Origin of the orbital period change in contact binary stars

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Abstract

The evolution of contact binary star systems in mass asymmetry (transfer) coordinate is considered. The orbital period changes is explained by an evolution in mass asymmetry towards the symmetry (symmetrization of binary system). It is predicted that a decreasing and an increasing orbital periods are related, respectively, with the non-overlapping and overlapping stage of the binary star during its symmetrization. A huge amount of energy $\Delta U \approx 10^{41}$ J is converted from the the potential energy into internal energy of the stars during the symmetrization. As shown, the merger of stars in the binary systems, including KIC 9832227, is energetically an unfavorable process. The sensitivity of the calculated results to the values of total mass and orbital angular momentum is analyzed.

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I. INTRODUCTION

Overcontact, contact, and near-contact binaries, forming di-star compounds with the average distances between the stars of the same order as the sum of their radii [1–4] are of great interest for stellar evolution. The compact binary stars are a good laboratory for a wide variety of astrophysical phenomena, for example, the mass transfer between stars. The information on formation and evolution of compact binaries is required to understand the processes observed in isolated stars. The observations of the stages of evolution in these binaries provide a verification of our understanding of the inner structure and dynamical interaction of stars.

It has been found that the orbital period of W-type overcontact binary GW Cep ($q = M_{1i}/M_{2i} = 0.37$, $\eta_i = (M_{1i} - M_{2i})/(M_{1i} + M_{2i}) = 0.46$, $M_{1i}$ and $M_{2i}$ are the masses of stars in the binary) is decreasing with time [5]. For the overcontact binaries VY Cet ($q = 0.67$, $\eta_i = 0.20$) and V700 Cyg ($q = 0.65$, $\eta_i = 0.21$), a cyclic oscillations have been found to be superimposed on a secular period increase. This effect has been explained either by the strong external perturbation, i.e. by a close-by third object, or by the magnetic activity cycles. For the EM Lac ($q = 0.63$, $\eta_i = 0.23$) and AW Vir ($q = 0.76$, $\eta_i = 0.14$) binaries, the periods show a secular increase. It has been concluded that the period variations of a W UMa-type binary star is correlated with the mass ratio $q$ and the mass $M_{1i}$ of the primary component [5]. The low mass ratio $q$ in binaries usually results in a decreasing period, while the periods of larger-$q$ systems are increasing.

Because mass transfer is an important observable for close binary systems in which the two stars are nearly in contact [1–7], it is necessary to study the influence of evolution of these stellar systems in the mass asymmetry coordinate $\eta = (M_1 - M_2)/M$ ($M_k (k =1,2)$ are the stellar masses and $M = M_1 + M_2$) on the orbital period variations. This is the main aim of the present article. As in our previous works [6, 7], we analyze the total potential energy $U(\eta)$ and the orbital period as functions of $\eta$ at fixed total mass $M = M_1 + M_2$ and orbital angular momentum $L = L_i$ of the system.
II. THEORETICAL METHOD

The total potential energy of the di-star system

\[ U = U_1 + U_2 + V \]  

is given by the sum of the potential energies \( U_k \) \((k = 1, 2)\) of two stars and star-star interaction potential \( V \). The radiation energy is neglected because the absolute values of the gravitational energy and the intrinsic kinetic energy are much larger than the radiation energy. The energy of the star "\( k \)" is

\[ U_k = -\omega_k \frac{G M_k^2}{2 R_k}, \]

where \( G, M_k, \) and \( R_k \) are the gravitational constant, mass, and radius of the star, respectively. Employing the values of the dimensionless structural factor

\[ \omega_k = 1.644 \left( \frac{M_\odot}{M_k} \right)^{1/4} \]

and radius

\[ R_k = R_\odot \left( \frac{M_k}{M_\odot} \right)^{2/3} \]

of the star from the model of Ref. [3], we obtain

\[ U_k = -\omega_0 G M_k^{13/12} / 2, \]

\[ \omega_0 = 1.644 \frac{M_\odot^{11/12}}{R_\odot}, \]

where \( M_\odot, R_\odot \) are mass and radius of the Sun, respectively. Because the average density

\[ \rho_k = M_\odot \rho_\odot / M_k \] (\( \rho_\odot \) is the average density of the Sun) increase with decreasing the mass \( M_k \) of star [3], the structural factor \( \omega_k \) depends on \( M_k \) in Eq. (3). The change of \( \eta \) from 0 to 1 leads to the change of \( \omega_1 \) by about of 16%. Note that, in general, the dimensionless structural factor \( \omega_i \) is determined by the density profile of the star. In the present paper, we employ the values of the structural factor of the stars from the model of Ref. [3]. This model well describes the observable temperature-radius-mass-luminosity relations of stars, especially binary stars, the spectra of seismic oscillations of the Sun, distribution of stars on their masses, magnetic fields of stars and etc. The stellar radii, masses, and temperatures are expressed by the corresponding ratios of the fundamental constants, and the individuality
of stars is determined by two parameters - by the charge and mass numbers of nuclei, from which the star is composed [3].

Because the two stars rotate with respect to each other around the common center of mass, the star-star interaction potential contains, together with the gravitational energy of interaction $V_G$ of two stars, the kinetic energy of orbital rotation $V_R$:

$$V(R) = V_G + V_R = V_G + \frac{L^2}{2\mu R^2},$$

(5)

where $L$ is the orbital angular momentum of the di-star which is conserved during the conservative mass transfer and $\mu = \frac{M_1 M_2}{M}$ is the reduced mass. At $R \geq R_t = R_1 + R_2$ and $R \leq R_t$,

$$V_G(R) = -\frac{GM_1 M_2}{R}$$

(6)

and

$$V_G(R) = -\frac{GM_1 M_2}{2R_t} \left[ 3 - \frac{R^2}{R_t^2} \right],$$

(7)

respectively [11]. Here, $R_t$ is the touching distance. From the fix point conditions $\partial V / \partial R |_{R=R_m} = 0$ and $\partial^2 V / \partial R^2 |_{R=R_m} > 0$, we find the relative equilibrium distance between two stars corresponding to the minimum of $V$:

$$R_m = \frac{L^2}{G\mu^2 M}$$

(8)

at $R_m \geq R_t$ or

$$R_m = \left( \frac{L^2 R_t^3}{G\mu^2 M} \right)^{1/4}$$

(9)

at $R_m \leq R_t$. Finally, one can derive the expression for the star-star interaction potential

$$V(R_m) = -\frac{GM_1 M_2}{2R_m}$$

(10)

at $R_m \geq R_t$ or

$$V(R_m) = -\frac{GM_1 M_2}{R_t} \left[ \frac{3}{2} - \frac{R_m^2}{R_t^2} \right]$$

(11)

at $R_m \leq R_t$. In Eqs. (10) and (11), $R_m$ is the semi-major axis of the elliptical relative orbit. Note that for the di-star systems considered, $v(R_m) = \left( \frac{G M}{R_m} \right)^{1/2} \ll c$, where $v$ and $c$ is
the velocities of orbital motion and light, respectively, and one can neglect the relativistic effects. Since \( GM_k/R_m \ll c^2 \), the gravitational field can be considered weak. Because of these facts, we use the Newtonian law of gravity.

Using the mass asymmetry coordinate \( \eta \) instead of masses \( M_1 = \frac{M}{2}(1 + \eta) \) and \( M_2 = \frac{M}{2}(1 - \eta) \), we rewrite the final expression [1] for the total potential energy as

\[
U = -\frac{GM_2^3}{2R_\odot} \left( \alpha [(1 + \eta)^{13/12} + (1 - \eta)^{13/12}] + \beta_1 [1 - \eta^2]^3 \right)
\] (12)

at \( R_m \geq R_t \) or

\[
U = -\frac{GM_2^3}{2R_\odot} \left( \alpha [(1 + \eta)^{13/12} + (1 - \eta)^{13/12}] \right.
\]

\[\left. + \frac{1 - \eta^2}{(1 + \eta)^{2/3} + (1 - \eta)^{2/3}} \left[ \frac{3}{2} - \frac{\gamma}{[1 - \eta^2][(1 + \eta)^{2/3} + (1 - \eta)^{2/3}]^{1/2}} \right] \right) \] (13)

at \( R \leq R_t \), where

\[
\alpha = 1.644 \left( \frac{M}{2M_\odot} \right)^{13/12},
\]

\[
\beta_1 = \frac{GM^5}{64L^2} = \frac{GM_\odot^5}{2L^2} \left( \frac{M}{2M_\odot} \right)^5,
\]

\[
\beta_2 = 2 \left( \frac{M}{2M_\odot} \right)^{4/3},
\]

and

\[
\gamma = \frac{2^{7/3}LM_\odot^{1/3}}{(GR_\odot M^{11/3})^{1/2}}.
\]

Here, we assume that the orbital angular momentum \( L \) and the total mass \( M \) are conserved during the conservative evolution of the di-star in the mass asymmetry coordinate \( \eta \). The orbital angular momentum \( L \) is calculated by using the experimental masses \( M_{k,i} \) of stars and period \( P_{\text{orb},i} \) of their orbital rotation [5, 8]. As seen from Eq. (13), the stability of the binary star system depends on the orbital angular momentum \( L \) the total mass \( M \), and the structural factor.

To obtain the period \( P_{\text{orb}} = \frac{2\pi}{\omega_{\text{orb}}} = \frac{2\pi\mu R_m^2}{L} \) of orbital rotation with frequency \( \omega_{\text{orb}} \), we use the relation between \( L \) and \( R_m \). At \( R_m > R_t \) and \( R_m \leq R_t \), we have the periods

\[
P_{\text{orb}}^> = 2\pi \left( \frac{R_m^3}{GM} \right)^{1/2}
\] (14)

and

\[
P_{\text{orb}}^< = 2\pi \left( \frac{R_t^3}{GM} \right)^{1/2},
\] (15)
respectively. As seen from formulas, at initial $|\eta_i|$ smaller than the position $|\eta_b|$ of barrier of the potential energy $U$ (see Sect. III) and $R_m > R_t$ ($R_m \leq R_t$), the system moves towards the symmetric configuration and, respectively, $\eta$ decreases, $R_m$ decreases ($R_t$ increases), and, finally, the $P_{\text{orb}}^{>}$ decreases ($P_{\text{orb}}^{<}$ increases).

III. APPLICATION TO CLOSE BINARIES

Many di-star systems have different $M$ and $L$ (Table I) and, correspondingly, the potential energy shapes. The potential energies (driving potentials) $U(\eta)$ of the close di-star systems versus $\eta$ are presented in Figs. 1 and 2. In the calculations, we assume that the orbital angular momentum and the total mass are conserved during the conservative evolution of the di-star in mass asymmetry coordinate $\eta$. For all binary systems shown, the potential energies have the barriers at $\eta = \pm \eta_b$ and the minimum at $\eta = \eta_m = 0$. The barrier in $\eta$ appears as a result of the interplay between the total gravitational energy $U_1 + U_2$ of the stars and the star-star interaction potential $V$. These values have different behavior as a function of mass asymmetry: $U_1 + U_2$ decreases and $V$ increases with changing $\eta$ from 0 to $\pm 1$. One should stress that the driving potentials $U(\eta)$ for the di-star systems looks like the driving potentials for the microscopic dinuclear systems. The collective coordinate $\eta$, treated a separate dynamical degree of freedom, plays a comparable important role in macroscopic object as well as in microscopic dinuclear systems. Note that the same conclusion was drawn in Refs. [6, 7].

The evolution of di-star system depends on the initial mass asymmetry $\eta = \eta_i$ at its formation. The original di-star is asymmetric and $|\eta_i| < |\eta_b|$, then it is energetically favorable to evolve in $\eta$ towards a configuration at $\eta = 0$, that is, to form a symmetric di-star system. The matter of a heavy star can move to an adjacent light star enforcing the symmetrization of di-star without additional driving energy. The symmetrization of asymmetric binary star leads to the decrease of potential energy $U$, i.e. the transformation of the potential energy into internal energy of the stars. A huge amount of energy $\Delta U \approx 10^{41}$ J is released and converted during the symmetrization (see Figs. 1 and 2). The resulting symmetric di-star is created at large excitation energy. So, the binary star systems are the sources of energy in the Universe. Note that for all binary stars considered the energy of one single star ($|\eta| = 1$) is larger than the energy of the symmetric binary ($\eta = 0$) which means that the merger of
stars in the binary system is energetically an unfavorable process.

A spectacular recent case is KIC 9832227 which was predicted \([12]\) to be merge in 2022, enlightening the sky as a red nova. For the KIC 9832227 (\(|\eta_i| = 0.63, \, |\eta_b| = 0.84\)), we predict that a fast merger is excluded (see Fig. 2). This di-star is driven instead towards the mass symmetry. It should be stressed that the observational data of Ref. \([13]\) negate the merger of KIC 9832227 in 2022.

For the KIC 9832227, we studied the change of potential energy surface with the variations of the total mass \(M\) and orbital angular momentum \(L\) of system (Figs. 2 and 3). In this way, we take effectively into consideration the losses of mass and angular momentum of the binary star during its evolution. The mass or angular momentum may be removed from the system via stellar wind or gravitational radiation. As seen, a loss of the orbital angular momentum at a fixed mass \(M\) increases the symmetrization rate. The simultaneous losses of \(M\) and \(L\) (\(\Delta M \sim \Delta L\)) by 20% or 50% weakly influence the shape of potential energy. The decrease of \(M\) at fixed \(L\) has a more pronounced effect (the depth of the minimum in \(U(\eta)\) decreases) but the evolution to the global symmetric minimum is still energetically favored. So, the realistic simultaneous losses of \(M\) and \(L\) almost do not influence the rate of symmetrization of the system. Note that a loss of the orbital angular momentum at the fixed mass increase the rate of symmetrization of the system.

For KIC 9832227 system (\(|\eta_i| = 0.63\)), the mass is transferring from the heavy star to light companion, and the relative distance \((R_m > R_t = R_1 + R_2)\) between two stars and the period \(P_{\text{orb}}^>\) of the orbital rotation are decreasing. The evolution in \(\eta\) leads to the touching configuration \((R_m = R_t = R_1 + R_2)\) of stars at some critical mass asymmetry \(|\eta| = |\eta_i| \approx 0.45\) (Fig. 4). Further evolution in \(\eta\) leads to a configuration with partial overlap \((R_m < R_t)\) of the stars. So, at \(|\eta| \leq |\eta_t|\) the period \(P_{\text{orb}}^<\) of the orbital rotation is slightly increasing because \(P_{\text{orb}}^< \sim R_t^{3/2}\) and \(R_t\) increases with decreasing \(\eta\). Thus, once the system has crossed the point \(\eta = \eta_t\) and the phase of partial overlap is entered, the evolution of period changes abruptly. The other contact binaries considered in Fig. 4 show a similar behavior of period.

For the binary GW Cep (\(|\eta_i| = 0.46\), \(|\eta_i| > |\eta_t|\), the system moves towards mass symmetry and the orbital period is decreasing with time. For the almost symmetric EM Lac (\(|\eta_i| = 0.23\) and AW Vir (\(|\eta_i| = 0.14\) binaries, \(|\eta_i| < |\eta_t|\) and the periods show the secular increase. So, one can conclude that the period variations of a W UMa-type binary star is correlated with the mass asymmetry evolution towards the global symmetric minimum. At
low mass ratio \( q \), i.e. or the large mass asymmetry \( \eta \), binaries usually show a decreasing period because \( |\eta_i| > |\eta_t| (R_m > R_t) \), while the periods in systems with high \( q \) (or small \( \eta \)) are increasing because \( |\eta_i| < |\eta_t| (R_m < R_t) \).

IV. SUMMARY

For all contact di-star systems considered, the potential energies have symmetric barriers at \( \eta = \pm |\eta_b| \) and the minimum at \( \eta = \eta_m = 0 \). The di-star system is initially formed with \( \eta = \eta_i \) and \( 0 < |\eta_i| < |\eta_b| \). The two stars start to exchange matter and the system is driven to the symmetric di-star configuration (towards a global minimum of the potential landscape). The mass asymmetry coordinate is governing the symmetrization driven by the mass transfer process of two stars. The losses of the total mass and orbital angular momentum weakly influence the symmetrization of system. Note that the merger of the binary star systems considered is energetically not favored.

The orbital period changes can be plausibly explained by an evolution in mass asymmetry \( \eta \) towards the symmetry. We predicted that a decreasing and an increasing orbital periods are related, respectively, with the non-overlapping \((|\eta_i| > |\eta_t|, R_m > R_1 + R_2)\) and overlapping \((|\eta_i| < |\eta_t|, R_m < R_1 + R_2)\) stage of the binary star during its symmetrization. Thus, the observations of changing periods allows us to distinguish between these two stages of the binary star systems.

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TABLE I: The observed data $M_1/M_\odot$, $M_2/M_\odot$, and $P_{\text{orb}}$ are from Refs. [5, 8, 12].

| Di-star   | $M_1/M_\odot$ | $M_2/M_\odot$ | $P_{\text{orb}}$ (days) |
|-----------|---------------|---------------|-------------------------|
| AH Aur    | 1.68          | 0.28          | 0.4941                  |
| AP Aur    | 2.05          | 0.50          | 0.5694                  |
| DN Aur    | 1.44          | 0.30          | 0.6169                  |
| AW Vir    | 1.11          | 0.84          | 0.3540                  |
| AW UMa    | 1.38          | 0.14          | 0.4387                  |
| HV UMa    | 2.84          | 0.54          | 0.7108                  |
| KIC 9832227 | 1.40       | 0.32          | 0.4583                  |
| HV Aqr    | 1.31          | 0.19          | 0.3734                  |
| GX And    | 1.23          | 0.29          | 0.4122                  |
| RR Cen    | 2.09          | 0.45          | 0.6060                  |
| EM Lac    | 1.06          | 0.67          | 0.3891                  |
| GW Cep    | 1.06          | 0.39          | 0.3188                  |
| V700 Cyg  | 0.92          | 0.60          | 0.3400                  |
| V870 Ara  | 1.34          | 0.11          | 0.3997                  |
FIG. 1: The calculated total potential energies $U$ vs $\eta$ for the indicated overcontact binary star systems. The arrow on $x$-axis shows the corresponding initial $\eta_i$ for binary star.
FIG. 2: The same as in Fig. 1 but for other indicated close binary star systems.
FIG. 3: The calculated total potential energies $U$, the relative $R_m$ (solid line) and touching $R_t = R_1 + R_2$ (dotted line) distances between components in the units of the Sun radius $R_{\text{sun}} = R_\odot$ vs $\eta$ for the binary star KIC 9832227. The notations (0.8M, 1L), (0.5M, 1L), (0.8M, 0.8L), and (0.5M, 0.5L) mean that the calculations are performed with the losses of total mass $M$ and orbital angular momentum $L = L_i$ by (20%, 0%), (50%, 0%), (20%, 20%), and (50%, 50%), respectively. The arrow on x-axis shows the corresponding initial $\eta_i$ for binary star.
FIG. 4: The calculated relative \( R_m \) (solid line) and touching \( R_t = R_1 + R_2 \) (dotted line) distances between components in the units of the Sun radius \( R_{\text{sun}} = R_\odot \) vs \( \eta \) for the indicated binary star systems.