}^{(\text{sm})} = g^2 T$ for the smooth part of electron-phonon transport scattering rate. The oscillatory part, which represents the effect of the Landau quantization, reduces to

$$
\Gamma_{\text{ph}}^{(\text{osc})} = 8\lambda g^2 T \int_0^{2\pi} \frac{d\varphi d\varphi'}{4\pi^2} \sin^2 \varphi - \cos^2 \varphi' \times \cos \left[ 2\pi \sin \varphi - (\epsilon_{ph} - \epsilon_{dc} \cos \varphi) \right].
$$

The behavior of $\Gamma_{\text{ph}}^{(\text{osc})}(\epsilon_{dc}, \epsilon_{ph})$ is different in parametric regimes, illustrated in Fig. 1f. In the limit of weak electric fields, $\epsilon_{dc} \ll 1$, the current is linear in the applied dc field and

$$
\Gamma_{\text{ph}}^{(\text{osc})}(0, \epsilon_{ph}) = 2\lambda^2 g^2 T [J_0(\epsilon_{ph}) - J_2(2\pi \epsilon_{ph})].
$$

The oscillations with $\epsilon_{ph} = \omega_c/\omega_\pi$ in Eq. (7) are driven by commensurability between the phonon energy $\omega_\pi = 2\omega_p$ and the period of DoS oscillations $\omega_c$, see Fig. 1f. For $\epsilon_{ph} > 1$, Eq. (7) gives $\Gamma_{\text{ph}}^{(\text{osc})}(0, \epsilon_{ph}) = (4\lambda^2 g^2 T/\pi \sqrt{\rho_{\text{ph}}} \cos(2\pi\epsilon_{ph}) - \pi/4)$. This regime was studied previously by Raichev [16]. The difference in the oscillations phase of our result and that of Ref. [17] is due to a different choice of the photon scattering form-factor in narrow vs. wide quantum wells.

A strong Hall field tilts the Landau levels and changes the commensurability conditions for electron-phonon scattering as shown in Fig. 1b-d, resulting in oscillations with combined parameters $\epsilon_{\pm} = \epsilon_{dc} \pm \epsilon_{ph}$. Equivalently, this effect can be
viewed as a result of the Doppler shift of the phonon modes in the frame moving with the Hall velocity $v_H$ in $y$ direction. In the moving frame, the electric field is absent while the phonon speed changes as $s \to s - v_H \cos \varphi_+$. This theoretical result quantitatively reproduces the experimental evolution of PIRO with increasing $\tau_0 \equiv \tau_0$.

This behavior can also be inferred from the analytical results for $\Gamma_{\phi \rho}^{(osc)}$, Eq. (5), obtained for low temperatures $T \lesssim 2p_F$, but still $T \gtrsim \omega_c$. For $\epsilon_{dc} < \epsilon_{ph}$, we have

$$\Gamma_{\phi \rho}^{(osc)}(\epsilon_{dc}, \epsilon_{ph}) = \frac{4 \lambda^2 g^2 \omega_c \epsilon_{ph}}{\pi^2 \epsilon_{dc}} \exp\left(\frac{-2 |\epsilon_-| \omega_c}{T}\right) \times \left[\sqrt{\frac{1}{2} \sin 2 \pi \epsilon_- + \sqrt{\frac{1}{2} e^{-2 \epsilon_{dc} \omega_c/T} \sin 2 \pi \epsilon_+}\right].$$

In the supersonic regime, $\epsilon_- = \epsilon_{dc} - \epsilon_{ph} > 0$, we obtain

$$\Gamma_{\phi \rho}^{(osc)}(\epsilon_{dc}, \epsilon_{ph}) = \frac{2 \lambda^2 g^2 \omega_c \epsilon_{ph}}{\pi^2 \epsilon_{dc}} \left[\frac{T}{\omega_c} + \sqrt{\frac{2}{\pi}} e^{-2 \epsilon_{dc} \omega_c/T} \sin 2 \pi \epsilon_+\right].$$

A similar behavior also occurs with the smooth component of the electron-phonon rate $\Gamma_{\phi \rho}^{(sm)}$: it vanishes in the subsonic regime and saturates in the supersonic regime, see Fig. [3]. For $T = 0$, the smooth component

$$\Gamma_{\phi \rho}^{(sm)}(\epsilon_{dc}, \epsilon_{ph}) = \frac{8 g^2 \omega_c \epsilon_{ph}}{3 \pi^2} \left[\arccos \frac{\epsilon_{ph}}{\epsilon_{dc} - \sqrt{\epsilon_{dc} - \epsilon_{ph}}} - \frac{\epsilon_{dc} - \epsilon_{ph}}{\sqrt{\epsilon_{dc} - \epsilon_{ph}}}ight]$$

for $\epsilon_{dc} > \epsilon_{ph}$ and $\Gamma_{\phi \rho}^{(sm)}(\epsilon_{dc}, \epsilon_{ph}) = 0$ otherwise. At finite but low temperature, $\Gamma_{\phi \rho}^{(sm)}(\epsilon_{dc}, \epsilon_{ph})$ remains small in the subsonic sector and grows continuously across the sound barrier. Such contrast in low-temperature behavior of electron-phonon scattering rates $\Gamma_{\phi \rho}^{(osc)}$ and $\Gamma_{\phi \rho}^{(sm)}$ in the subsonic and supersonic regimes can be understood from the structure of Eq. (2). At low temperatures $T \lesssim 2p_F$, the occupation number of phonon modes with energy $\omega \sim 2p_F$ is exponentially small. Therefore, the electron scattering off thermal phonons becomes negligible. At the same time, the spontaneous emission of phonons depends only on the combination $f(\epsilon_{in})[1 - f(\epsilon_f)]$ of electron occupation numbers $f(\epsilon)$ at initial ($\epsilon_{in}$) and final ($\epsilon_f$) energies. For emission processes, this combination is also exponentially small in equilibrium, but may remain finite in a system subject to electric fields. When
FIG. 3: (a)-(c): Oscillatory part of the phonon-assisted differential resistivity \( r_{xx}^{\text{osc}} \) in units \( 2\lambda^2 g^2 \rho \omega_c \) calculated using Eqs. [4], [5] for the case of high \([ T = 5\omega_c, \text{ panel (a)}] \), intermediate \([ T = 0.5\omega_c, \text{ panel (b)}] \), and low \([ T = 0.25\omega_c, \text{ panel (c)}] \) temperature. Panel (d): Smooth part of phonon-assisted scattering rate \( r_{\text{ph}}^{(1)}(\epsilon_{\text{dc}}, \epsilon_{\text{ph}}) \) at zero temperature, measured in units of \( g^2 \omega_c \). Eq. (14).

the Hall drift exceeds the speed of sound, \( \epsilon_f \) for electron scattering with increase in electrostatic energy is larger than \( \epsilon_i \) even for the process with phonon emissions, see Fig. [11], and the spontaneous phonon emission takes place.

Conclusions. We developed a theory of phonon-assisted magneto-oscillations in strong dc electric fields. In the limit of a weak Hall field, \( \epsilon_{\text{dc}} \ll 1 \ll \epsilon_{\text{ph}} \), or in the limit of quasi-elastic phonon scattering, \( \epsilon_{\text{ph}} \ll 1 \ll \epsilon_{\text{dc}} \), the theory is consistent with the known results for PIRO [17] and HIRO [7]. In the nonlinear dc-response \( \epsilon_{\text{dc}} < \epsilon_{\text{ph}} \), the theory reproduces the evolution of PIRO observed in recent experiment, cf. Ref. [10] and Fig. 2. We find that the phase of oscillations changes across the sound barrier at \( \epsilon_{\text{dc}} = \epsilon_{\text{ph}} \), where the Hall drift velocity \( v_H = eE/B \) reaches the speed of sound \( s \), which also warrants an experimental verification. In the supersonic regime \( \epsilon_{\text{dc}} > \epsilon_{\text{ph}} \), the amplitude of oscillations saturates at finite value for very low temperatures \( T \ll \omega_c \), while in the subsonic sector \( \epsilon_{\text{dc}} < \epsilon_{\text{ph}} \) the phonon-assisted current decays exponentially with lowering \( T \), as the number of thermal phonons decreases. At zero temperature, the smooth part of the phonon-assisted nonlinear conductivity also exhibits saturation to a finite value in the supersonic regime, while vanishes in the subsonic regime.

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\[ \text{[16]} \text{We consider interaction with 2D isotropic deformational phonons justified if } z\text{-component of relevant phonon momentum is negligible (i.e. the width of the quantum well where 2DEG is confined } b \gg k_F^z / s / T). \text{ In this case, } g^2 = mD^2 / pbs^2 \text{ in terms of the mass density } p \text{ and deformational-potential } D \text{ of the host crystal. General case will be presented elsewhere.} \]

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