End Track Following Control Method for Multi-axis Machining of Linear Cutting Tools

Chen Guo-Bin¹, Lu Xing-hua²*

¹Guangzhou Huali Science and Technology Vocational College, Guangzhou, 511325, China
²Huali College Guangdong University of Technology, Guangdong Guangzhou, 511325, China

*Corresponding author E-mail: xhlu@gdut.edu.cn

Abstract: In the process of linear tool multi-axis machining, it is prone to local interference, resulting in poor curvature matching of the machining end trajectory, tool path tracking control performance is not good, based on critical tool axis vector decomposition, a linear tool end path following control method for multi-axis machining is proposed. The tool axis spatial distribution model of complex curved surface channel parts is constructed, and the local interference suppression is carried out by using two-dimensional feasible pendulum tool domain analysis method. The curve and surface are the critical elements on the multi-axis machining interference surface of linear cutter. The intersection curve equation between the pendulum cutter plane and the surface to be processed is constructed, and the optimal tool parameter selection is realized by using the discrete characteristic point distribution of the surface of the critical cutter axis. Combined with the critical tool axis vector decomposition method, the end trajectory following control of linear tool multiaxial machining is realized. The simulation results show that the output stability of the end track following control of linear tool multiaxial machining is good by using this method. The precision and efficiency of multi-axis machining are improved significantly.

1. Introductions
Multi-axis machining of linear cutting tools has important application value in the machining of complex curved surface channel parts, and it is widely used in the machining of integral impeller, impeller and other aerospace parts. Linear tool path is still the most widely used NC code format in industry at present. At the joint point of linear path, the tangential direction and curvature of the tool are not continuous. This kind of discontinuity often leads to the fluctuation of feed speed, and it is easy to cause machine tool vibration in the process of actual machining, which seriously reduces the machining efficiency and surface quality, so it is necessary to follow and control the end trajectory of linear cutting tool in multi-axis machining. In order to improve the efficiency of multi-axis machining of linear cutting tools and the stability of machining process, the path disturbance of small line segments is easy to occur in the process of controlling the end trajectory of linear tool multi-axis machining, in order to overcome the problems caused by small line segments. The path of small segment can be faired, and the continuity of multi-axis machining path of linear cutting tool can be improved by combining with the tracking control of end track of small segment, so as to improve the stability and efficiency of end trajectory control of multi-axis machining of linear cutting tool, and improve the quality of processing[1].
For the three-axis and multi-axis NC machining, there have been some systematic research results to improve the machining performance by controlling the end trajectory of the cutter path\textsuperscript{[2]}. However, for the five-axis machining, at present, there are two kinds of tracking control methods: fitting fairing, transposition fairing. Fitting fairing uses high-order spline curves to approximate or interpolate discrete small line segments. The end trajectory following control algorithm of fitting fairing is difficult to control the fitting accuracy, and the calculation is large\textsuperscript{[3]}. It is very difficult to use in real time computing environment. The idea of end track following control of transposition fairing is to use arc or high order continuous spline curve in small line segment connection point\textsuperscript{[4]}. In reference\textsuperscript{[6]}, two curves are used for tool tip point and tool axis respectively, one point for interference suppression, the tracking control of the end of the tool machining is realized. The order of each curve is the same. The trajectory of the tip point and the point on the cutter shaft is G2 continuous, but the synchronization parameterization is not realized. The velocity and acceleration of each servo axis are not continuous at the junction point. At the same time, the coupling control method cannot guarantee that the direction vector of the cutter axis is a unit vector. The tracking control performance of the end trajectory is not good. TULSYAN transfers the tip point and the cutter axis vector separately, but the end trajectory following control method cannot guarantee the unitization of the tool axis vector. In addition, the Jacobian iteration is needed to solve the multivariate nonlinear equations, and the velocity programming method also has an iterative process. It takes a lot of time from the terminal trajectory following control to the trajectory interpolation calculation. In reference\textsuperscript{[6]}, In the machine tool coordinate system, the terminal trajectory of the translational axis and the rotational axis are controlled by the Bézier curve of degree 3 respectively, and the trajectory interpolation is performed. Because of the rotation of the rotating axis, the trajectory of the tip of the cutter is a space curve deviating from the theoretical trajectory. The machining accuracy is reduced and the rotating shaft and the moving shaft are different transmission modes. The speed planning is not suitable for considering both of them and the output error of the end trajectory following control is large.

In order to solve the above problems, this paper presents a tracking control method based on critical tool axis vector decomposition for multi-axis machining of linear cutting tools. The tool axis spatial distribution model of complex curved surface channel parts is constructed, and the local interference suppression is carried out by using two-dimensional feasible pendulum tool domain analysis method. Combined with the critical tool axis vector decomposition method, the end trajectory following control of linear tool multiaxial machining is realized.

2. Construction of spatial distribution model of cutter axis and suppression of local interference

2.1 Spatial distribution model of linear cutting tool multi-axis machining

In order to realize the end track following control of multiaxial machining of linear cutting tools, it is necessary to construct the tool axis spatial distribution model of the parts with complex curved surface channel, and to transform the tip point and the cutter axis vector respectively. The control point tracking design of the curve is carried out by using the adjacent linear path planning method. The tool tip point and the cutter axis vector of the five-axis linear path are controlled by the cubic Bézier curve respectively\textsuperscript{[7]}. In order to improve the smoothness of end trajectory following control in linear tool multiaxial machining, considering the position and direction of cutting tool relative to workpiece, the end trajectory smoothing model of linear tool multiaxial machining is established, and the tool axis vector is represented by two Euler angles. The spatial distribution model of linear multi-axis machining tool shaft is obtained by using 7-segment S-type velocity planning method, which is shown in Figure 1.
According to the spatial distribution model of linear tool multi-axis machining shown in figure 1, each connecting rod of tool shaft is consolidated into a coordinate system, and the pendulum cutter plane frame is carried out between adjacent coordinate systems. It is obtained that the magnetic field density of multi-axis machining is $B$, the magnetic field is modulated adaptively by the smooth feedback of the machining surface, and when the flux coefficient of the output pendulum cutter plane is $I$, the flux leakage coefficient is:

$$\eta = \frac{P_o}{T_i (Z_m + R_p)} = \frac{\alpha_i M_{\mu_i}^2 M_{\mu_i}^2 L_i^2 L_o^2 R_o}{h_i (\alpha_i M_{\mu_i}^2 L_i^2 R_i + \alpha_i^2 M_{\mu_i}^2 M_{\mu_i}^2 R_o + h_i R_p)}$$

(1)

Wherein: $h_i = \alpha_i M_{\mu_i}^2 L_i^2 + R_i R_i^2 + R_R R_M^2$. The isometric transformation of the source surface is carried out, and the interference suppression is carried out according to the spatial distribution model of the linear tool's multi-axis machining tool axis [8].

2.2 Local interference suppression based on two-dimensional feasible pendulum tool analysis

The local interference suppression is carried out by using the method of 2-D feasible pendulum tool field analysis. The NURBS surface is used to describe the surface $S_0$ and the potential interference surface $S_1$, whose equations are respectively expressed as $S_i(u, v)$, $S_j(u, v)$, and every point on the source surface. According to the rotation transformation in the direction of their external normal vector $n$, the trajectory distribution of the spherical cutter with radius $R$ is obtained as follows:

$$\alpha_i (V) = 0.1 (25 - V) / (exp((25 - V)/10) - 1)$$

(2)

$$\beta_i (V) = 4 exp(-V/18)$$

(3)

$$\alpha_i (V) = 0.07 exp(-V/20)$$

(4)

$$\beta_i (V) = 1 / (exp(-V + 30)/10) + 1$$

(5)

$$\alpha_i (V) = 0.01 (10 - V) / (exp(10 - V)/10 - 1)$$

(6)

$$\beta_i (V) = 0.125 exp(-V/80)$$

(7)

According to the above parameter analysis, the critical cutter axis vector control model is obtained as follows:

$$F_m = \frac{B l_m}{\mu_0 \mu_i}$$

(8)

$$A_x = l_x \beta \pi (r_x + l_x)$$

(9)

$$\mathcal{R} = \frac{1}{\mu_0 \mu_i \beta \pi l \rho} \left( \frac{1}{\mu_4} \int_{r_i}^{r_i + l_i} \frac{d_r}{r} + \int_{r_i}^{r_i + l_i} \frac{d_r}{r} + \frac{1}{\mu_2} \int_{r_i}^{r_i + l_i} \frac{d_r}{r} \right)$$

(10)

The interference intensity of the end trajectory of tool size is $\mu_0 = 4 \pi \times 10^{-7}$ H/m, $\mu_i$ and $\mu_2$ are the feasible range and permeability of the pendulum tool at the tangent contact $C$.

All the critical cutter axis unit vectors $T$ at tangent contact $C$ are shifted to the center of the sphere $O W$. The results of the interference solution in the feasible pendulum cutter region are expressed as follows:
\[ \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & x_{t1} & \cdots & x_{t,n-1} \\ 1 & x_{s1} & \cdots & x_{s,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{e1} & \cdots & x_{e,n-1} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{n-1} \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{pmatrix} \] (11)

The fuzzy control of linear tool multiaxial machining under nonlinear elastic-plastic strain is carried out, and the state equation of interference suppression of discrete stress field is obtained:
\[ x(t) = (x_o(t), x_i(t), \ldots, x_{i-1}(t))^T \] (12)

Thus, the local interference suppression of two-dimensional feasible pendulum tool analysis is realized[9].

3. Optimization of end track following control for multiaxial machining of linear cutting tools

3.1 Discrete Eigenpoint Distribution Model of critical tool Shaft on curved Surface

On the basis of constructing the spatial distribution model of the tool shaft of the passage parts with complex curved surface and adopting the method of 2-D feasible pendulum tool domain analysis to suppress the local interference, the optimization design of the end track following control of the multiaxial machining of linear cutting tools is carried out. In this paper, a tracking control method based on critical cutter vector decomposition is proposed for multi-axis machining of linear cutting tools. The curve and surface are the critical elements on the multi-axis machining interference surface of the linear cutting tool, and the smooth weight of the tool and the surface to be processed is obtained.

\[ w_j(t + 1) = w_j(t) + \alpha(e_j)[x(t) - w_j(t)] \] (13)

Wherein, \( j \in (f_j, NE_j(t)) \), the weighted coefficient \( \omega_0 \) of the constraint surface of tool length is calculated, the vector edge is reconstructed based on the space constraint method of fault surface. The smooth logarithmic function of linear tool end following control under section space constraints is obtained as follows:
\[ p_v = \frac{\omega_0 M^2 L^2 - M^2 L^2 L^2 R^2}{(\omega_0 M^2 L^2 + R R L^2 + R R M^2)^2} \] (14)

The same coordinate on the two-dimensional projection plane contains multiple heavy values. According to the fault points on each grid node, the multi-value case of the interpolation points of the layer on the grid node is determined, and the following control equation of the tool attitude under the action of the rotary surface is obtained as follows:
\[ \frac{Y(s)}{R(s)} = \frac{G_c(s)G_e(s)e^{-rs}}{1 + G_c(s)G_e(s)} \] (15)

Wherein: \( Y(s) \) — the distance between the tool shaft line and the return curved surface; \( R(s) \) — control parameters of tool attitude restricted by upper and lower constraint surfaces; \( e^{-rs} \) — the fuzzy correction compensation parameters of the whole surface to be processed.

3.2 End track following control output of linear tool multi-axis Machining

The equation of the intersection curve between the pendulum cutter plane and the surface to be processed is built, and the tool posture is received. Under the restriction of the lower constraint surface, the optimal tool parameters are selected by using the discrete feature point distribution of the critical cutter axis. The linear distribution formula of the tool parameters is obtained as follows:
\[ \omega L_p - \frac{1}{\omega C_p} = 0 \Rightarrow C_p = \frac{1}{\omega^2 L_p} \] (16)
\[ \omega L_i - \frac{1}{\omega C_i} = 0 \Rightarrow C_i = \frac{1}{\omega^2 L_i} \] (17)
To find the optimal tool parameters that can realize the existence of no collision interference in each feature point, the global optimization problem \( \min \{ f(x) \} \) is considered, and the updating formula of the tool length solution model is obtained as follows:

\[
\begin{align*}
    v_{id}^{t+1} &= a v_{id}^t + c_1 r_1 (p_{ad} - x_{id}^t) + c_2 r_2 (p_{gd} - x_{id}^t) \\
    x_{id}^{t+1} &= x_{id}^t + v_{id}^{t+1}
\end{align*}
\]  

(18)

Taking the minimum value of the maximum tool size at all feature points and the maximum of the shortest tool length at all feature points as the final optimization result, the path error coefficient \( k_c \) and span coefficient \( k_\beta \) on the surface to be machined are considered, and the cutter axis vector is considered. The cutter axis vector \( T_2 \) and \( ZW \) axes can be expressed as:

\[
T_2 = \frac{\pi k_c k_\alpha k_\beta B l_1 l_2}{\ln \left( \frac{r_c + l_3 + l_m}{r_c - l_m} \right)}
\]  

(19)

\[
ZW = 1 - \frac{1}{0.9 r_c / (\beta p(l_3 + l_m))^2 + 1}
\]  

(20)

\[
k_\beta = \frac{\alpha(\beta, k_c)}{k_c}
\]  

(21)

The optimal tool parameter selection is realized by using the discrete feature point distribution of the critical cutter axis. The optimized parameters are obtained as follows:

\[
\alpha = \min(\beta, k_c) [k_c + (1 - k_c) \tanh(\delta |\beta - k_c|)]
\]  

(22)

Wherein, \( k_c < 1 \), \( \delta \) is empirical value.

4. Simulation experiment and result analysis

In order to test the performance of this method in realizing the end track following control of linear tool multi-axis machining, the simulation experiment is carried out. The algorithm is simulated and tested with UG software and Matlab 7 software. The trajectory deviation error is measured by three-axis electronic compass LSM303DLH. The tool radius \( R_0 = 0 \), \( \Delta R = 0.05 \), balance coefficient \( \alpha = 0.15 \), tool length constraint surface are taken from the upper end of the impeller, respectively. According to the tool parameter optimization algorithm proposed in this paper, the bottom end face and the three surfaces of the rotary surface formed by the tip curve of the blade are offset 10mm. Given the shortest effective tool length is 76mm, the linear tool multi-axis machining can be obtained. The critical cutter axis vector distribution at a single point is shown in Fig. 2.

![Critical cutter axis vector distribution at single point](image)
According to the vector model of multi-axis machining given in figure 2, the relationship between the radius variation of the spherical cutter and the feasible pendulum cutter region is analyzed. The result is shown in figure 3.

![Fig. 3 Relationship between linear tool radius and feasible pendulum cutter region](image)

Figure 3 shows that, with the increase of tool radius, the probability of collision interference will increase, and the feasible pendulum cutter field will be reduced. The end trajectory following control method designed in this paper is used for multi-axis machining, and it is used in VERICUT software. Simulation results show that the machining results under different tool diameters are shown in figure 4.

![Fig. 4 Multi-axis machining model of linear cutting tools with different tool diameters](image)

(a) Tool diameter 12 mm  
(b) Tool diameter 14 mm

The analysis and simulation results show that the trajectory length is reduced by 188.2 mm, the machining efficiency is increased by 12.74% and the machining time is shortened by 13.50%. The method of this paper is used to control the trajectory following of the multi-axis machining end of the linear cutting tool and to improve the precision and efficiency of the multi-axis machining.

5. Conclusions

In order to improve the efficiency of multiaxial machining and the stability of machining process, the tracking control of the end trajectory of linear tool multi-axis machining is carried out. In the course of controlling the end trajectory of linear tool multi-axis machining, small lines are easy to appear. In order to overcome the problem caused by small line segments, this paper presents a method of trajectory following control for multi-axis machining of linear cutting tools based on critical tool axis vector decomposition, and constructs the tool axis gap of the passage parts with complex curved surfaces. Two-dimensional feasible pendulum tool domain analysis method is used to suppress local interference in the model of interspace distribution. The curved surface is the critical element on the multi-axis machining interference surface of the linear cutter. The intersection curve equation between the pendulum cutter plane and the surface to be processed is constructed. The optimal tool parameter selection is realized by using the discrete characteristic point distribution of the critical cutter axis. The combined critical tool axis vector decomposition method is used to realize the end track following control of linear tool multiaxial machining. The simulation results show that the output stability of the method is more stable than that of the linear tool multi-axis machining end track following control. The accuracy and efficiency of multi-axis machining are improved significantly.
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