Cryptocurrency with Fully Asynchronous Communication based on Banks and Democracy

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Abstract. Cryptocurrencies came to the world in the recent decade and attempted to put a new order where the financial system is not governed by a centralized entity, and where you have complete control over your account without the need to trust strangers (governments and banks above all). However, cryptocurrency systems meet many challenges that prevent them of being used as an everyday coin. In this paper we attempt to take one step forward by introducing a cryptocurrency system that includes many important properties. Perhaps the most revolutionary property is its deterministic operation over a fully asynchronous communication network, which has been mistakenly considered sometimes to be impossible. By avoiding any temporal assumptions, we get a system that is robust against arbitrary delays in the network, and whose latency is only a function of the actual communication delay. The presented approach is based on what we are already used to – banking and democracy. Our banks, just like normal banks, keep their clients’ money and perform their clients’ requests. However, because of the cryptographic scheme, your bank cannot do anything in your account without your permission and its entire operation is transparent so you don’t have to trust it blindly. The democracy means that every operation performed by the banks (e.g., committing a client transaction) has to be accepted by a majority of the coin holders, in a way that resembles representative democracy where the banks are the representatives and where each client implicitly delegates his voting power (the sum of money in his account) to his bank. A client can switch banks at any moment, by simply applying a corresponding request to the new bank of his choice (at the cost of paying commission). The presented approach employs the advantages of centralization while still providing a complete trustless and decentralized system. By employing concepts from everyday life and attaining high throughput and low latency for committing transactions, the hope is that this paper will lay the foundations for a cryptocurrency that can be truly used as a daily basis coin.

Keywords: cryptocurrency, Bitcoin, consensus, altcoin, asynchronous

1 Introduction

In its most basic form, cryptocurrency can be seen as a digital coin (in contrast to physical coin) accompanied by cryptographic tools that provide several benefits. The most basic requirement from digital money is that no one will be able to spend money from your account, but you. This is achieved using digital signature mechanism, where every account is accompanied by a secret key, known only to the account owner, and where every request for transfer of money from one account to another (a transaction) must be signed using the secret key of the former account in a way that everyone can verify it, but no one is able to fake it, assuming that the account owner manages to keep the secret key for himself. Perhaps the more problematic part in cryptocurrency is how to maintain the information concerning the account balances. In the common cryptocurrency scheme, the idea is to deploy a decentralized and trustless system, where there is no single entity that can be trusted to tell you how much money you (or someone else) have. Instead, the amount of money you have should be determined by consensus among the other users of the cryptocurrency – everyone should agree, somehow, that you have that specific amount of money. Moreover, this vague consensus should be reached regarding all the transactions (money transfers) that are committed. This is required in order to prevent double spending, where one can pay the same coin more than once, as each payee doesn’t know that the money was also paid to others. Once there is a consensus on one of that user’s payments, the
corresponding payee can be sure that he got the money (because everyone agrees that the money is his). A second payment with the same coin will not be accepted, as the corresponding (second) payee will see that it is not in the consensus. The big question is how you can reach such consensus in a setting where you can trust no one, and where it is not generally defined who the other players are. This is also known as the “Consensus in the Permissionless Model” problem [13].

And then Bitcoin [12] has emerged, offering a solution to this problem and putting the cryptocurrency in the headlines. However, although it is the first decentralized coin, and despite its glorious success (for the moment), Bitcoin has many known flaws. Among its flaws are the time it takes a transaction to be accepted [9], its limited transactions throughput [14], and finally, the great energy consumption involved in keeping it alive [10]. There are numerous works and other cryptocoins that are trying to fix these flaws. Especially, there are strong environmental and economical reasons to alleviate its energy consumption that follows from its innovative mechanism for solving the “Consensus in the Permissionless Model” problem. More information concerning Bitcoin and other protocols appears in Section 2.

The common denominator for probably all of the numerous different cryptocoins is that there are roughly two groups – users and administrators. The users are the simple persons/clients that want to hold coins and use them for any type of trade or investment. The administrators’ role is to make sure that there is consensus concerning the current balance of the user accounts and concerning the committed user transactions. The administrators must invest resources such as computation, storage and bandwidth in order to manage the consensus. They usually have a simple incentive to do so – they receive coins for their work (either by commission from transactions, or by creating money from thin air, i.e., stamping new money). The coin they receive plays a double role – it provides both revenue for their investment, and an incentive to keep the stability of the coin, as otherwise the coin value will decrease and the administrators’ true gain will decrease as well (the coins they will receive shall have lower value). The relationship between the users and administrators is usually not well defined. It is only required of the administrators to form some kind of a network (so they can speak with each other in order to reach consensus) and of the users to be able to transmit their requests to at least one of the administrators. As we shall see, in the approach presented in this paper there is a straightforward relationship between the users and the administrators.

We should be careful, however, when using the word consensus. In distributed computing theory, the consensus problem is where we have a set of nodes, each node holds an input value, and the nodes must all agree on a single input value (input value of one of the nodes). This is similar to our case in cryptocurrency, where finally all nodes (administrators) should agree which transaction has been accepted (where a user might send conflicting transactions to different nodes). Many works on cryptocurrency (e.g. [3,10,17,18]) mention the FLP impossibility result [6], that states that a consensus problem cannot be solved when the communication channels between the nodes are asynchronous, and where nodes might fail (by stop responding). A communication channel is considered asynchronous if a message that is sent through the channel can suffer from arbitrary delay. The advantage of assuming asynchronous communications is that (1) security is proven even where unexpected arbitrary delays are introduced and (2) the latency in the system is only a function of the actual communication delay, which can result in lower latency. Following the FLP result, it might be deduced that a (deterministic) cryptocurrency system cannot assume asynchronous communications. The first immediate result presented in this paper shows that this is wrong – we can implement a cryptocurrency system assuming asynchronous communications. It follows that the cryptocurrency problem is not as hard as the classical consensus problem.

To be more exact, perhaps most if not all the existing cryptocurrencies are working on asynchronous communications. The way they avoid the FLP impossibility is by eventual agreement. In this eventual agreement an admin might accept a certain transaction, and later revert its acceptance. The agreement is achieved “eventually” in the sense that a transaction that was accepted “long ago” by one valid admin, will be with high probability accepted by all other valid admins and won’t be reverted. One problem with such attitude is that acceptance of a transaction is not truly final. A seller that trades some good in exchange for coin, can not tell if he got the money. All he can do is to wait for long enough period after the corresponding transaction was first accepted, and then assume that if it wasn’t reverted so far, then it will stay accepted
forever. In this paper we don’t assume such “weak acceptance” that can be later reverted. We discuss a cryptocurrency where a transaction, once accepted by a valid node, won’t be reverted.

More formally, each node (admin) shall accept user transactions (where acceptance is non-revertible), and the following requirements must apply:

- **Agreement**: A transaction accepted by a valid node will be accepted by all valid nodes.
- **Positive-Balance**: At every time point, and for every valid node, applying the set of transactions that were accepted by that node results in non-negative balance in all the accounts.
- **Termination**: Every transaction that is sent from a user to a valid node must be either accepted or rejected by that node.
- **Rejection Restriction**: A node can reject a transaction only if the transaction is not valid or conflicting with another issued transaction.

The last requirement of rejection restriction is provided to avoid a trivial solution where all transactions are rejected. However, it is not well defined here, because describing transactions as conflicting or invalid is dependent on the transaction structure. For example, in Bitcoin-style transactions, where each transaction consumes previous accepted transactions, a transaction is valid if all the previous transactions were accepted and if it is properly signed. Two transactions under such settings are considered conflicting if they try to consume the same previous transaction. As will be described next, we define the transactions in a bit different manner (although we could have probably used Bitcoin-style transactions just as well).

So, what is the secret that allows us to operate under asynchronous settings? The secret appears in the requirements above. The problem in consensus is in accepting a single transaction out of a set of conflicting transactions. Following the above requirements, if there is a set of conflicting transactions, we don’t have to accept on one of them – we can reject them all. That makes sense as conflicting transactions are result of invalid behavior of a user, so a user that behaves invalidly should be aware that its transactions might be rejected.

And there is one more little secret. We don’t use a blockchain. A blockchain means that all transactions are ordered in a list, where everyone accept on that order. Accepting on such total order is as hard as solving the classical consensus problem, and so it is unsolvable under asynchronous communications according to the FLP.

### 1.1 Banking and Democracy

We shall now describe the basis of the approach presented in this paper, that solves the cryptocurrency problem according to the requirements defined above. From now on, when we mention consensus, we do so in its simple meaning, that is a set of entities that agree on something.

Recall that the administrators are the ones that manage the consensus, concerning the existing balance and accepted transactions. However, we claim that the consensus should be between the users. I.e., it is the users who should be interested in the consensus. Without consensus there is no value to the coins they hold (as there is no agreement on the amount of coins each user holds). The more coins you hold, the more responsibility you have for its future (as you will lose more if its value will drop). The way we can coordinate between all the different users is by means of democracy – the majority of the “people” determines. Note, however, that the “people” in our case are not exactly the users, but rather the coins. As coins belong to users, we will provide each user voting power that is proportional to the amount of coins he owns. The problem is that the users are too numerous, and asking all of them to vote on each decision (i.e., transaction) is not practical. Recall that in a democracy the people don’t need to accept every rule, they just need to choose representatives to make the decisions for them (or on their behalf). Thus, our users simply need to choose representatives. The most trivial representatives are the administrators, whose job is to maintain the consensus. We shall now discuss the identities of the administrators.

In the real world we don’t keep the money ourselves (at least most of our money). Instead, we let someone (the bank) save it for us. When we want to use this money we simply address our bank (either directly or
indirectly). Life was good and simple, and then came the cryptocurrency fellows and said “whoa, you need no banks! Your money can be in your hands! (by the agreement of the rest)”. However, recall that cryptocurrencies still have those admins that manage the agreement. While real banks physically keep our money, the administrators only have to manage the information about how much money each user has. In today’s world, where most of our money is anyway just numbers (and not something physical), is there really a difference between the two? Well, there is one important difference. The banks are centralized entities, whom we must trust, while the administrators are not. We don’t need to trust the administrators because their way of action is transparent, and in fact each of us can become an administrator (or at least can gather the information that they get) and check for himself the correctness of the consensus.

While the common cryptocurrency scheme has moved away from banks, we return to them. Our banks, just like administrators in other cryptocoins, will be responsible for managing the consensus. And just like other administrators, their way of action will be completely transparent so they don’t have to be trusted. From the user side, this scheme is just as in real life: Each user’s money will be virtually deposited in a bank (an administrator) of his choice, and for every operation that the user wants to perform he should send his request directly to his bank (and pay an appropriate commission). Of course the user must also have the option to switch banks, so he won’t lose his money in case his bank fails or otherwise ignores his requests. This scheme introduces several advantages that follow from the centralized nature of banks. From the user side, it means the user has a single and known address for all of his issues. From the administrators (the banks) side, it means simple and defined power distribution (each bank has the power according to the total amount of money of its clients). Another important advantage follows from our consensus mechanism, that will be now described shortly.

Let’s assume I ask my bank to transfer money from my account to another. As we are living in a democracy, the transfer must be accepted by the holders of the majority of the coin (i.e., a group of coin holders that together posses the majority of the money must accept this transfer). In our approach each bank represents its clients and ‘votes’ on their behalf. Roughly speaking, each bank gets its voting power according to the sum of money in the user accounts it manages. Once the banks agree on a transaction, we can see it as if the money holders themselves agreed on that transaction (as each money holder delegated his voting power to his bank). So far we have only described the basic principle of our consensus – a set of banks that represent majority of the coin holders must agree on each transaction. The question is how do we implement it? In other coins there is usually a single global ledger that lists all the transactions, and everyone (should) agree on this ledger. By observing the agreed ledger, everyone can tell, out of two (accepted) transactions, which one comes before the other. I.e., there is an agreement on the total order of the accepted transactions. This most famous ledger is the “blockchain” – a chain of blocks, where each block contains a set of transactions. However, such agreement on the total order of the transactions is superfluous. For example, if Alice transfers 20$ to Charlie, and Bob transfers 30$ to Charlie, there is no importance which transfer comes first. This total ordering of the transactions is a source of many problems, as it brings together all the administrators into a single condensed point – appending a new block to the blockchain. The obvious question in this context is who will be the one that defines the next block? The solution of Bitcoin (for example) to this question causes a grave waste of energy, with big latency and small throughput for committing transactions. Other solutions might mitigate some of these problems, but no solution can be optimal in the sense that such global blockchain is more than what is truly required. In fact, the total order implied by the blockchain can be used to solve the classical consensus problem. Thus, according to the FLP result, it cannot be implemented over asynchronous communications. However, as is proved in this paper, we can avoid the total order, and solve the cryptocurrency problem under asynchronous communications.

Instead of using a global ledger (i.e., a blockchain), in our approach we let each bank manage its own ledger – a separate ledger(/blockchain) for each bank. The transactions that appear in such “private blockchain” of a given bank are usually transactions that are committed by the clients of the same bank. I.e., it lists mostly transactions that transfer money from that bank’s clients to other clients (other clients either of the same bank or of other banks). There might also be a transaction of a user of another bank, in case that user asks to leave its original bank and move to this bank. If we want to compute the balance of a

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1 Most often we address our credit card company, and they address our bank.
client, observing only the blockchain of his bank is not enough, as it doesn’t list money that he receives from clients of other banks. The information about such money transfers is found in the blockchains of the other banks (the banks whose clients transferred that money). As banks must be able to compute such account balances, they must hold all of the existing blockchains. In fact, each block in a given blockchain will contain pointers to blocks in other blockchains (to the most recent blocks, at the time of creating that block). More information appears in Section 3.

So, when I send a transaction to my bank, my bank will check this transaction to make sure that I have the required money to spend, then it will probably group it with transactions of other users into a block, and it will chain this block to its private blockchain. Next, my bank will send this new block to all the other banks, in order to receive their acceptance for the block. Once a majority of banks (by means of voting power) sent their acceptance for the validity of the block, my bank has the proof that the block is valid. More information concerning the exact protocol appears in section 4.

In the common cryptocurrency scheme, each user chooses a pair of private and public keys. The private key is used by the user for encoding digital signatures, and the public key is used for verifying the user’s digital signatures and as the account number. In our approach however, we want the user’s account number to be related to a specific bank of his choice. For this purpose, each bank has to choose a pair of private and public keys itself (just like the simple users), and a user’s account number is in fact a combination of his bank’s public key and his own public key. The bank’s public key can be seen as the “branch number”, while the user’s public key is the “inner account number” in that specific bank.

Let us summarize our new cryptocurrency scheme: First, when I “open” a new account, I do it using my ordinary private and public keys (just as in other cryptocurrencies). But, I also choose a bank and use its public key as my “branch number”, so that my full account number comprises of “branch number” and “account number”, just as in real life. When I want to issue a transaction, I send it to my bank. My bank is in charge of making sure other banks acknowledge this transaction, i.e., that the representatives (the banks) of the majority of the money holders accept it. Of course I shall pay commission on that transaction. And finally, if I don’t like my bank, then I can address another bank and ask it to switch banks.

The presented approach has the following advantages:

- Cryptocurrency with irrevocable acceptance of transactions.
- Agreement mechanism without superfluous energy consumption.
- Works under fully asynchronous communications.
- Just as in real life, you address all of your requests directly to your bank.
- As each bank updates its own ledger, we can have higher transaction throughput with lower latency. As a result of the lower latency, this coin can be used as an everyday payment method.
- The banks have incentives to give their clients good service, or otherwise the clients will move to other banks (and less clients means lower commissions).

2 Related Works

Maybe the greatest challenge of cryptocurrency is the consensus on the money balance and/or committed transactions. The most prominent approach to this problem, introduced by Bitcoin [12], is the famous blockchain. The difficult question is how a new block is added to a given chain, in a way that everyone will agree that this block is indeed the next block in the chain. The first approach, Bitcoin’s approach, is by a race based on computational power. The chance one has to win such a race is roughly equal to its computation power divided by the overall computation power that participate in this race. By assuming that most of

\footnote{According to [5], even if most of the computation power is in good hands, it still might be not enough for achieving fair results.}
the computational power is in “good” hands, that will recognize the true winner of a race, it is argued that it will benefit everyone to recognize that winner as well, thus, reaching consensus. This concept is called Proof-of-Work (PoW). The great downside of PoW is its resource consumption.\(^3\)

In order to alleviate the resource consumption, the idea of Proof-of-Stake (PoS) \(^8\) was born, where instead of deciding the next block by stochastic means, based on ownership of external resources (such as computation power), we can decide in a deterministic fashion based on inherent resources – the coin itself. The idea is that if you have more of the coin, you can get the chances to create more blocks. While this solves the energy consumption problem, it doesn’t truly solve the other downsides of Bitcoin - high latency and low throughput for committed transactions.

Achieving high throughput and low latency is problematic when relying on a single main blockchain that everyone must agree upon. The problem with the blockchain is that it cannot be concurrently modified by different admins. This is a result of its “chain” structure, where the only way to extend it is by adding a new block on top of it, so only one admin at a time can add a new block. This causes a great congestion, that limits the possible throughput and introduces high latency. In order to solve this problem, one option is to do stuff outside the main chain (e.g. \([14]\)), and update the main chain only when necessary (less frequent updates shall lead to less congestion). Another option, however, is to completely avoid a main blockchain (e.g. \([15,11,16,7,3]\)). While blockchain has the convenient feature that it defines a total order of all the committed transactions, this convenience comes with a great cost, and is not truly necessary: If both Alice and Bob transfer money to Charlie, then it doesn’t matter which transfer is considered the first. Of course we still need some sort of partial order. For example, if Alice wishes to transfer some money to Bob, we want to see that Alice received enough money \textbf{before} that, so she indeed has enough money in her account to perform that transfer. For keeping only partial order, we can use a DAG (directed acyclic graph). Instead of agreeing on a single blockchain, the admins will agree on this graph. Each node in the graph can be, for example, similar to a block in the blockchain (containing user transactions), only that such a block can now have several parent and child blocks, in contrast to blockchain where each block has exactly one parent and, eventually, one child.

Note that a blockchain can also be simulated by a DAG that contains all the blocks that were ever created, and where there is an edge from each block to its parent (as every block in a blockchain contains an hash of its parent). However, the only relevant set of nodes (blocks) in such a DAG are the nodes that appear along the longest path in this graph. These nodes construct the longest chain of blocks, thus, the actual blockchain. Clearly a DAG is not helpful in this case.

A DAG will be effective only if nodes that were created concurrently, on different branches, won’t obscure each other, and can both have effect. Maybe the simplest use of DAG is as a set of parallel blockchains that are connected between them. For example, every admin can have a blockchain of its own (a chain of blocks that are all belong to that admin), and every block in such blockchain can reference blocks from blockchains of other admins (e.g., the “gossip” graph in \([3]\)). In such case there is no competition between admins, and no congestion, as every admin can freely create new blocks in its own chain. However, the problem is that conflicting transactions might appear on different blockchains, in blocks that have been created concurrently. Conflicting transactions are transactions that cannot be applied together. For example, in Bitcoin this means two transactions that consume the same previous transaction. More generally, if Alice has 40\$ and she issues two transactions in which she transfer 20\$ to Bob and 30\$ to Charlie, then clearly the two transactions cannot live together. When using a single blockchain it was easy – one of these two transactions had to appear before the other, so the second transaction will become invalid. But what should we do if these two transactions appear in two different blocks that were added concurrently to a graph at two different locations (two different blockchains in our case)? Different DAG-based cryptocoins deal with such problem in different ways. A simple solution is that transactions of the same “type”, that might be conflicting, are allowed to be introduced only at a specific location in the graph, so they cannot appear concurrently at different locations. E.g., if our graph is indeed a set of parallel blockchains, then we can divide the transactions according to

\(^3\) It should be mentioned though that the requirement for resource consumption plays another important role – it makes the option to alter history unlikely, as one has to invest more resources than all the resources that have been used, starting from the point of required change in history.
their source account, such that transactions of a specific account are allowed to appear only in blocks at a specific blockchain [7,4].

In the above paragraph we mentioned two possible properties of a DAG: (1) That there will be a separate blockchain for every admin, so there will be no congestion, and (2) that conflicting transactions could not be issued on different blockchains. Combining both of these properties can have a good effect. However, if even one admin will stop responding, that means that some set of transactions cannot be issued, which is clearly not an option. Another option, of [11], is that instead for every admin, there is a blockchain for every user. In such case, if we could have trust all the users, it could be ideal. However, as a malicious user can create conflicting blocks in its chain, there yet must be some sort of agreement on the actual blockchains.

Our solution indeed applies a blockchain for every admin (bank), and users should submit transactions to the specific admin they chose (their bank), so that conflicting transactions are not supposed to appear in different blockchains. However, the users can also choose to submit a transaction to another bank, when they want to transfer their account to that bank. Thus, it is not really guaranteed that there won’t be conflicting transactions (though, they should be less frequent – only when users switch banks).

Because there might be conflicting transactions, we cannot immediately accept the transactions in every new block that is introduced to the graph (even the bank that issued that block cannot immediately accept its transactions). I.e., there is a delayed acceptance of transactions. In PoW based protocols this delayed acceptance is in means of actual time, where the more time you have waited before accepting a block, the more chance that you won’t have to later revert your acceptance. It is also common to either accept or reject a complete block instead of specific conflicting transactions. In our case, we delay acceptance on a block’s transactions up to the point that this block is in consensus. The consensus can be observed directly from the graph itself. Moreover, if we reject, then we reject specific (conflicting) transactions and not complete blocks. See Sections 3 and 4 for more details.

Concerning the consensus mechanism, we employ ‘democracy’ where a single coin means a single vote and transactions are accepted by a majority. The voting power of each coin is in fact delegated to the bank in which it is deposited. The idea of delegating voting power to an administrator seems to be, unfortunately for the author, not new, see e.g. [11,1,2]. The advantages of delegating the voting power is that we can apply consensus by ‘democracy’, where the coin holders are effectively those that decide. Maybe the most prominent problem in delegating power is the ‘indifferent user’. The problem is when users don’t really care to whom they delegate their power, or when the users aren’t forced to delegate their power at all. The result might be that most of the honest users delegate their power to non functioning or malicious administrators, or even don’t delegate their power at all (in which case the rest of the users, that many of them might have malicious intentions, virtually hold more power than their true share). In our approach the users cannot truly be indifferent. The reason is the close connection between the users and the administrators, or more precisely the ‘intimate’ connection each user has with one specific administrator - his bank. As our users must choose a bank, and as they can get service only from that bank, they cannot delegate their voting power to a bank that is not functioning or that was proved to be malicious (a bank that acts maliciously will be ignored by the rest of the banks, so it won’t be able to give services to its users).

While some of the concepts, such as using a DAG and delegating the voting power, appear also in earlier works (and particularly they both appear in Nano [11]), they are integrated in this work in an innovative way that allows us a to apply non revertible transactions, to operate under asynchronous communications and to avoid the indifferent user problem (all of which do not apply in Nano, for example).

3 Settings

In this section we define the block-graph (a DAG) that is used instead of a blockchain. Recall that a blockchain contains the entire history of the coin – all the accepted user transactions. By applying all the transactions in the blockchain, we can have the current balance of each account. The block-graph has a similar role as the blockchain – it contains the entire history of the coin (the issued transactions, not necessarily all of them are accepted).
Recall that a blockchain can also be described as a graph. The nodes of this graph are the blocks, and there is an edge from each block to the block that comes before it (because each block in the blockchain contains an hash of the previous block, which is kind of a pointer to the previous block). In our block-graph, each bank has such “blockchain-graph” of its own. I.e., there is a set of blockchains where each blockchain belongs to a different bank. A blockchain that belongs to a specific bank contains only blocks that were issued by that bank. In particular, those blocks must be digitally signed by that bank. However, instead of blocks we talk about nodes, where each block from the original blockchain is split into several nodes in the graph. The beginning of a block is a start node that contains the transactions to be committed. A block ends at an accept node. Between a pair of start node and accept node there might be additional nodes, update nodes, that contain references to nodes in the chains of other banks. All the nodes between a pair of start and accept nodes are considered as a single block. The update nodes connect the blocks of the different banks.

Unlike Bitcoin, there is no money creation in our coin. Instead, there is a finite amount of coin in the system. To define the initial coin distribution, we use an init node. The init node contains a list of account numbers and a positive amount of money for each account in the list.

See Figure 1 for an example of a block-graph.

Each node in the graph (except the init node) contains the following information:

- The bank number that owns that node.
- The sequence number of the node in its bank’s chain.
- Hash of the previous node in its chain (or the hash of the init node if it is the first node in the chain). This previous node is considered the parent of the current node.
- Corresponding digital signature of the owner bank.

Excluding the init node, there are three types of nodes. Each node might also contain information according to its type:

- A start node contains user transactions the bank wants to apply.
- An update node contains references to nodes of other banks.
- An accept node contains no additional information.

See Figure 2 for example of nodes.

When a bank wants to issue a set of transactions it received from its users, it creates a start node that contains those transactions, and send it to all other banks. Another bank that receives that node can create an update node that references that start node (and possibly references new nodes of other banks as well) and it sends that new update node to all other banks. A reference to a node is basically its hash together with its true id (i.e., the id of the bank that issued it, and that node’s sequence number in its bank’s chain).

When a bank creates such update node, this means that it acknowledges the node it references. Acknowledgment applies even with indirectly references, for example when an update node references another update node that references another node. More generally, every node acknowledges all the nodes that it can reach to (by means of paths in the graph). For example, in Figure 1, the top accept node of $B_2$ acknowledges all the nodes in the graph excluding the two top nodes of $B_2$.

Back to the first bank, that created the start node, that bank has to wait for other banks to acknowledge its start node, so that this node will be in “consensus”. To show that other banks have indeed acknowledged its node, it has to acknowledge their acknowledgments. It does so by creating an update node of its own, that references the update nodes of the other banks (directly or indirectly). Once that bank has enough banks that have acknowledged its original start node, and it has acknowledged those banks’ acknowledgments, it can tell everyone to accept the transactions that are listed in this start node. It does so by creating an accept node, that marks the end of the block. This accept node doesn’t need to contain any additional information, as it is a complementary node to the original start node, and a new start node cannot be issued before the previous start node received a corresponding accept node.

We conclude this scheme by formally defining the structure of a block-graph:
**Fig. 1.** A block-graph with an *init* node (bottom) and with “blockchains” of three banks: $B_1$, $B_2$ and $B_3$, where every such blockchain is constructed by *start* nodes, *update* nodes and *accept* nodes. Edges from nodes to their parents appear in black, and edges from *update* nodes to their referenced nodes in gray. Note that the first node of every blockchain (the downmost node in every column) references the *init* node as its parent. Dotted blue rectangles mark complete blocks (a set of nodes from *start* node to *accept* node).

**Fig. 2.** Nodes example. An *init* node (top left) describes initial coin distribution, a *start* node (bottom) lists issued transactions and an *update* node (top right) references nodes of other banks. Note that the parent of the *start* node (in this example) is the *init* node, and the parent of the *update* node is the *start* node.

**Definition 1 (block-graph).** A graph $G$ is considered a block-graph if all of the following applies:

- Every node of $G$ has to be a *start* node, *accept* node, *update* node or *init* node.
- There must be exactly one *init* node in $G$.
- The parent of every node, except the *init* node that has no parent, must also appear in $G$, and it must belong to the same bank (unless the parent is the *init* node).
- For every *update* node in $G$, all its referenced nodes also appear in $G$.
- There is an edge from each node to its parent, and edges from each *update* node to its referenced nodes. There must be no other edges in $G$.
- $G$ has no cycles.

- Let $v$ be a node in $G$. We consider the (longest) sequence of nodes that starts at $v$ and where every other node in the sequence is the parent of its predecessor. Ignoring the *init* node and the *update* nodes in this sequence, we must get an alternating sequence of *start* nodes and *accept* nodes where the last node is a *start* node.

Note that the last condition means that a single bank shall have no *accept* node without a matching *start* node that comes before it, and no *start* node that comes after another *start* node without an *accept* node in the middle. *Update* nodes can appear everywhere. A simple property of block-graphs, that we shall use ahead, is described in the following lemma:

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[^4]: A cycle means there is a cycle of cryptographic hash values, where each value is the hash of its previous, and we assume that it is impossible to produce such a cycle.
Lemma 1. Let \( G \) be a block-graph, and let \( G' \) be a subgraph of \( G \). If every node in \( G' \) has the same number of outgoing edges as in \( G \), then \( G' \) is also a block-graph.

Proof. All the requirements of block-graphs are trivially true for \( G' \), except the second and last requirements. We start with the last requirement. Let \( v \) be a node in \( G' \). The parent of \( v \) also appears in \( G' \), as there is an outgoing edge from \( v \) to its parent. But then also the parent of \( v \) has an edge to its own parent, and so on. Thus, we get the same sequence of nodes as in \( G \), and so the requirement must hold. Following the above case, that sequence of nodes must end at some point (as there are no cycles in \( G' \)). The only node that has no parent is the init node. Thus, the init node also appears in \( G' \).

Before a bank accepts a node it received from another bank (by referencing it with an update node), it must check its validity. Some of the conditions for validity are specific to the node’s type, and some are general. We start with the general conditions.

Let \( v \) be a node. We say that a block-graph \( G \) represents \( v \) if \( v \) appears in \( G \), and if there are paths in \( G \) from \( v \) to every other node in \( G \). Recall that \( v \) contains references to other nodes (their hashes) – a reference to its parent, and possibly references to nodes of other banks, if \( v \) is an update node. If we know those referenced nodes, then we can consider also the references that they contain, and so on, until we have a complete block-graph, which is exactly the representing graph of \( v \). If \( v \) has no representing graph, then it is clearly not valid (it references, directly or indirectly, an impossible node). If \( v \) does have a representing graph, we assume it is unique. I.e., we assume that no node has more than one representing graph. The reason is that the references each node has to other nodes are defined by cryptographic hash, and we assume that it is impossible to create two different nodes with the same hash. Moreover, note that such two nodes must both belong to the same bank and be with the same sequence number, and possibly have additional requirements in order to fit in the right place in the graph.

Considering the representing block-graph of \( v \), we remove \( v \) from this graph, and to the resulting graph we call the subgraph of \( v \). Note that the representing graph contained no cycles (as it is a block-graph), so there could be no edge from another node to \( v \) in that graph. Thus, the conditions of Lemma 1 applies for the subgraph of \( v \), and so it is also a block-graph. The subgraph of \( v \) is considered valid only if every node in its subgraph is valid. If that subgraph is not valid, then \( v \) is not valid.

Assuming that the subgraph of \( v \) is valid, we check \( v \)'s contents according to its type. If \( v \) is an init node, then it is trivially valid. If \( v \) is an update node, then it could also be trivially valid, but in order to avoid waste of information we should make sure that an update node doesn’t reference nodes that are already appear in its parent’s subgraph. Later on in this section, we introduce an additional requirement from update nodes that intends to deal with malicious banks.

Before considering the requirements of start nodes and accept nodes, we define some important concepts, namely the transactions structure and the way we compute the balance of each account. A simple transaction is a coin transfer request of some amount, from one account to a second, digitally signed by the first account’s private key.\(^5\) Just as we chained the nodes of each bank, we chain the transactions of each user. Each transaction contains the following data: The source account number, the destination account number, the amount to be transferred, the sequence number in the user’s transactions chain and a corresponding digital signature.

Note that in the common case a user should send his signed transactions to his own bank. If he wants to switch banks, however, he should be able to submit to another bank – his new bank, a transaction that transfers all his money to that bank. Moreover, there are cases where the user should be able to resubmit to another bank the same transaction that it has already submitted to its original bank. An example is if the original bank of the user has stopped responding after receiving that transaction from the user, but before it managed to accept it. For simplicity, we assume for now that a user can submit every transaction to every bank of his choice.

Next, we define how to compute the balance of each account according to a given block-graph. Recall that the block-graph contains an init node that encodes the initial balance, and transactions that appear in start nodes. When computing the total balance, we consider only accepted transactions, i.e., transactions

\(^5\) We can also employ Bitcoin’s transaction mechanism, where each transaction consumes previous transactions.
that appear in start nodes that have matching accept nodes. We can get the total balance by starting with the initial balance, and then applying the accepted transactions. Note however that the same transaction might appear in more than one start node (in start nodes of different banks), so we must make sure that we apply it only once. Moreover, there might be different transactions of the same user with the same sequence number that appear in different start nodes (of different banks). Such transactions are conflicting, and we don’t want to apply them together. Recall that the set of nodes between a start node and a matching accept node (including the start node and the accept node themselves) are considered a single block. We say that one block, \( b_1 \), acknowledges another block, \( b_2 \), if the accept node of \( b_1 \) acknowledges the start node of \( b_2 \) (i.e., the subgraph of the accept node of \( b_1 \) contains the start node of \( b_2 \)). For example, in Figure 1, the bottom block of \( B_2 \) and the block of \( B_1 \) are both acknowledge each other (note the update node that appears below that block of \( B_2 \)). On the contrary, the block of \( B_3 \) acknowledges that bottom block of \( B_2 \), but not vice versa.

Assume that there are pair of conflicting transactions that appear in two blocks \( b_1 \) and \( b_2 \). If \( b_1 \) acknowledges \( b_2 \), then we don’t apply the conflicting transaction from \( b_1 \) as it “knows”, at the moment of accepting its transactions, that there is already another conflicting transaction. If both \( b_1 \) and \( b_2 \) acknowledge each other, then both transactions won’t be apply. The problem is if no block acknowledges the other. We shall soon consider that case, but we first conclude the total balance computation:

**Definition 2 (Total balance).** Given a block-graph, we take the initial balance according to the init node, and then for every accept node we apply every transaction that appears in its matching start node, as long that the accept node’s subgraph doesn’t contain a conflicting transaction, and as long that the same transaction wasn’t already applied before that during the computation.

We can now define the validity requirement for start nodes. Let \( v \) be a start node that belongs to bank \( B \) and whose subgraph, \( G \), is valid. \( v \) will be valid if all the transactions that it contains are valid. Let \( t \) be a transaction that appears in \( v \), that belongs to the user \( u \) and transfer amount of \( m \) to another account. Roughly speaking, \( t \) is considered valid if it is indeed the next transaction in the transaction-chain of \( u \), and if \( u \) has a balance of at least \( m \). We start with deciding if \( t \) is indeed the next transaction in \( u \)’s chain. First, we consider all the accepted transactions of \( u \) in \( G \) (ignoring identical transactions). I.e., all the transactions of \( u \) that are considered when computing the total balance according to \( G \). Assume there are \( N \) such transactions, their sequence numbers must cover all the range of 1 to \( N \), or else \( t \) will be considered invalid, as that user has no valid transaction chain. Assuming the sequence numbers of \( u \)’s accepted transactions indeed cover all the range of 1 to \( N \), the sequence number of \( t \) must be \( N + 1 \). Next, there must be no other, different, transaction of \( u \) with the sequence number of \( N + 1 \). There might be however another transaction that is identical to \( t \), as long that it appears in a start node of another bank, \( B’ \) (where \( B’ \neq B \)). In this case this means that \( u \) has resubmitted \( t \) to \( B \), maybe because \( B’ \) stopped responding. Note that there cannot be a matching accept node for that start node of \( B’ \) (in \( G \)), because then this identical transaction would either be accepted, and then \( t \) is not valid because it should have a bigger sequence number, or otherwise it would be rejected, which means that there is another, different, transaction with the same sequence number (\( N + 1 \)), so \( t \) is, once again, invalid. The second requirement for \( t \)’s validity is that \( u \) has a balance of at least \( m \). We decide the balance of \( u \) by computing the total balance according to \( G \) (the subgraph of the start node \( v \)). If \( t \) fulfills both requirements it is considered valid.

Next, we need to define the validity requirements of an accept node. Let \( v_a \) be an accept node that belongs to bank \( B \), and let \( v_s \) be the start node that comes before \( v_a \) in \( B \)’s chain. The creation of \( v_a \) signals that the transactions of \( v_a \) should be accepted. \( B \) can create \( v_a \) only once that \( v_s \) is in consensus. Consensus in our system is defined by coin possession. Each user has a voting power that is proportional to his balance, and that voting power is delegated to the user’s bank. \( v_a \) is considered valid if it is evident from its subgraph that there is a set of banks, \( S \), that together hold voting power above a predefined threshold, and that they all acknowledged \( v_s \). More precisely, a bank \( B’ \) is in \( S \) if (and only if) it has a node \( v \) such that (1) \( v \) has acknowledged \( v \) and (2) \( v \) has acknowledged \( v_a \). Recall that a node \( v_1 \) acknowledges another node, \( v_2 \), if \( v_2 \) appears in \( v_1 \)’s subgraph. Recall also that the sequence of nodes of \( B \) starting from \( v_s \) and ending at \( v_a \) are considered a single block (see Fig. 1). If \( B’ \in S \) we say that \( B’ \) supports \( v_a \)’s block. To finish the definition of
validity requirements of an accept node, we need to define how to compute the voting power of each bank, and the threshold of required supporting voting power. We shall soon do so.

When computing the total balance, we mentioned that there is a problem if two blocks that contain conflicting transactions don’t know of each other. In such case, the two conflicting transactions will be both applied when computing the total balance, and that might result in an account that has a negative balance. Such account has managed to spend more than what it originally had, and unlike in real life, no one can come to that account and ask the money back. To avoid this problem, we want to make sure that all the blocks know each other. This brings us to the following definition.

**Definition 3 (Proper graph).** We say that a block-graph is proper if for every pair of accept nodes, at least one of them acknowledges the start node of the other.

We shall prove later that computing the total balance of a block-graph that is proper and valid results in non-negative balance for all the accounts. Thus, we have big interest in proper graphs.

How can we make sure that a (valid) block-graph will remain proper? I.e., that for every two accept nodes, at least one of them acknowledges the start node of the other? Roughly speaking, we can get this effect by making sure that every block (/accept node) is supported by the voting power of majority of the banks. In this case, every two different blocks (/accept nodes) will be obliged to share some common supporting voting power. I.e., there must be at least one bank that supports both blocks. Let \( B \) be such bank, that supports two blocks, \( b_1 \) and \( b_2 \), and assume it first acknowledges \( b_1 \) and then acknowledges \( b_2 \). When \( b_2 \) acknowledges \( B \)'s acknowledgment on \( b_2 \) itself, it acknowledges also the acknowledgment of \( B \) on \( b_1 \), and so it indirectly acknowledges \( b_1 \). As a result, the two blocks are indeed connected.

We shall now define how to compute the voting power distribution. Every block-graph defines a specific voting power distribution between the banks. In its most basic form, the voting power of a bank is the amount of money belong to the users of this bank. Thus, to compute the voting power distribution, we start by computing the total balance according to the graph. Recall that computing the total balance considers only accepted blocks (i.e., completed blocks – start nodes that are followed by matching accept nodes). This is not enough, however. We might have a problem with “open blocks” – start nodes that don’t have a matching accept node. If a transaction in such open block transfers money from one bank to another (or, more precisely, between users of these banks), then the transferred money is “in transit” between the two banks. We name the bank from which money is transferred as the “source bank”, and the bank that receives the money as the “destination bank”. Of course the money is truly belong to the users, but a corresponding voting power is delegated to the banks. The true concern is when such transaction appears in a start node of a bank which is not the source bank. In such case, some banks might see extensions of this graph, where that open block is already closed, and the money no longer belongs to the source bank, but to the destination bank. Yet, the source bank might not be aware of this “closure”, and it might keep using that money as part of its own voting power. The result is that the same money (/voting power) might be used to support different blocks, that might not be aware of each other, and this might result in a non proper graph. To avoid this problem, we define such money to be shared between the two banks (the source and destination banks). A shared money can support a block only if all the banks that share it support that block.

An exception is the case where such problematic transaction, that appears in an “open” block, is doomed to be rejected because there is already another (or identical) transaction of the same user with the same sequence number that is accepted according to this graph. On the contrary, there might be several conflicting problematic transactions, in different open blocks. In this case, the money will be shared between the source bank and all possible destination banks. More formally we define the following set of problematic transactions:

**Definition 4 (Uncertain transactions set).** Let \( G \) be a block-graph. We define the set of uncertain transactions in \( G \) as follows. We start with the set of transactions that appear in start nodes that don’t have matching accept nodes in \( G \). Let \( t \) be a transaction from this set, that appears in an “open” start node that belongs to bank \( A \) and that transfers money from an account in bank \( B \) to account in bank \( C \). The set of uncertain transactions includes \( t \) exactly if:

- \( A \neq B \neq C \) (possibly \( A = C \)), and
There is no other transaction of the same user with the same sequence number as \( t \) that is applied when computing the total balance according to this graph.

The money that is transferred by uncertain transactions is exactly the money that we cannot be sure to which bank its corresponding voting power should be delegated – the source bank or the destination bank. Thus, we will define that money (or, more precisely, the corresponding voting power) as shared between the two banks. As we shall now prove, if a user has several uncertain transactions, then they all have the same sequence number. If our graph is proper (and remains proper), this means that at most one of these transactions might be later accepted, and so that money will belong either to only one of the destination banks (if one of the transactions gets accepted) or to the source bank (in case all the transactions get rejected). Thus, we define that money as shared between the source bank and the destination banks of all those conflicting transactions (as exactly one of them might get that money). Note that those conflicting transactions might wish to transfer different amounts of money, so we simply take the biggest amount among those amounts of money.\(^6\) We shall now prove what we claimed above:

**Lemma 2.** Let \( G \) be a block-graph whose start nodes are all valid, and let \( t_1 \) and \( t_2 \) be two transactions that appear in \( G \). If \( t_1 \) and \( t_2 \) are uncertain transactions of the same user, then both have the same sequence number.

**Proof.** Let \( t_1 \) and \( t_2 \) be two uncertain transactions in \( G \) that belong to the same user, \( u \). Assume by way of contradiction that they have different sequence number. Without loss of generality, assume that \( t_1 \) has a lower sequence number. By the assumptions, the start node where \( t_2 \) appears must be valid. According to the validity requirements of a start node, there are in its subgraph (thus, in \( G \)) accepted transactions of \( u \) with all the sequence numbers between 1 and one below the sequence number of \( t_2 \). Thus, there is an accepted transaction of \( u \) with the same sequence number as \( t_1 \), and so \( t_1 \) cannot be an uncertain transaction (according to the uncertain transactions definition).

We shall now prove what we claimed above:

**Definition 5 (Voting power distribution).** Let \( G \) be a block-graph. We define the voting power distribution of \( G \) as follows. We start by computing the total balance, according to \( G \). The initial voting power of each bank is the sum of money of its users.

Next, we consider the uncertain transactions in \( G \), and divide them according to their issuing user. For each user we take the transaction that wishes to transfer the biggest sum of money. That sum of money is decreased from the voting power of his bank, and is granted to a coalition that consists of his bank and the banks of all the users that might get money according to that user’s set of uncertain transactions.

We can now finish defining the requirements for an accept node, \( v_a \), to be valid. Recall that a bank \( B' \) supports \( v_a \)'s block exactly if \( v_a \) acknowledges a node of \( B' \) that acknowledges \( v_a \) (the start node of \( v_a \)'s block). We compute the voting power that supports \( v_a \)'s block by computing the voting power distribution according to the subgraph of \( v_a \), and summing the total voting power of banks that support its block (according to its subgraph) including the voting power that is shared between several banks, as long that all the banks that share it support that block as well. That sum of supporting voting power must be above a predefined threshold that we shall soon consider. As we shall see, this threshold depends on what we assume about the banks nature.

After defining the validity requirements of nodes, we say that a block-graph is valid if all of its nodes are valid. Our next goal is to prove that a valid block-graph is also proper. Before we can do so, we must consider the banks nature. The first problem is with malicious banks. Recall that a valid bank must create

\(^6\) A more accurate way to define the shared voting power is as follows. First, find the transaction that wishes to transfer the lowest amount of money, and define its money has shared between all the banks from the uncertain transactions set of the user. Then, find the transaction with the next lowest amount of money, and define the difference between the two transactions as shared between all the banks from the set above, removing the previous transaction. Keep doing so until no transactions left. When using this method, each bank will have a “shared” part that is equal to the amount that it can actual get, and no more than it.
a continuous sequence of nodes (just like users with transactions). I.e., each node is the child of the previous node. A malicious bank, however, might create two parallel (conflicting) nodes – different nodes with the same sequence number, just as malicious user might create two conflicting transactions with the same sequence number. The problem with such conflicting nodes is that they might seem valid for themselves (when observing their subgraphs), and different banks might adopt a different node of the conflicting pair. Only when a bank is aware of the two conflicting nodes it knows that the bank that issued them is malicious. To deal with malicious banks, the voting power threshold required for an accept node to be valid must be set to an higher value.

Even if banks are not malicious, and don’t create conflicting nodes, they might still pose a problem if they stop responding. Non responsive banks might make it impossible for accept nodes to gather the required support. To deal with non responsive banks the threshold must not be set too high.

Assuming there are no malicious banks, and that the voting power of non-responsive banks is always less than half of the total voting power, we can set the required threshold to be a simple majority (i.e., half the total voting power), and then prove that every valid graph is also proper. However, an assumption that there are no malicious banks is not practical. In the next subsection we extend our definitions in order to deal with malicious banks, and formally prove that valid block-graphs will be proper.

3.1 The case of malicious banks

A bank is considered malicious if it creates conflicting nodes – different nodes with the same sequence number. When the valid banks find out that there are two conflicting nodes, they know that the bank that created them is malicious, and they should ignore its future nodes. More precisely, if a block-graph contains conflicting nodes of a given bank, then this bank is considered malicious according to this graph. I.e., the graph itself holds the proof that the bank is malicious.

We deal with malicious banks by defining update nodes to be invalid if it is evident from their subgraph that their own bank is malicious. I.e., a malicious bank cannot reference nodes of banks that already know that it is malicious, as it will make its update nodes invalid and no other bank will accept such invalid nodes. Another optional, cosmetic, requirement (has no practical importance) is that update nodes (of valid banks) won’t reference nodes of banks that they already know that are malicious. More precisely, if it is evident from the subgraph of the parent of an update node (the node that comes before that update node in its bank’s chain) that a specific bank is malicious, then this update node should not directly reference a new node of that bank.

Recall that we want graphs to be proper, so that conflicting transactions won’t be accepted together. With malicious banks around, we must make sure they don’t turn our graphs to non proper. The “easiest” way for a malicious bank to turn a graph to be non proper is by issuing two conflicting start nodes, each accompanied with an accept node of its own. Note that because of the new rules for update nodes validity, the two created blocks cannot acknowledge each other, and so we get a non proper graph. However, a bank cannot create a (valid) accept node at its will, as it needs the support of the majority of the voting power (above the predefined threshold). We claim that the voting power of a non malicious bank can support at most one of such conflicting accept nodes. This is true because if such non malicious bank, \( \bar{B} \), will acknowledge both accept nodes of the malicious bank, \( B \), then it will be evident from \( B \)'s later acknowledgment that \( \bar{B} \) is malicious, so \( \bar{B} \) won’t be able to validly reference it. (A more formal proof appears later on.) Thus, by defining the threshold of the required voting power to be high enough, and by assuming that the voting power in the hands of malicious banks is limited, we prevent malicious banks the option of creating such two conflicting accept nodes (that are based on different start nodes).

But, what prevents a malicious bank from creating two conflicting accept nodes for a single start node? I.e., after gathering the required support for a legitimate start node, a malicious bank might create two conflicting, and valid, accept nodes. Assume that the malicious bank \( \bar{B} \) indeed creates such two conflicting accept nodes \( v_1 \) and \( v_2 \) for a single start node \( v \). Moreover, assume that there is a transaction \( t \) that appears in \( v \) that is accepted at \( v_1 \) but rejected at \( v_2 \) (because there is an update node that comes before \( v_2 \), but not before \( v_1 \), that references another block that contains a transaction that is conflicting with \( t \)). The result is that some of the banks might see \( v_1 \), so they shall think that \( t \) should be applied, while other might see \( v_2 \).
and think that $t$ is canceled. The problem in the above case is not the temporary situation where some banks accept an extra transaction ($t$) while others don’t, as the latter banks shall sooner or later see also $v_1$, that accepts $t$, and then they should apply it as well. Instead, the true problem is that this state can lead to creation of a valid graph that is not proper. The reason is that $v_1$ and $v_2$ induce different voting power distribution. Considering the voting power that corresponds to the amount of money that is transferred by $t$, that voting power belongs to the destination bank, according to $v_1$, and to the original bank according to $v_2$. Thus, each of the banks might think that this voting power is in its own hands, and each of them might use it to support a different block. As a result, two different blocks might enjoy the support of the same voting power without knowing of each other, which might produce a non proper graph.

In order to solve this problem we separate the actual acceptance/rejection of a transaction (at an accept node) from the decision of whether it would be accepted or rejected. We introduce a new node to the block-graph – a close node, that must appear exactly once between every pair of start node and accept node. The validity requirement of a close node is exactly as the requirement of an accept node, i.e., it must have a supporting voting power above a predefined threshold. More precisely, let $v_s$ be a start node, and let $v_c$ and $v_a$ be the close and accept nodes respectively that come after $v_s$. We define two set of banks, $S_c$ and $S_a$. A bank $B'$ is in $S_c$ if and only if it has a node $v$ such that (1) $v_c$ has acknowledged $v$ and (2) $v$ has acknowledged $v_s$. A bank $B''$ is in $S_a$ if and only if it has a node $v''$ such that (1) $v_a$ has acknowledged $v''$ and (2) $v''$ has acknowledged $v_s$. $v_c$ will be valid if the sum of voting power of the banks in $S_c$, computed according to $v_c$’s subgraph, is above a predefined threshold, and $v_a$ will be valid under the same condition when considering the set of $S_a$, and the voting power distribution according to $v_a$’s subgraph.

Using the close node, a malicious bank might still create two conflicting accept nodes, but both of them will be based on the same close node (just as before that, two conflicting accept nodes were based on the same start node, or otherwise they wouldn’t get the required supporting voting power). Next, we change the definition of the total balance computation, such that a transaction is considered conflicting (and get rejected) only if it is evident from the corresponding close node (instead of the accept node):

**Definition 6 (Total balance #2).** Given a block-graph, we take the initial balance according to the init node, and then for every accept node we apply every transaction that appears in its matching start node, as long that the subgraph of its matching close node doesn’t contain a conflicting transaction, and as long that the same transaction wasn’t already applied before that.

Now, two conflicting accept nodes that are based on a single close node have no effect. The same transactions will be accepted or rejected in both. Note that a malicious bank might also create conflicting close nodes (based on a single start node), but as long that it will be able to complete only one of them with a matching accept node, there will be no effect. This motivates updating the definition of proper graph:

**Definition 7 (Proper graph #2).** We say that a block-graph is proper if (1) for every pair of close nodes, at least one of them acknowledges the start node of the other, and (2) for every pair of accept nodes, at least one of them acknowledges the close node of the other.

The first requirement, concerning close nodes, is equal to the original requirement from proper graph, that makes sure that there is a connection between every two blocks, so that conflicting transactions won’t be applied together. The second requirement, concerning accept nodes, comes to deal with malicious banks. If a malicious bank creates two distinct close nodes with two matching accept nodes, there must be a connection between its two conflicting chains, or the graph won’t be proper. However, such connection is impossible in valid graphs, because it means that the malicious bank acknowledges its own malice. More precisely, an update node in one of its conflicting chains must reference (directly or indirectly) a node from the other chain, and such update node is considered invalid in our new terms. We shall now prove the following lemma.

**Lemma 3.** Computing the total balance of a block-graph that is valid and proper results in non-negative balance for all the accounts.

Before proving it, we start with a technical auxiliary lemma:
Lemma 4. Let $G$ be a valid and proper block-graph. We can construct a sequence of graphs with the following properties:

- The first graph in the sequence contains only a single node - the init node of $G$.
- Each graph extends the previous graph in the sequence by exactly one node.
- The last graph is equal to $G$.
- All the graphs in the sequence are valid and proper block-graphs.

Proof. Recall that block-graphs contain no cycles. Thus, we can define a topological ordering on the nodes of $G$. For every node of $G$, all the nodes in its subgraph come after it in the topological order. We construct the sequence of graphs such that the $n$th graph contains the $n$ last nodes in the topological order, and the edges that connect them. This sequence clearly answers the first three requirements.

Every graph in this sequence is indeed a block-graph according to Lemma 1. It is also valid, because if one of its nodes is invalid, then it will be invalid also in $G$. Assume by way of contradiction that one of these graphs, $G'$, is not proper. That means that there are in $G'$ a pair of accept nodes that don’t know the close nodes of each other, or a pair of close nodes that don’t know the start nodes of each other. The same pair also appear in $G$, and have the same subgraphs in $G$ and $G'$. Thus, they also don’t know the close or start node of each other in $G$ and so $G$ is not proper, in contradiction with the assumption that it is.

We shall now prove the main lemma:

Proof. Let $G$ be a valid and proper block-graph. We construct the sequence of graphs according to the above lemma. Note that every graph in this sequence is valid and proper. We shall prove by induction that the current lemma applies for each of these graphs, and so it applies also to $G$ (the last graph in the sequence). The first graph contains only the init node of $G$. In an init node all the sums of money are positive, so the lemma clearly holds for the first graph.

Assume that the lemma is true for the $n$th graph in the list, $G_1$. We prove it is also true for the $(n+1)$th graph, $G_2$. Note that $G_2$ extends $G_1$ by only a single node. If this node is a start node, close node or update node, then there is no difference in the balance computation between $G_1$ and $G_2$ so the lemma holds for $G_2$ as well.

If this node is an accept node, then we can compute the total balance according to $G_2$ by starting from the balance of $G_1$, and then apply the transactions that should be accepted according to the new accept node. We denote the new accept node by $v_a$ and its matching start node by $v_s$. Let $t$ be a transaction that appears in $v_s$, accepted in $v_a$ and was not applied in the balance computation of $G_1$. Note that $t$ is one of those transactions that we will apply after we already computed the balance according to $G_1$. Let $u$ be the user (the account) that transfers money according to $t$. As $v_s$ is valid, the balance that $u$ has according to the subgraph of $v_s$ is no less than the money that should be transferred in $t$. We denote $v_s$’s subgraph by $G_s$. $u$ can “lose” money between $G_s$ and $G_1$ only if it has accepted transactions in $G_1$ that aren’t accepted yet in $G_s$. However, we claim that all the accepted transactions of $u$ in $G_1$ are already accepted in $G_s$. Thus, the balance of $u$ according to $G_1$ is no smaller than the balance of $u$ according to $G_s$, and after applying $t$ in $G_2$ the balance of $u$ remains non negative, as claimed.

We still need to prove that indeed all the accepted transactions of $u$ in $G_1$ are already accepted in $G_s$. As $v_s$ is valid, we can see in its subgraph ($G_s$) accepted transactions of $u$, with all the sequence numbers up to one less than the sequence number of $t$ (each transaction with a different sequence number). Assume by way of contradiction that there is an accepted transaction of $u$ in $G_1$ that doesn’t appear in $G_s$. If that transaction has a sequence number smaller than the sequence number of $t$, then it is conflicting with another transaction, which is impossible as $G_1$ is proper. It cannot have an equal sequence number to $t$, as it cannot be conflicting with $t$, from the same reason as above (because $G_2$ is proper), and it cannot be identical to $t$, because $t$ is first accepted in $v_a$ (recall that $t$ wasn’t applied in the balance computation of $G_1$). If it has a bigger sequence number, then the start node at which that transaction appears must have in its subgraph an accepted transaction with the sequence number as $t$, which we have just proved to be impossible.

We want graphs to always be proper, where conflicting transactions cannot be accepted together, and where balances are kept non negative. As we shall now prove, valid graphs will be proper indeed, assuming
the voting power of malicious banks is limited. To define the limited voting power of malicious banks, we first label each bank as either “malicious” or “non-malicious”, where only banks labeled as “malicious” might produce conflicting nodes. Voting power is considered “valid” if it belongs to a non-malicious bank, or shared by only non-malicious banks. Otherwise, the voting power is considered “invalid”.

In classical distributed systems theory we assume a fixed set of servers or nodes. Dealing with malicious (more commonly called byzantine) nodes usually involves an assumption that more than two thirds of the nodes are non-malicious (or, non byzantine). This is similar in our case, only that there is no importance here to the number of the nodes (i.e., banks), but rather to the voting power distribution. The voting power distribution should always be such that more than two thirds of the total voting power is in the hands of non-malicious banks. Following that assumption, we need to define the threshold of required voting power for creation of valid accept and close nodes to be at least two thirds of the total voting power.

We are now ready for the main claim of this section:

**Lemma 5 (Proper graphs).** Let $G$ be a valid block-graph. If in every subgraph of $G$ that is proper more than two thirds of the voting power is valid, and if the threshold of required voting power for creation of valid accept and close nodes is two thirds of the total voting power, then $G$ is proper.

**Proof.** Let $G$ be a valid graph, that answers the conditions of the lemma. Assume by way of contradiction that $G$ is not proper. We will find a subgraph of $G$ that will also be non proper, and where there is exactly a single pair of either (1) accept nodes that don’t know the close nodes of each other, or (2) close nodes that don’t know the start nodes of each other. We will prove that it is impossible for these two nodes to gather the required supporting voting power, so one of them must be invalid, which means that $G$ is also invalid, contradicting the assumption that $G$ is valid. Thus, $G$ must be proper.

We call a pair of such accept nodes or close nodes an ignorant pair. There must be at least one such pair in $G$, or else $G$ is proper. If there is only one such pair, we are done. Assume there is more than a single such pair. We can get subgraphs of $G$ by removing nodes from $G$. However, if we remove a node we must also remove all the nodes whose subgraphs contain the removed node, or otherwise we will remain with a subgraph that is not a block-graph. Note that by removing nodes, the set of ignorant pairs can only get shrinked (new ignorant pairs cannot be created that way).

Let us consider two pairs of ignorant nodes. Let $v_a$ and $v_b$ be the nodes of one pair, and $v_1$ and $v_2$ the nodes of the other pair. Concerning those nodes, there are two options: (1) They are all different, or (2) two of them – one from each pair, are the same. Assume first they are all different. For at least one of these nodes, it must be that the other nodes don’t know of it (don’t acknowledge it), or otherwise there will be a cycle in the graph and block-graphs have no cycles. We shall remove that node, and so we remain only with the ignorant pair that doesn’t contain this node. Assume otherwise that $v_a = v_1$ (w.l.o.g.). In this case either $v_2$ doesn’t know $v_b$ or vice versa, or otherwise there will be a cycle in the graph. $v_1$ (equivalently $v_a$) doesn’t know both $v_2$ and $v_b$ as they both form ignorant pairs with it. Thus, we can safely remove (at least) one of $v_2$ and $v_b$, and once again we remain with only one of the two ignorant pairs. We will keep doing so repeatedly for every two ignorant pairs, and finally we shall remain only with a single pair, as required.

Let $G'$ be the subgraph of $G$ that we found, that contains only a single pair of ignorant nodes. Let $v_a$ and $v_b$ be the two nodes in this pair. We want to prove that the sum of voting power that supports $v_a$ and $v_b$ is less than $4/3$ of the total voting power, so they cannot both have the required voting power, and one of them must be invalid. Note that $v_a$ and $v_b$ might be two accept nodes that don’t know the close node of the other, or two close nodes that don’t know the start node of the other. The proof is identical in both cases, so we assume that they are two accept nodes, unless stated otherwise.

We shall consider three different subgraphs of $G'$: (1) $G_a$, the subgraph of $v_a$, (2) $G_b$, the subgraph of $v_b$, and (3) $G_{ab}$, the intersection of $G_a$ and $G_b$. It can be easily seen (following Lemma 1) that the three subgraphs are valid and proper block-graphs (recall that the only ignorant pair in $G'$ contains $v_a$ and $v_b$, and they don’t appear in any of the above graphs). We compute the voting power distribution according to each of these graphs. According to the assumptions, more than $2/3$ of the voting power in each of these distributions belongs to non-malicious banks.

We claim that a non-malicious bank might either support $v_a$ according to $G_a$, or support $v_b$ according to $G_b$, but not both. Proof: Assume by way of contradiction that a non-malicious bank, $B$, supports both
nodes, that means that \( B \) has one or two \textit{update} nodes whose subgraphs contain the \textit{close} nodes of \( v_a \) and \( v_b \) (assuming w.l.o.g that \( v_a \) and \( v_b \) are \textit{accept} nodes). In return, \( v_a \) and \( v_b \) have these \textit{update} nodes in their own subgraphs (\( G_a \) and \( G_b \) respectively). Let \( v_1 \) and \( v_2 \) be the (earliest) \textit{update} nodes of \( B \), such that the \textit{close} node of \( v_a \) appears in the subgraph of \( v_1 \), and the \textit{close} node of \( v_b \) appears in the subgraph of \( v_2 \). Without loss of generality, assume that \( v_1 \) appears before \( v_2 \) in \( B \)'s chain, or that \( v_1 = v_2 \). This means that \( v_1 \) appears in \( v_2 \)'s subgraph. As \( v_b \) has \( v_2 \) in its subgraph, that means that it also has \( v_1 \) in its subgraph, and so it also has the \textit{close} node of \( v_a \) in its subgraph, contradicting the assumption that \( v_a \) and \( v_b \) form an ignorant pair.

Recall that a voting power is “valid” if it belongs to a non-malicious bank, or shared by group of non-malicious banks, and “invalid” otherwise. Considering \( G_{ab} \)'s voting power distribution, assume that there is \( x \) amount of valid voting power, and \( y \) of non-valid voting power. Note that \( x + y \) is the total voting power in the system. For simplicity we assume that \( x + y = 1 \). According to the assumption we have that \( y < 1/3 \). The valid voting power belongs to non-malicious banks. As we have seen, a non-malicious bank might support at most one of \( v_a \) and \( v_b \). Invalid voting power, however, might belong to banks that support both nodes. Thus, the sum of voting power that supports \( v_a \) and \( v_b \) according to the voting power distribution of \( G_{ab} \) is at most \( x + 2y \). Note that \( x + 2y = (x + y) + y = 1 + y < 1 + 1/3 = 4/3 \), as required. We want to prove that this is also true when considering the correct supporting voting power – of \( v_a \) according to \( G_a \), and of \( v_b \) according to \( G_b \). For that cause we must consider the changes in the voting power distribution when moving from \( G_{ab} \) to \( G_a \) and \( G_b \). We shall prove that the valid voting power in \( G_{ab} \), \( x \), is still divided between the nodes, so the sum of \( x + 2y \) is still relevant (as an upper bound).

The major change in the voting power distribution will be due to new \textit{accept} nodes, in \( G_a \) and \( G_b \), that will change the total balance distribution. Such new \textit{accept} nodes can appear only for matching \textit{start} nodes that already appear in \( G_{ab} \). I.e., there cannot be a completely new block (from \textit{start} node to \textit{accept} node) that appears in \( G_a \) or \( G_b \). Proof: Assume by way of contradiction that such new block appeared in \( G_a \). That means that the new \textit{accept} node appears in the subgraph of \( v_a \), but that its new, matching, \textit{start} node doesn’t appear in the subgraph of \( v_b \) (or else it would also appeared in \( G_{ab} \)). Because the only ignorant pair in \( G' \) are \( v_a \) and \( v_b \), it follows that:

1. If \( v_b \) is an \textit{accept} node, then the new \textit{accept} node must know \( v_b \)'s \textit{close} node. Thus, \( v_a \) also knows \( v_b \)'s \textit{close} node.

2. If \( v_b \) is a \textit{close} node, then the new \textit{close} node (that must appear in the new block) must know \( v_b \)'s \textit{start} node. Thus, \( v_a \) also knows \( v_b \)'s \textit{start} node.

In both cases we get contradiction to the original assumption that \( v_a \) and \( v_b \) are ignorant pairs. Thus, a new \textit{accept} node can appear only if its \textit{start} node already appears in \( G_{ab} \).

Except for new \textit{accept} nodes (for existing \textit{start} nodes), might also appear new \textit{start} nodes (without matching \textit{accept} nodes, but possibly with matching \textit{close} nodes, in case \( v_a \) and \( v_b \) are \textit{accept} nodes\textsuperscript{7}). The only effect that such new open blocks might have on the voting power distribution is that existing amounts of voting power will be shared with additional banks because of new uncertain transactions. Such sharing cannot increase the supporting voting power of \( v_a \) or \( v_b \), because if that voting power didn’t support them in the first place, then sharing it with additional bank won’t change it. However, if that voting power supported them beforehand, then now it might not support them, in case the new bank doesn’t support them. Thus, such new \textit{start} nodes might only decrease the supporting voting power of the nodes. As we are interested in the maximal sum of supporting power, we will ignore such new \textit{start} nodes, and consider only the effect of new \textit{accept} nodes (for existing \textit{start} nodes).

Recall that when computing the voting power distribution, we start with computing the balance of each user. It can be seen as if each user has amount of voting power that equals to his balance, and each user delegates his voting power to his bank. However, if the graph contains uncertain transactions that belong to a specific user, then the bank of that user has to share some of this voting power with other banks that might receive that power according to those transactions. To conclude, we can divide the total voting power

\textsuperscript{7} If \( v_a \) and \( v_b \) are \textit{close} nodes, then from the same reasoning as before, we will get that a new matching pair of \textit{start} node and \textit{close} node cannot appear in \( G_a \) or \( G_b \), or else \( v_a \) and \( v_b \) wouldn’t be ignorant pair.
We are interested in voting power that was valid in $G_{ab}$. Such valid voting power is based on balances of users that sit in non-malicious banks. That is because other voting power will be either belonged exclusively or shared by a malicious bank and so will be invalid. Even if the user sits in a non-malicious bank, the shared voting power that he contributes will be invalid in case one of the banks that share it is malicious. We shall consider the valid voting power that each user (that sits in non-malicious bank) contributes in $G_{ab}$, and we shall see what happens with this voting power in $G_a$ and $G_b$. Recall that we defined $x$ to be the sum of valid voting power in $G_{ab}$. $x$ is exactly the sum of all the contributions of valid power from those users.

Let $u$ be a user of a non-malicious bank $B$. Concerning $B$, we have already proved that it might either support (exactly) one of $v_a$ or $v_b$, or not support any of them. Assume that it supports $v_a$. In this case it cannot have a new accept node that appears in $G_b$. Proof: Assume by way of contradiction that a new accept node of $B$ does appear in $G_b$. Denote that new accept node by $v_1$. That means that $v_1$ appears in the subgraph of $v_b$. It cannot appear also in $G_a$, as otherwise it would appeared also in $G_{ab}$, and then it won’t be “new”. As $B$ supports $v_a$, according to $G_a$, then one of its nodes in $G_a$ has the matching start node/close node of $v_a$ (according to $v_a$’s type) in its subgraph. As this node appears in $G_a$ while $v_1$ does not, then it must come before $v_1$ in $B$’s chain (and recall that $B$ is non-malicious, so it has no forks). Thus, $v_1$ has the matching start node/close node of $v_a$ in its subgraph, and so does $v_b$, contradicting the assumption that $v_a$ and $v_b$ are ignorant pairs.

Recall that we are currently considering what happens with the valid voting power in $G_{ab}$ that contributed a user $u$ (in bank $B$). Assume, once again, that $B$ supports $v_a$. This means that the non-shared part of the voting power that $u$ contributes supports $v_a$, and its shared voting power either supports $v_a$ or not, depending on the identities of the other banks. The voting power of $u$ cannot support $v_b$, because we have already seen that $B$ cannot support both $v_a$ and $v_b$. The question is if the voting power that belonged to $u$ in $G_{ab}$ might be later used to support $v_b$ in $G_b$. This is possible only if a transaction of $u$ is accepted in $G_b$ (and not in $G_{ab}$) such that some of its money is transferred to another bank, that does support $v_b$. Denote that transaction by $t$. We proved above that a new accept node of $B$ cannot appear in $G_b$. Thus, if $t$ is accepted in $G_b$, and transfers money outside of $B$, then it must be an uncertain transaction in $G_{ab}$. This means that the voting power that will be transferred is shared, according to $G_{ab}$. If that voting power goes to a malicious bank, then this power was invalid in $G_{ab}$ (as it was shared with a malicious bank), and is not of our interest. Assume otherwise that it goes to a non-malicious bank, $B'$. We want this voting power to support $v_b$. This means that $B'$ should support $v_b$. As $B'$ is non-malicious, it cannot also support $v_a$. As $t$ is an uncertain transaction in $G_{ab}$ that attempts to transfers money to $B'$, the corresponding voting power (in $G_{ab}$) does not support $v_a$. The question is if that voting power still won’t support $v_a$ when considering the voting power distribution at $G_a$. The only way that voting power might indeed support $v_a$ is if $t$ will cease to be “uncertain” in $G_a$ (but not by being accepted), so that amount of money will be no longer shared with $B'$. This can be either because $t$ was rejected at a new accept node, or otherwise that another transaction with the same sequence number was accepted.

We start with contradicting the first case, where $t$ is rejected in $G_a$. Note that $t$ as to be both accepted in $G_b$ and rejected in $G_a$ by two conflicting accept nodes for the same start node at which $t$ appears in $G_{ab}$. Clearly that means that $t$ appears in a malicious bank’s block. For it to be both accepted and rejected, there must be also two conflicting close nodes, where one of them sees another conflicting transaction in another block, and the other does not. The result is that we have two conflicting close nodes with two matching (and conflicting) accept nodes for the same bank. They all appear in the union of $G_a$ and $G_b$, and so they also appear in $G'$. Each of the accept nodes cannot have in its subgraph the close node of the other, as this results in invalid graph (a malicious bank can not identify its own malice). Thus, they form a pair of ignorant nodes in $G'$. The only ignorant nodes in $G'$ are $v_a$ and $v_b$. However, those two conflicting accept nodes cannot be $v_a$ and $v_b$, as they appear in $G_a$ and $G_b$ which are the subgraphs of $v_a$ and $v_b$ respectively (and according to our definition of subgraph, a node does not form part of its own subgraph). Thus, it cannot be that $t$ is accepted in $G_b$ and rejected in $G_a$. 

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The second case is where another transaction of \( u \), different than \( t \) but with the same sequence number, is accepted in \( G_a \). As \( t \) gets accepted in \( G_b \), we have two conflicting transactions that get accepted in \( G' \) (which contains both \( G_a \) and \( G_b \)). This means that the two \( \text{start} \) nodes that contain those conflicting transactions have a matching pair of ignorant \( \text{close} \) nodes (or else at least one of the transactions wouldn’t be accepted) and there is also a matching pair of \( \text{accept} \) nodes (that might either form an ignorant pair or not). However, the only ignorant pair in \( G' \) are \( v_a \) and \( v_b \). Thus, this is possible only if those ignorant \( \text{close} \) nodes are exactly \( v_a \) and \( v_b \). But, in such case they don’t have the matching \( \text{accept} \) nodes (where the conflicting transactions get accepted) in \( G_a \) and \( G_b \), and so this is not possible.

Recall that we considered above the valid voting power of a user (\( u \)) whose (non-malicious) bank (\( B \)) supports \( v_a \). We have seen above that if some of its voting power will support \( v_b \) according to \( G_b \), then it won’t support \( v_a \) in \( G_a \).

The next case is if \( B \) supports neither \( v_a \) nor \( v_b \). In this case the voting power of \( u \) in \( G_{ab} \) couldn’t support \( v_a \) nor \( v_b \), as \( B \) doesn’t support them. The question is if that voting power might later support both of them – \( v_a \) in \( G_a \) and \( v_b \) in \( G_b \). This is only possible if \( u \) has a transaction that transfers money to a bank, \( B'' \), that supports both. This is true because (1) two conflicting transactions cannot be accepted, as it requires an ignorant pair of \( \text{close} \) nodes, and the only such pair that exist is the pair of \( v_a \) and \( v_b \), and (2) two consequent transactions of the same user cannot be accepted, because the \( \text{start} \) node that contains the later transaction must know the \( \text{accept} \) node that accepted the first transaction, and so the block that accepts the second transaction is a new block in \( G_a \) or \( G_b \) (both its \( \text{start} \) node and \( \text{accept} \) node don’t appear in \( G_{ab} \)), which we proved to be impossible. If that \( B'' \) supports both \( v_a \) and \( v_b \), then it is malicious (as non-malicious banks can support at most one of them). If the transaction that transfers the money to \( B'' \) does not appear in \( B \), then it is an uncertain transaction in \( G_{ab} \), and then that voting power is considered invalid in \( G_{ab} \), and is not of our interest. Assume otherwise, that this transaction appears in a block of \( B \). For this voting power to support both \( v_a \) according to \( G_a \) and \( v_b \) according to \( G_b \), a corresponding \( \text{accept} \) node must appear both in \( G_a \) and in \( G_b \). As \( B \) is not malicious, it can have only one such \( \text{accept} \) node, that can appear at most in one of \( G_a \) or \( G_b \) (or else it would appear also in \( G_{ab} \)). Thus, at most one of \( v_a \) or \( v_b \) can be supported by this voting power.

The result is that the valid voting power from \( G_{ab} \) of every user that sits in non-malicious bank can support at most one of \( v_a \) or \( v_b \) (in \( G_a \) and \( G_b \) respectively). This is what we claimed, and so the total supporting voting power is less than \( 4/3 \), and at least one of \( v_a \) and \( v_b \) cannot be valid.

\( \square \)

4 The Protocol

In the previous section we described the block-graph which stands in the center of our cryptocurrency. In the base of the block-graph stands the \( \text{init} \) node that defines the initial balance. Recall that a block-graph contains exctly one \( \text{init} \) node. Thus, different \( \text{init} \) nodes define completely distinct monetary systems.

We consider a single monetary system, that starts with a specific \( \text{init} \) node. Every bank that take part in the monetary system holds a block-graph that in its base there is that \( \text{init} \) node. As time passes, each bank might add to its block-graph new nodes that it creates or receives from other banks. We define a bank as valid if (1) it doesn’t create conflicting nodes, (2) it never removes existing nodes from its graph, and (3) it follows the defined protocol.

In the introduction we defined several requirements from a cryptocurrency in which acceptance of transactions is non-reversible. We shall now redefine these requirements with our notions concerning the admins that run the cryptocurrency (the banks) and the transaction structure:

– **Agreement**: A transaction accepted by a valid bank will be eventually accepted by all valid banks.

– **Positive-Balance**: At every point, and for every valid bank, applying the set of transactions that were accepted by that bank results in non-negative balance in all the accounts.

– **Termination**: Every transaction that is sent from a user to a valid bank must be either accepted or rejected by that bank.
Rejection Restriction: A valid bank can reject a transaction only if the transaction is not valid, according to the bank’s block-graph at time of submitting the transaction, or if the same user has submitted a different transaction with the same sequence number (at any point in time).

The protocol we use is rather straightforward:

- When receiving a transaction from a user, make sure it is valid, and add it to a list of waiting user transactions.
- If you can create a start node (your previous start node has a matching accept node) and your list of waiting transactions is not empty, then extract the transactions from the list and create a start node that contains the ones that are still valid, according to your current graph (assuming that at least one of those transactions is valid).
- If you have an open transactions block (a start node with no matching close node, or close node with no matching accept node) and you can validly create an accept/close node, then create it.
- When receiving a node from another bank, check if it is already included in your block-graph. If not, then check if the nodes it references are included in your block-graph. If not, ask the other banks for these nodes. If (or once) your block-graph includes all the nodes that are referenced by the node you received (but doesn’t include that node itself), then make sure that (1) the node is valid, and (2) that adding it to your graph won’t make your graph improper. If it answers both criteria, then add it to your block-graph, and create a new update node that references it.
- After creating a node in one of the above cases, send this node to all other banks.

We want to prove that the protocol fulfills the above requirements. We start with the rejection restriction requirement. A transaction \( t \) might be rejected, by the bank that receives it, \( B \), in three cases. The first case is at the moment that \( B \) receives \( t \), and observes that it is invalid according to its current block-graph. The second case is when \( B \) creates its next start node, and then it observes that \( t \) is invalid. As \( t \) was valid at the moment that \( B \) received it, then there are accepted transactions of the user that created \( t \) with all the sequence numbers up to (not including) the sequence number of \( t \), and it had a balance which is higher that the amount that should be transferred by \( t \). Thus, the only reason that \( t \) won’t be valid is because the same user has submitted a different transaction with the same sequence number. The last case is if \( t \) appears in a start node that has a matching close and accept nodes, and its close node’s subgraph contains a conflicting transaction. All of the above cases are legible according to the rejection restriction requirement.

The positive-balance requirement holds by Lemma 3 because the graph of every valid bank is always proper (as we don’t accept nodes that cause it to be improper).

Agreement holds if every accept node that is accepted by one valid bank, will be accepted by all the other valid banks. Such accept node must be valid, or else it wouldn’t have been accepted by the first valid bank. Thus, it won’t be accepted by another bank only if it will make its graph improper. If we assume that the voting power distribution of every graph, at every time point, is such that more than two thirds of the voting power is valid (i.e., in the hands of non malicious banks), then adding a new, single, valid node won’t make this graph improper, following Lemma 5.

After agreement we left only with the termination requirement. While practically termination should pose no concern, it is theoretically problematic. We call it the problem of migrating power, and it generally applies to every cryptocurrency system where some resource is used as the base for decisions. The problem with resources is that they might migrate. One day they are in one hand, the second they are in another. If the lazy user sends his transaction to an admin that already lost its power, then that admin will have to forward it to the one that is now in power (that holds a big share of resources). But, what if by the time this transaction reaches the second admin, the power have already migrated to another admin? And more generally, if the speed of migrating power is faster than the communication delay from those admins that have already lost the power, then a user that sent his transaction to an “old” admin will never get a response.

8 In a practical implementation we don’t have to respond with an update node if this won’t deliver any important information for the protocol. E.g., there is no importance to acknowledge an acknowledgment to a previous acknowledgment of your own node.
There is an option to bypass this problem by conditioning acceptance of blocks with agreement of admins that already lost their power. This is, however, a bad design, as admins that lost their power will probably soon be non responding.

In practice this problem doesn’t seem to make sense, and anyway a user that sees that his transaction doesn’t get to the right locations can resubmit his transaction to the current power owners. The problem with the migration of power is that it migrates infinitely, going through infinite number of admins. Now, let’s be realistic, our coin won’t live forever. In that finite time that it does, there is a finite set of banks that will ever had any voting power, and in particular a finite set of valid banks. Thus, if we assumed before that two thirds of the voting power is always in the hands of valid banks, then we can further assume that there is a finite set of valid banks that together always hold more than two thirds of the voting power. Following this assumption, we shall prove that termination holds.

We start with the following claim: Let \( B \) be a valid bank, that created a \textit{start} node, but didn’t create yet a matching \textit{close} node. We claim that eventually \( B \) will create the matching \textit{close} node. The same claim (with the same proof) is that if \( B \) created a \textit{close} node with no matching \textit{accept} node, then eventually it will create that matching \textit{accept} node.

**Proof:** According to the protocol, \( B \) will create the \textit{close} node at the moment that it can. It can create such node only if the sum of voting power that supports the matching \textit{start} node is at least two thirds of the total voting power. When \( B \) created the \textit{start} node, it sent it to all other banks. Now, recall that by the assumption there is a finite set of banks that more than two thirds of the voting power is always in their hands. We denote this set by \( S \). The banks in \( S \) will eventually accept that \textit{start} node of \( B \) (once they accepted the nodes that it references), as it is valid, and it won’t make their graph improper (as we have seen before). After they accept it, they will create corresponding \textit{update} nodes, that reference this \textit{start} node, and they will send their \textit{update} nodes back to \( B \). In return, \( B \) will create new \textit{update} nodes that reference those \textit{update} nodes from the banks of \( S \). At this point \( B \) has the support of all the banks from \( S \). Considering the voting power distribution, more than two thirds of the voting power is in their hands, so \( B \) has the required supporting voting power, and it will create a \textit{close} node.

Now we can easily prove that the termination requirement holds. **Proof:** Let \( B \) be a valid bank that receives a transaction from its user. According to the protocol, \( B \) will put the transaction in its next \textit{start} node, or otherwise it will reject it (if it is non valid). The only thing that might prevent \( B \) from creating its next \textit{start} node is if it already has an open \textit{start} node without a matching \textit{close} node or \textit{accept} node. According to the above claim, \( B \) will eventually create matching \textit{close} node and \textit{accept} node, and then it will also create a new \textit{start} node that will contain the required transaction. Once again, according to the above claim, \( B \) will eventually create matching \textit{close} node and \textit{accept} node to the new \textit{start} node. Once it created the \textit{accept} node, the transaction will be considered either accepted or rejected.

The result is that our monetary system achieves all the requirements we defined for a deterministic, non-reversible, cryptocurrency system.

### 4.1 Implications

The protocol and settings defined above are rather limited, and intended to provide only the minimum that is required in order to have a cryptocurrency that achieves the requirements we defined under asynchronous communications. We provide here some details and implications we ignored.

The first case concerns allegedly malicious users. According to the block-graph definition, a user that submitted conflicting transactions might not be able to submit any additional transaction, which makes its money unusable. This can be seen as a punishment to such user, but it might be a too harsh punishment, as it might be an honest mistake, with no malicious intentions. To overcome it, we can allow a user to submit a “group of transactions” with a single sequence number. Such group won’t be considered as conflicting with any subset of the transactions it contains.

Another concept that wasn’t discussed above is the commission. We have mentioned in the introduction that a user will pay commission for each of his issued transactions. The simplest approach is that the commission will be a fixed percentage of the transaction sum, that is given to the bank of the user that
issued the transaction. Another option is to divide the commission between the user’s bank (probably about an half of the commission) and the banks that supported the block at which that transaction appeared (as they all took part in the agreement process). If we want to encourage participation of as many banks as possible, then we can define the commission percentage to be variable, where this percentage gets bigger as more banks (voting power) support the block. As the issuing bank receives a fixed share of the commission, the commission it receives will be bigger as more banks support its block. This incentivizes the issuing bank to ask for the acceptance of as many banks as it can.

However, sharing the commission between the banks that supported the block might cause problems if the same transaction appears in blocks of different banks, as it is not clear then according to which block we divide the commission. Such case is possible if the user submitted a transaction to his bank and got no response, so he sent it also to another bank, and as a result both banks might create blocks that contain this transaction. In order to solve this problem we can define that the commission distribution is computed only according the block of the original bank of the user. If that bank stops responding before it manages to accept its block, then the amount of commission that should have been taken remains effectively as inaccessible money.

5 Conclusions

In this paper we define a cryptocurrency system that is ideal from several reasons. First, we want a cryptocurrency that does not cause great waste of energy, and that has low latency in accepting transactions. High latency and great waste of energy are part of the solution of Proof of Work to the problem of deciding the identity of the next block in the blockchain. The block-graph, on the other hand, provides effectively a blockchain for each bank separately, so there is no problem in deciding the next block (node) – each bank can freely add a new node to its own blockchain, without competing with other banks (a competition that takes both energy and time). The only restriction is when a bank wants to close/accept its block, and then it has to wait for replies from other banks telling they support its block. Thus, the delay in the system is only due to message transmission times and the time it takes to perform the basic required computations of validating/applying signatures. The hope is that such delay will be on the order of seconds, which might make it usable as an everyday payment method.

Another requirement from the cryptocurrency system is to be deterministic, where once a transaction is accepted, it is irrevertible. The problem with the blockchain is that it is not monotonic. A chain of blocks in the blockchain might be later ignored, in favor of another, parallel, longer chain. Thus, transactions might be accepted, and later rejected. In our coin, once transactions are accepted, they remain accepted.

Like every cryptocurrency, we have to manage some sort of consensus. In our case the consensus is achieved based on the coin holdings, instead of computation power. More coin means more voting power, where the coin of a user is delegated to its bank, like in representative democracy where the banks are the representatives. One of the most important features in this coin is that the consensus is achieved even under asynchronous communications. While seems impossible because of the FLP result, it is indeed possible because the consensus required in a cryptocurrency system is not equivalent to the classical consensus. In classical consensus we must accept a single value out of conflicting ones, and in our case we can choose not to accept any of the conflicting values (reject them all).

Security in our case is very plain and simple. By employing a deterministic protocol, that works even when communication is asynchronous, the only way to undermine the security is if more than one third of the voting power will be in malicious hands, or, more precisely, in your hands (it doesn’t help if the other malicious banks don’t cooperate with you).

5.1 Future Work

The settings and protocol described in this paper are used to formally prove the existence of such asynchronous deterministic cryptocurrency system, with all the good features we discussed. There are yet many open challenges for implementing such a system. For example, we might want to spare memory, and not to
remember the entire block-graph. The question is what can we forget, and under what conditions. Another challenge is with large number of banks. The more banks we have, the bigger the requirement for memory and bandwidth. At some point, if there will be banks that won’t have strong enough hardware, and their voting power on the other hand will be significant (so they are required for the consensus process to complete), it might cause extra delay in the system.

An existing downside in many cryptocurrencies including Bitcoin and the coin presented in this paper is that the system is completely transparent. I.e., every transaction that is accepted is visible to the public, so that everyone can see the source and destination accounts, and the sum of money that was transferred. Yet, the identities of the account owners are of course generally unknown. An interesting direction in our case is if we can limit the transparency to the level of banks, so that the only entities that truly knows the transaction source, destination and sum of money (all together) are the banks of the issuing and receiving clients.

An important concept in cryptocurrencies at which consensus is based on coin possession, such as in Proof of Stake, is that malicious entities will be financially damaged because of their acts. Otherwise, different entities (and especially strong entities, that posses a lot of coin) might maliciously attempt to maneuver the currency, without getting harmed. This is the famous nothing at stake problem. Note that this is never true in practice, because an unstable coin will have a lower value, so such entities that posses a lot of coin have an interest in keeping the coin credible. Yet, if the coin value is decreased, this will harm also non malicious parties. Thus, the best practice is indeed financially damaging those malicious entities.

Note that in our case banks should have a private account, to where they receive the commission on the users’ accepted transactions. Transactions that transfer private money of a bank can be issued only in blocks of that banks itself, and cannot appear in blocks of other banks. Recall that a malicious bank (that made a malicious act) won’t mange to create new blocks, as it cannot reference nodes of other banks that are already aware to its malice so it cannot gain the required support for a block. Thus, a malicious bank won’t be able to use its private money (and once it turned out to be malicious, it won’t receive any more commission). If we can incentivize a bank to keep money in its private account, this can be used as means to make sure the bank follows the protocol, as if it won’t, then it will lose that money. This will be similar to real banks, that by regulatory requirements, in order to ensure their stability, must have some capital of their own (such stability is not relevant to our banks, as they cannot use their clients’ money, as opposed to real world banks).

The most reasonable way to incentivize the banks to keep private money is by limiting their voting power in case they don’t have enough private money. I.e., we can define a percentage of private money out of the voting power of a bank that the bank must have in order to use its full voting power. If it doesn’t have the required percentage, its voting power will be decreased proportionally. A more research is required in order to make sure that the system still complies with the agreement and termination requirements.

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